Optimal PMU Placement Solution: Graph Theory's and ILP Simulating

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Abstract
In this study, new methods are proposed using graph theory (Depth First; Graph Theoretic Procedure and Annealing Method) and intelligence linear programming (ILP) to solve the problem of setting the optimal phase measurement unit (PMU) (OPP) to observe the complete network. The proposed approach assures maximum measurement redundancy too. A decision matrix consists of some unique criteria from the concept of network graph theory and this helps to define zero injection buses (ZIBs) buses using ILP software. The contribution of zero injection buses (ZIBs) in PMU placement problem has been considered. The proposed technique is further analyzed for complete observability under single PMU loss or line outage cases. The proposed approach is tested on IEEE 14-bus, 30-bus, and 114-bus Algerian electrical network systems. To verify the computational efficiency and the higher redundancy of solutions of the suggested method whilst getting a comprehensive observation of every bus in the shortest time, the results were compared to some well-established methods stated in the literature.

Keywords: Optimal Phase Measurement Unit (PMU), Intelligence Linear Programming (ILP), Zero Injection Buses (ZIBs), Elapsed Time.

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Introduction

The electric power industry has witnessed rapid developments and transformations in recent years, so this development must be accompanied by the development of monitoring systems using phase measurements, allowing monitoring the electric power grid [1].In This Framework, when operating energy systems, they need careful analysis in order to assess their potential affects the security and reliability of the system [2].When installing on a network bus (node).Phasor measurement unit technology (PMU) the next step would be to improve the quality of system condition estimation, and to provide better operators information to maintain a high level of system reliability[3].The PMU measures the voltage of this bus and currents across a certain number of branches of the accident[4].The number of currents that can be measured depends on the number of channels of the device. PMUs are generally considered the most advanced synchronized measurement technology [5].When compared to previous solutions, PMUs offer the following major capabilities:

1) Location independent measurements synchronization using global positioning system (GPS)[6].

2) Direct measurements of voltage and current phase angles and increased accuracy, frequency, reliability and security of state measurements [7]. As such, the installation of PMUs can be seen as a major contribution to the overall resiliency of the critical infrastructure systems [8].

The placement sites are also limited by communication facilities and other physical constraints. solution for OPP problems, i.e., full observability of the network with a minimum number of PMUs and maximum measurement redundancy (MR)[9].The methods used for solving OPP problems can be broadly classified as topological, numerical, symbolic and hybrid.[10]Topological observability analysis has been followed in some papers. [11][12][13], where observability criteria have been defined as the construction of full rank spanning tree. However, the approach to constructing a full-order spanning tree is computationally complex and not foolproof. A numerical approach using orthogonal transformation was used to estimate the condition in[13][9] .Some other approaches with numerical methods are also very significant for observability analysis[9].A symbolic approach with reduced Jacobian Matrix has been used in [[14],[9]].In this context, a lot of optimization techniques have been proposed to solve the problem of optimal PMUs placement (OPP), such as: the depth first search (DeFS) [15], the minimum spanning tree algorithm (MST) [9], the simulated annealing (SA)[16], the tabu search (TS) [17], the genetic algorithms (GA) [18], the differential evolution (DE) [11], the immune algorithms (IA) [12], the particle swarm optimization (PSO) [19].the modified discrete binary particle swarm optimization (BPSO) [13] and the ant colony optimization (ACO) [20].

The hybrid technique has been used in very few papers, e.g. topological and linear algebra based numerical approaches have been combined [9], for flow islands determination and observability analysis of those super nodes (flow islands), respectively. Besides this the graph theoretic approach to determine coherent group of generators and installation of PMU for inter-area oscillation monitoring is quiet famous[13],; however, as coherent regions are dynamic in nature and the system may not be decomposable in meaningful clusters so these methods are not suitable as robust PMU placement technique. Some optimization technique, e.g. Tabu search [15, 16], [19, 20], and binary cuckoo algorithm [9][21][16]. has also been used to find the solution of OPP.

The contributions to this present paper are detailed as follows:

(i) Some unique criteria have been identified for placement of PMU which will give complete observability with minimum number of PMUs.

(ii) (ii) Deployment of PMUs using proposed method offers maximum MR

(iii) (iii) The observability analysis is also carried out considering zero injection buses (ZIBs) in the network

(iv) (iv) The proposed approach is straightforward and does not involve any iterative steps or complex equations. Thus, it takes very less memory space and computation time to give OPP solution for small as well as large-scale power systems.

Section 2 formulates the PMU placement problem which has been followed in this paper.
Section 3 describes some basics of graph theory for the better understanding of the approach.
Section 4 elaborates the details of the proposed methodology.
The results obtained from this method have been analyzed in Section 5.
Finally, Section 6 concludes the paper.

I. PMU placement problem:

Phasor measurement unit is a device that measures voltages and currents along with phasor values. In 1980s PMUs were introduced for the very first time [22]. The objective was to use PMUs in wide area monitoring system with a lot more applications that are still under research phase. In 1995 standards were made to use PMU in power system applications which were revised later with trademark IEEE standard [23][16]. The difference between SCADA and PMUs is that PMUs are much faster and help in monitoring in the shortest possible time with more precise controls from the point of view of application systems [24].

The following diagram shows the regulation of PMUs and the conventional measurement [25][26][27][28]. The PMUs receive two signals [29], a signal from the current transformer (CT) and a signal from a potential transformer (VT), and measurements simultaneously by receiving a signal from a global positioning position (GPS) via a (GPS) receiver.

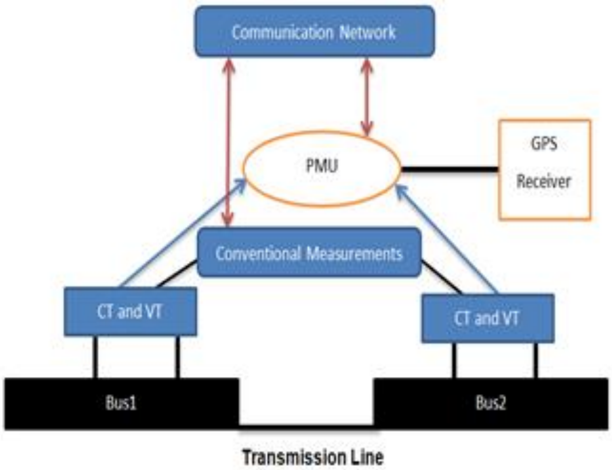


Fig01: Organigram of PMU and Conventional Measurement

II. Creating a Node –Incidence Matrix for Power System:

Using the power system topology, the various interconnections between the buses can be grouped into a matrix called a node occurrence matrix (A). The rule is simple: If node i is adjacent to node j, then $A_{ij}=1$,i≠j. Normally A is a large sparse matrix. For example, for the IEEE 14-bus system,

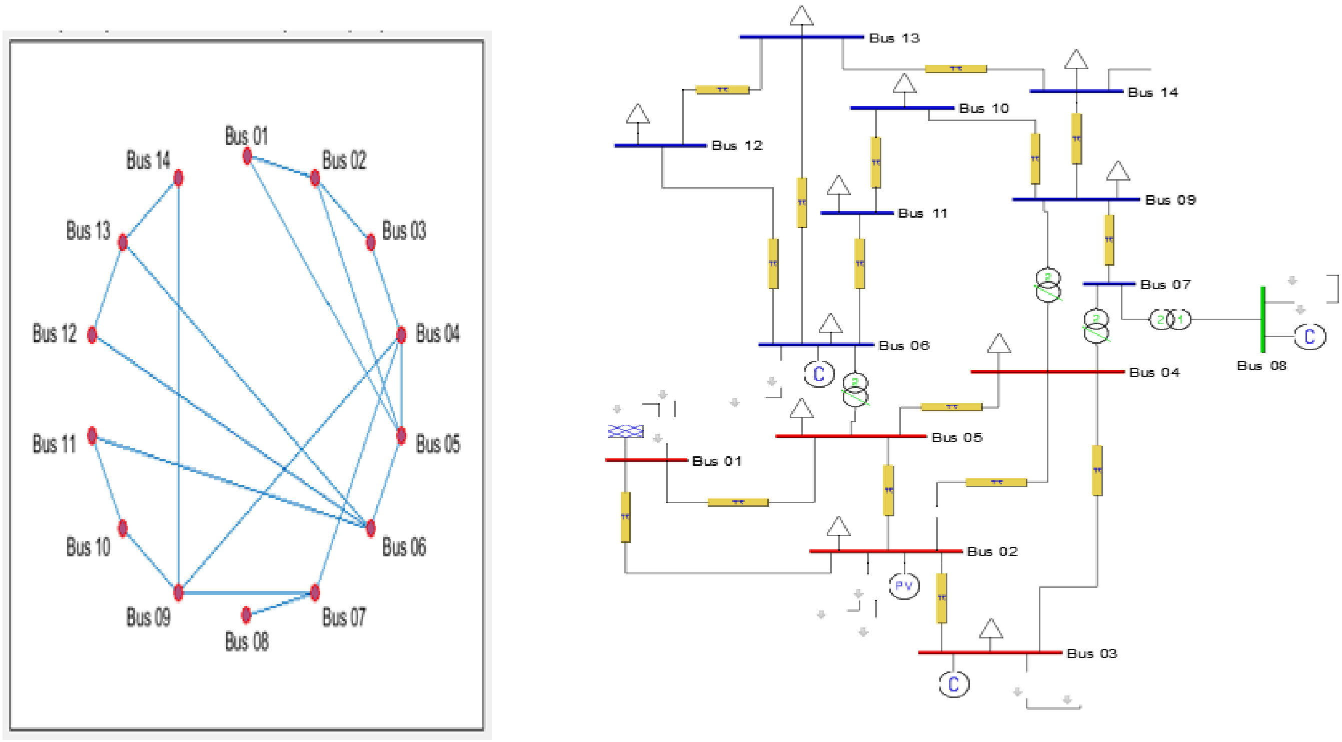


Fig 02: Example of the proposed placement for the IEEE 14-bus system.

$$A = \begin{bmatrix} 1&1&0&0&1&0&0&0&0&0&0&0&0&0 \\ 1&1&1&1&1&0&0&0&0&0&0&0&0&0 \\ 0&1&1&1&0&0&0&0&0&0&0&0&0&0 \\ 0&1&1&1&1&0&0&1&1&0&0&0&0&0 \\ 1&1&0&1&1&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&1&0&0&0&0&1&1&1&0 \\ 0&0&0&1&0&0&1&1&1&0&0&0&0&0 \\ 0&0&0&0&0&0&1&1&0&0&0&0&0&0 \\ 0&0&0&1&0&0&1&0&1&1&0&0&0&1 \\ 0&0&0&0&0&0&0&0&1&1&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0&1&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&1&1&0 \\ 0&0&0&0&0&1&0&0&0&0&0&1&1&1 \\ 0&0&0&0&0&0&0&0&1&0&0&0&1&1 \end{bmatrix}$$

It is easy to find out that there are 7 nodes with degree 3 or more:

-2 nodes with degree 3.

-4 nodes with degree 4.

-1 node with degree 5.

The measurement model is given as:

Z= h(x)+e

(1)

Z: measurement data; x: state of the system comprising of V and phase angle δ; except the phase angle at slack bus; h: nonlinear power flow equations; e: measurement noise.

$\min \sum_{i}^{n} Wj\ Xi$

(2)

Where

n: is the number of buses in the system; Wj: is the cost of installation of a PMU on bus i; Xi: is a vector of dimension n and has binary values de- fined as:

$$Xi = \begin{cases} 1\ PMU\ a\ is\ placed\ on\ bus\ i \\ 0\ a\ is\ not\ plaiced\ on\ bus \end{cases}$$

(3)

Subjected to the following constraints:

Bus 1: x1 +x2 +x5 ≥ 1

(4)

Bus 2: x1 + x2 + x3 + x4+ x5≥ 1

(5)

Bus 3: x2 + x3 + x4 ≥1

(6)

Bus 4: x2+ x3+ x4+ x5+ x7 + x8≥ 1

(7)

Bus 5: x1+ x2+ x4+ x5+ x6 ≥ 1

(8)

Bus 6: x5 + x6 + x11+ x12+ x13≥ 1

(9)

Bus 7: x4+ x7 + x8+ x9≥ 1

(10)

Bus 8: x7 + x8 ≥ 1

(11)

Bus 9: x4+ x7+ x9+ x10 + x14 ≥ 1

(12)

Bus 10: x9+ x10+ x11≥1

(13)

Bus 11: x6 + x10 + x11 ≥ 1

(14)

Bus 12: x6+ x12 + x13 ≥ 1

(15)

Bus 13: x6+ x12 + x13 + x14 ≥ 1

(16)

Bus 14: x9+ x13 + x14 ≥ 1

(17)

III. Proposed methodology:

Optimal PMU placement for normal operating condition: as it was explained before, the PMU installation at every node for the sake of confirming complete observability is rather unfounded, hence, all the possibilities for PMU installation and should be explored and one of them must be selected, which will give complete observability with a minimum number of PMUs[4][9]. Consequently, there may be one or more than one solutions and MR varies a lot depending on the solutions of OPP problem. As explained in Section 2, if it is possible to construct a full ranked spanning tree using direct PMU measurements then the system is completely observable[30][9]. In this paper, the ARi calculated from ([31][32][33]) is used to build the preference list of vertices to install the PMUs.

Several simulation tools have been used to analyze networks and find out the number of PMUs (such as MATLAB, OPP, ILP Power World, PSAT, Ect)[34][35].

We have chosen three programs to get the best results: software using MATLAB, ILP Power world, PSAT as a simulator.

1. Proposed method 1 (DeFS/ILP):

   The Depth-first search is an algorithm for going across or searching tree or graph data structures. The algorithm starts at the root node, explores the maximum possible the length of each branch before backtracking [36]. Accordingly, the fundamental idea is to initiate from the root or any arbitrary node, mark it, move to the adjacent unmarked node and maintain this loop until no unmarked adjacent node is observed. Then, other unmarked nodes sould be backtracked, checked and traversed as well. Finally the nodes in the path shall be printed. Then ILP program shall be used in order to reduce the number of nodes and thus obtain the lowest possible number of PMUs and observe the system in the shortest possible time.

2. Proposed method 2 (GTh/ILP):

   Topological Observability: Topology methods make use of the decoupled measurement model and graph theory. In these methods, the decision making process is based on logical operations [37]. As a result, they require only information about network connectivity, measurements type and their positions. In this paper, we use graph theory approach based topological analysis method based on PMUs according to the observability rules below:

   For a PMU installed bus, the voltage phasor of that bus and the current phasors of all incident branches to that bus are known. They are known to be direct measurements.

   If voltage and current phasors at one end of a branch are known, then voltage phasor at the other end of that branch can be obtained. These are called pseudo measurements.

   If voltage phasors of both ends of a branch are known, then the current phasor of this branch can be obtained directly. These measurements are also labelled pseudo measurements.

   For zero injection bus iin a N-bus system, we have:

   $$\sum_{j=1}^{N} Yij\, Vj = 0$$

   (18)

Therefore, if there is a zero-injection bus without PMU where all the incident branch current phasors are known but one, then the current phasor of the unknown one can be obtained using KCL equations.

If there is a zero-injection bus with unknown voltage phasor and voltage phasors of its adjacent buses are all known, then the voltage phasor of the zero-injection node can be found by node equations.

If there exists a group of adjacent zero-injection buses whose voltage phasors are unknown but the voltage phasors of all adjacent buses to the group are known, then the voltage phasors of zero-injection buses can be obtained through node equations. The measurements obtained from rules 4-6 are called extended measurements.

3. Proposed method 3 (AM/ILP):

Is a probabilistic method for roughly estimating a function's global optimum? In particular, it is a meta heuristic to approximate global optimization for an optimization problem in a big search space. When the search space is discrete (such as in the traveling salesman problem), it is frequently used [38]. Simulated annealing may be preferred to exact methods like gradient descent or branch and bound for issues where achieving an approximative global optimum is more crucial than finding a precise local optimum in a set amount of time. The algorithm's name and source of inspiration call for an intriguing feature connected to temperature variation to be incorporated into the algorithm's operating characteristics. As the simulation continues, the temperature must be gradually lowered because of this. The process begins by setting the initial value to a high number and then decreases it at each step in accordance with an annealing schedule that the user may choose, but which must conclude towards the end of the allocated time limit. Accordingly, the system is predicted to first stray towards a wide region of the search space containing promising solutions while ignoring minor details of the energy function, then veer towards low-energy regions that get progressively smaller, and finally move downward in accordance with the steepest descent heuristic. As the annealing schedule is extended, the likelihood that the simulated annealing procedure ends with a global optimal solution approaches 1. [10] This theoretical finding, however, is not very useful because it typically takes more time to guarantee a significant likelihood of success than it does to search the whole solution space.

IV. Case studies:

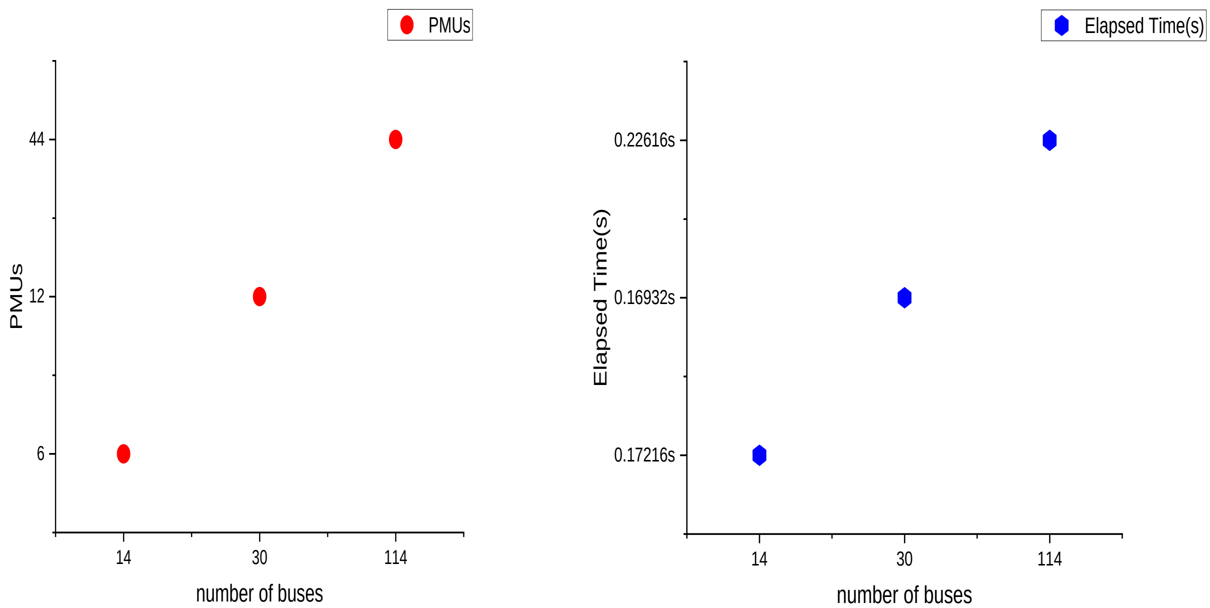
We provide and discuss the key outcomes from the application of the suggested methodology in this section. Methodology used to solve the issue of where to deploy project management units best in various case studies. We first analyze a very basic power system, such as the IEEE 14-bus and 30-bus, to provide insight into the key characteristics that set the proposed methods apart. Next, results obtained on large-scale systems, like the Algerian power distribution network's 114-bus, are reported to show the viability of the proposed method for practical power systems. The authors are confident that the theoretical framework created in this paper will be helpful to power system operators in both the short and long terms. In the short term, the main goal is to reduce the cost of PMUs, and in the long term, a massive pervasiveness of these devices in power networks will require semantic tools for information management. In the latter case, we anticipate that the application of the suggested paradigm will enable the system operators to extract structured knowledge from the network topology (i.e., the spanning measurement tree of full rank) that can be useful for implementing domain decomposition techniques aimed at solving multi area state estimation problems, deploying appropriate redundancy policies, and detecting sensor faults.

V. Results and Discussion:

a) Optimal PMU placement without ZIB:

Method DeFS/ILP: After applying this algorithm to a group of networks, the results obtained are included in the following table:

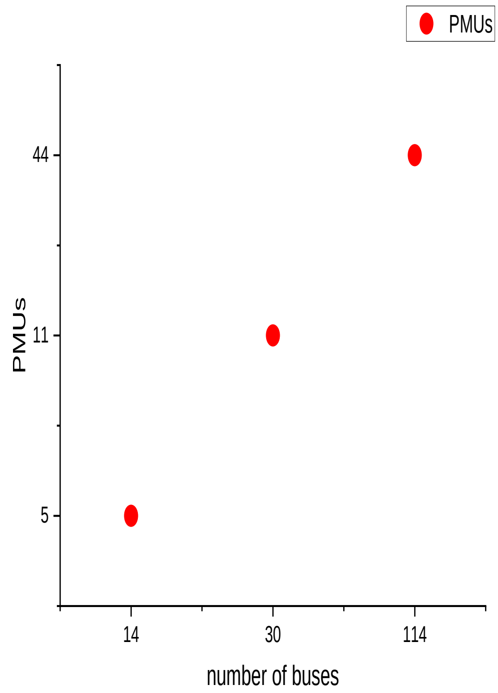
Number of buses	PMUs	PMU Location	Elapsed Time(s)	Meas. Currents	Pseudo-Meas. Currents
14	06	1 ;4 ;6 ;8 ;10 et 14	0.17216s	16	00
30	12	11 ;12 ;17 ;18 ;20 ;21 ;24 ; 26 ;27 ;3 ;5 ;6 .	0.16932s	33	00
114	44	102 ;103 ;104 ;109 ;112 ;113 ;13 ;14 ;15 ;2 ;20 ;22 ;26 ;3 ;31,33,38 ,39,40,42,43,45,47,49,53,57,59,6 3,69,7,70,72,73,75 ;76 ;78 ;79 ;8 1 ;84 ;87 ;89 ;93 ;95 ;98 .	0.22616s	140	00

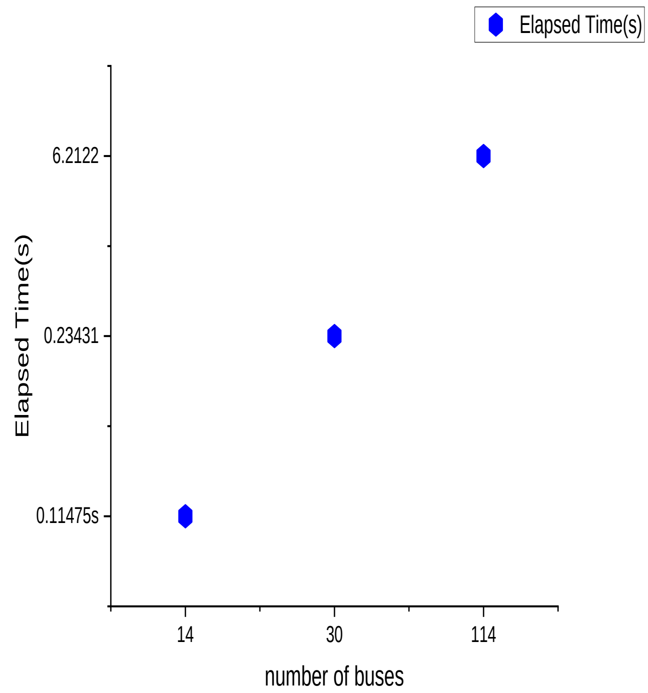


Calculation times for the proposed Depth First method for, IEEE 14-bus, IEEE 30-bus and 114-bus test systems being 0.17216s, 0.16932s, and 0.22616s,respectively. Calculation meas.currents 16, 33, and 140, respectively. Calculation Pseudo-Meas. Currents 00.

Method GTH/ILP :

Number of buses	PMUs	PMU Location	Elapsed Time(s)	Meas. Currents	Pseudo-Meas. Currents
14	5	1 ;4 ;6 ; 10 ;14.	0.11475s	15	5
30	11	11 ;12 ;17 ;18 ;20 ;21 ;24 27 ;3 ;5 ;6 .	0.23431	32	10
114	44	1 ;10 ;103 ;104 ;109 ;112 ;113 ; 12 ;16 ;20 ;22 ;26 ;31 ;33 ; 36 ;38 ;39 ;41 ;44 ;47 ;53 ;55 ;5 7 ;59 ;6 ;63 ;69 ;70 ;72 ;73 ; 75 ;76 ;78 ;79 ;82 ;84 ;85 ;88 ;8 9 ;9 ;95 ;97 ;99	6.2122	140	43

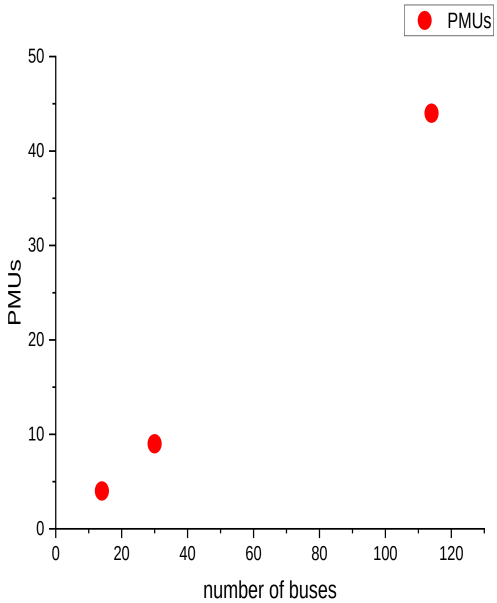


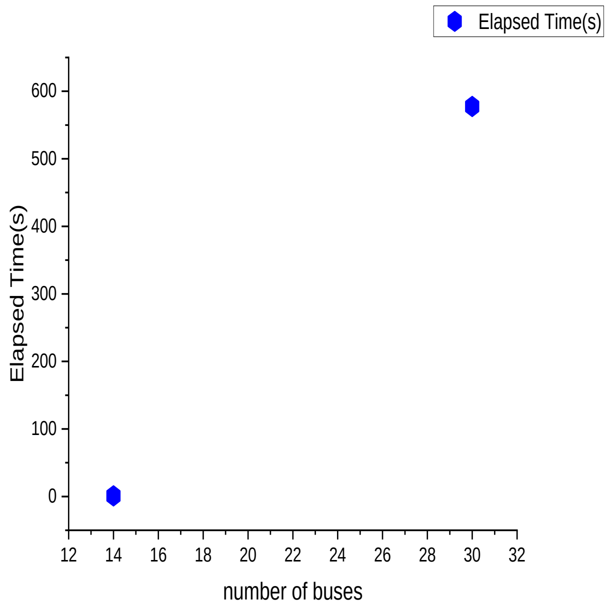


Calculation times for the proposed Graph Theoretic Procedure method for, IEEE 14-bus, IEEE 30-bus and 114-bus test systems being 0.11475s, 0.23431s, and 6.2122 respectively. Calculation meas.currents15; 32; and 140; respectively. Calculation Pseudo-Meas. Currents 05; 10; 43 respectively.

method AM/ILP :

Number of buses	PMUs	PMU Location	Elapsed Time(s)	Meas. Currents	Pseudo-Meas. Currents
14	4	4 ;5 ;6 et 9	0.8786	12	20
30	09	Bus1 ;10 ;12 ;19 ;24 ;29 ;7 ;8 ;9 .	9m 37.4673s	27	41
114	44	/	/	/	/





Calculation times for the proposed Annealing Method for, IEEE 14-bus, IEEE 30-bus and 114-bus test systems being 0.8786, 9m 37.4673s, and non-observable; respectively. Calculation meas.currents 12; 27; and non-observable; respectively. Calculation Pseudo-Meas. Currents 20; 41 and Non observable; respectively.

b) Optimal PMU placement considering effects of ZIB:

ZIBs are the buses from which no current is being injected into the power system. Virtually no generation or no load is connected to the ZIBs. If ZIBs are considered for optimal PMU placement, it can further reduce the required number of PMUs for complete system observability. For an N bus system having X number of ZIBs, it is possible to obtain voltage phasors of different X number of buses (ZIBs or non-ZIBs) if voltage phasors of remaining (N-X) are known. From observability rules mentioned earlier, it can be said that the zero-injection bus can be observed if all the adjacent buses are observed. Thus, taking into account the effect of zero injection buses, a modified matrix was developed by combining zero injection buses with one of the adjacent buses. In fig 02, bus-7 is zero injection bus and it merged with eighth bus and to become bus number 8'.

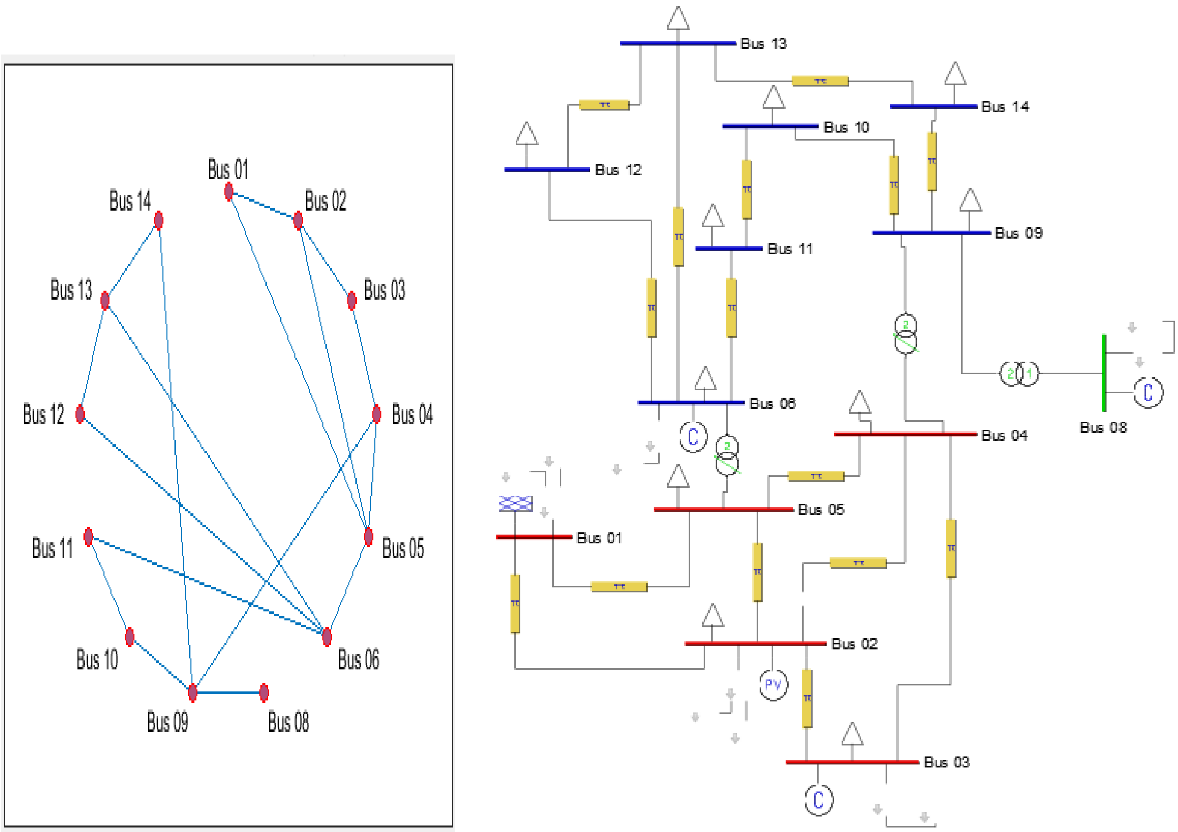


Fig.3 : zero injection bus merging method.

A' New matrix:

$$A' = \begin{bmatrix} 1&1&0&0&1&0&0&0&0&0&0&0&0 \\ 1&1&1&0&1&0&0&0&0&0&0&0&0 \\ 0&1&1&1&0&0&0&0&0&0&0&0&0 \\ 0&0&1&1&1&0&0&1&0&0&0&0&0 \\ 1&1&0&1&1&1&0&0&0&0&0&0&0 \\ 0&0&0&0&1&1&0&0&0&1&1&1&1 \\ 0&0&0&0&0&0&1&1&0&0&0&0&0 \\ 0&0&0&1&0&0&1&1&1&0&0&0&1 \\ 0&0&0&0&0&0&0&1&1&1&0&0&0 \\ 0&0&0&0&0&1&0&0&1&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&1&1&0 \\ 0&0&0&0&0&1&0&0&0&0&1&1&1 \\ 0&0&0&0&0&0&0&1&0&0&0&1&1 \end{bmatrix}$$

Starting with the new matrix, we get the following equations:

Bus 1: $x_1 + x_2 + x_5 \geq 1$

(19)

Bus 2: $x_1 + x_2 + x_3 + x_4 + x_5 \geq 1$

(20)

Bus 3: $x_2 + x_3 + x_4 \geq 1$

(21)

Bus 4: $x_2+ x_3+ x_4+ x_5 + x_{8'} \geq 1$

(22)

Bus 5: $x_1+ x_2+ x_4+ x_5+ x_6 \geq 1$

(23)

Bus 6: $x_5 + x_6 + x_{11}+ x_{12}+ x_{13} \geq 1$

(24)

Bus 8': $x_{8'} + x_9 \geq 1$

(25)

Bus 9: $x_4+ x_9+ x_{10} + x_{14} \geq 1$

(26)

Bus 10: $x_9+ x_{10}+ x_{11} \geq 1$

(27)

Bus 11: $x_6 + x_{10} + x_{11} \geq 1$

(28)

Bus 12: $x_6+ x_{12} + x_{13} \geq 1$

(29)

Bus 13: $x_6+ x_{12} + x_{13} + x_{14} \geq 1$

(30)

Bus 14: $x_9+ x_{13} + x_{14} \geq 1$

(31)

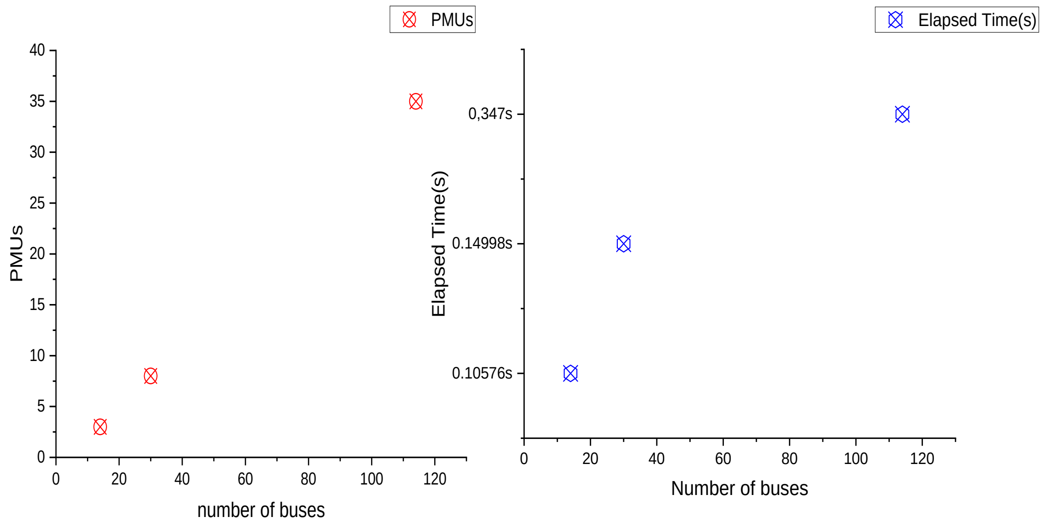
Ignored zero injection buses (ZIB): For systems 9, 14, 30, 57, 118 buses IEEE And 114 buses in the transportation network in Algeria and by using the OPP program, we got the following table:

Number of buses	Number of buses a ZIB	Location bus a ZIB
case9	03 buses	bus4; 6 and 8
case14	1 bus	Bus 7.
case30	06 buses	Bus 6;9; 22; 25; 27; and 28.
case39	10 buses	bus2; 5; 6; 10; 11; 13;14;17;19; and 22.
case57	15 buses	Bus 4; 7; 11; 21; 22; 24;26 ; 34; 36; 37; 39; 40; 45; 46; and 48.
case114	22 buses	Bus 2 ;14 ;16 ;18 ;27 ;28 ;31 ;42 ;44 ;46 ;48;58 ; 60 ;64 ;72 ;74 ;75 ;81 ;86 ;93 ;96 ; and 105.
case118	10 buses	bus5;9;30; 37; 38;63; 64; 68; 71; and 81.

Fig.4 number of buses a ZIB

Method DeFS/ILP:

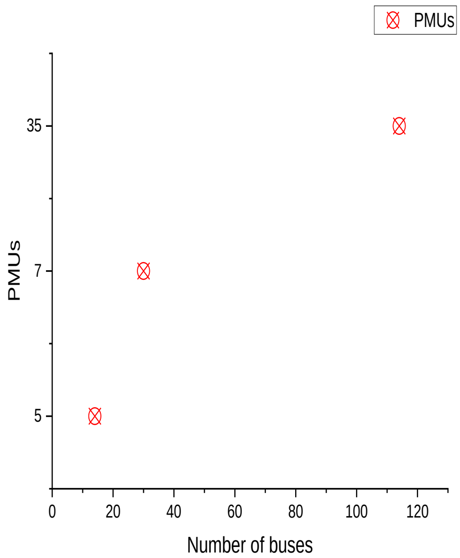
Number of buses	PMUs	PMU Location	Elapsed Time(s)	Meas. Currents	Pseudo-Meas. Currents
14	3	2 ;6 ;9	0.10576s	12	0
30	8	10 ;11 ;12 ;19 ;2 ;24 ;3 ;30	0.14998s	24	00
114	35	103 ;109 ;112 ;113 ;13 ;20 ;26 ; 3 ;30 ;37 ;38 ;39 ;4 ;41 ;43 ;50 ; 53 ;57 ;59 ;63 ;68 ;7 ;71 ;73 ;76 ;78 ;79 ;84 ;88 ;90 ;91 ;92 ;94 ; 97 ;99.	0.374	103	00

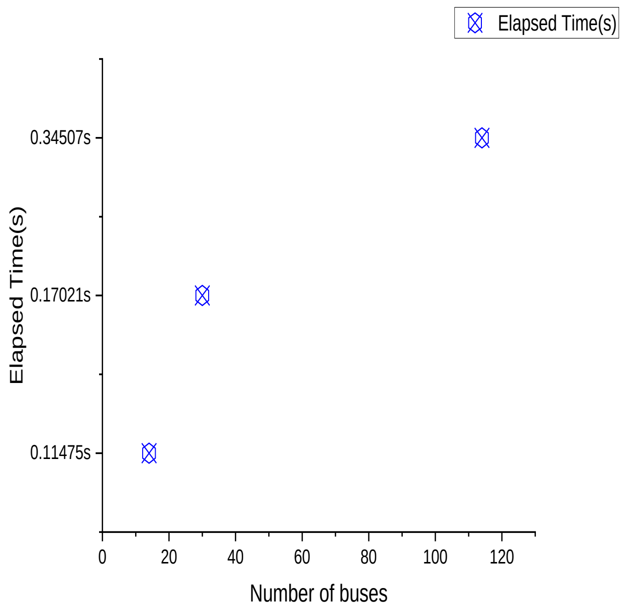


Calculation times for the proposed DF method a ZIB for, IEEE 14-bus, IEEE 30-bus and 114-bus test systems being 0.10576s, 0.14998s, and 0.22616s,respectively. Calculation meas.currents 16; 33; and 103; respectively.   Calculation Pseudo-Meas. Currents 00.

Method GTh/ILP:

Number of buses	PMUs	PMU Location	Elapsed Time(s)	Meas. Currents	Pseudo-Meas. Currents
14	5	1 ; 4 ;6 ;10 ;14	0.11475s	15	5
30	7	10 ;12 ;19 ;2 ;24 ;3 ;30	0.17021s	23	6
114	35	103 ; 109 ; 112 ; 113 ; 13 ; 20 ; 26 ; 3 ; 30 ; 37 ; 38 ; 39 ; 4 ; 41 ; 43 ; 50 ; 53 ; 57 ; 59 ; 63 ; 68 ; 7 ; 71 ; 73 ; 76 ; 78 ; 79 ; 84 ; 88 ; 90 ; 91 ;92 ;94 ;97 ;99.	0.34507s	103	00

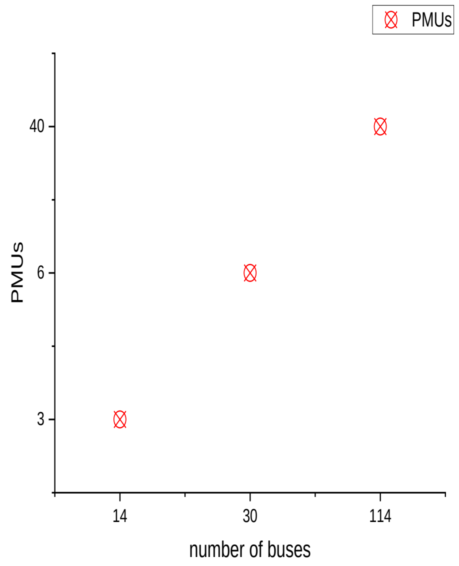


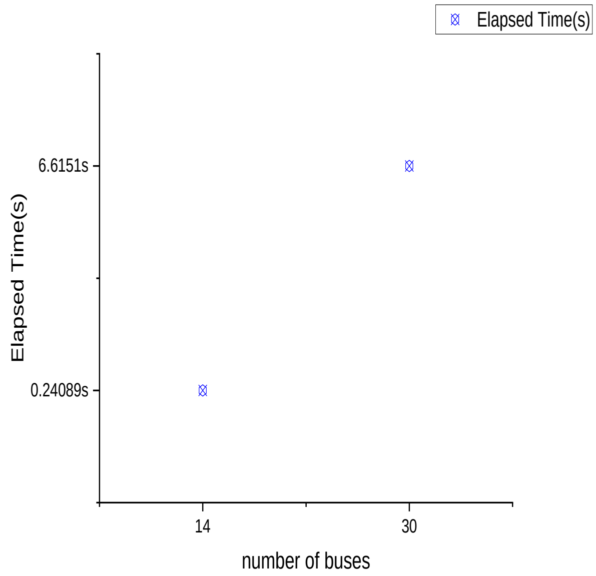


Calculation times for the proposed GTP method a ZIB for, IEEE 14-bus, IEEE 30-bus and 114-bus test systems being 0.11475, 0.17021, and0.34507s, respectively. Calculation meas.currents15, 23, and 103; respectively. Calculation Pseudo-Meas. Currents 00

Method **AM**/ILP:

Number of buses	PMUs	PMU Location	Elapsed Time(s)	Meas. Currents	Pseudo-Meas. Currents
14	3	bus2 ;6 ;9	0.24089s	12	8
30	06	Bus1; 10; 12; 18; 24; 30.	6.6151s	19	29
114	40	/	/	/	/





Calculation times for the proposed AM a ZIB for, IEEE 14-bus, IEEE 30-bus and 114-bus test systems being 0.24089, 6.6151s, and Non observable, respectively. Calculation meas-currents 12, 19, and Non Observable; respectively. Calculation Pseudo-Meas.Currents 00.

VI. Comparative evaluation of the methods proposed in this work with and without considering ZIBs:

1-Nombuer of PMUs:

	DeFS/ILP		GTh/ILP		AM/ILP	
	without ZIBs	with ZIBs	without ZIBs	with ZIBs	without ZIBs	with ZIBs
14 buses	6	3	5	5	4	3
30 buses	12	8	11	7	9	6
114 buses	44	35	44	35	/	/

2. Elapsed Time(s)**:**

	DeFS/ILP		GTh/ILP		AM/ILP	
	without ZIBs	with ZIBs	without ZIBs	with ZIBs	without ZIBs	with ZIBs
14 buses	0.17216s	0.10576s	0.11475s	0.11475s	0.8786	0.24089s
30 buses	0.16932s	0.14998s	0.23431s	0.17021s	9m 37.4673s	6.61510s
114 buses	0.22616s	0.37400s	6.2122s	0.34507s	/	/

Comparing the three offered theoriess in the previous two tables, we can see that the theory AM performs better in systems with fewer buses (14 and 30 buses). Regarding the final two hypotheses, they perform better in systems with a sizable number of buses (114 buses).
After comparing the options, we came to the conclusion that the Annealing Method is the best theory when there are few nodes in the system.
The findings of the two theories become Depth first and Graph Theoretic Procedure is considerably better and better as the system complexity and node count increase.

VII. Comparison of the results to other methods that considered the ZIB

methods \ systems	IEEE 14 BUS		IEEE 30 BUS		ALG- 114 BUS		Ref
	PMUs	Elapsed Time(s)	PMUs	Elapsed Time(s)	PMUs	Elapsed Time(s)	
DeFS/ILP	3	0.10576s	8	0.14998s	35	0.37400s	
GTh/ILP	5	0.11475s	7	0.17021s	35	6.2122s	
AM/ILP	3	0.24089s	6	6.61510s	/	/	
GWO	3	/	7	/	36	/	[2]
ABC	3	0.264 s	7	0.823 s	/	/	[3]
BK, AK	3	0.660 s	7	0.830 s	/	/	[4]
BIP	3	1.160 s	7	1.240 s	/	/	[4]

In this table, the goal is to compare the optimal number of PMUs required after each algorithm as well as the number required to manage each system. From this table, we can say that the proposed algorithms make it possible to obtain competitive results. On the other hand, as any strategy allows. To obtain approximately the same results in cases, it is difficult to judge the superiority between these methods. Unless a deep comparative study is conducted taking into account other aspects and limitations, especially execution time and simplicity of algorithm computing. However, it is important to clarify that each approach. It has its originality and effectiveness. In addition, we touched on Meas. Currents and Pseudo-Meas. Currents in each system in order to facilitate good and total system control.

Conclusions: This study employs Depth-First, Graph Theoretic, Annealing Method, Intlin-Prog software, and the Observation Capacity Redundancy Index to examine the performance of IEEE standard buses and the real power system of the Algeria114 bus (SORI). Before applying the techniques to a 114-bus system in Algeria, they are first being evaluated on IEEE standard buses to compare the outcomes to those reported in the literature. The simulation results show a modest difference between the road performance of IEEE standard buses and the ALG-114 bus

system. However, the various adopted algorithms give the optimal number and optimal location of the project management units on the three systems with the highest SORI. In addition to, The system gives the least number of PMU positions for the possibility of full monitoring of the energy system. Therefore, the number of branches in the power system plays an important role in determining the optimal number of project management units for the possibility of full monitoring of power systems and SORI. If the number of branches does not enable the system to comprehensively monitor the system, then it resorts to measuring currents in imaginary branches for the overall monitoring of this system (Pseudo-Meas. Currents). The results of the 114-bus system in Algeria show the strategic positions and the locations where the project management units should be located to increase the monitoring and control of the system. The study also suggested ZIB modeling by looking at the intelligence linear programming (ILP).
The proposed approach has been tested on IEEE standard buses and simulation results prove its efficacy with those methods in the literature.

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