# Distribution Characteristics of Multivariate Financial Assets Return based on Mathematical Model

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**Abstract: For the non-normality and time variability of the distribution of multivariate financial assets return, a dynamic model of the distribution of multivariate financial assets return based on mathematical model is constructed in this paper. AR(1)-DCC(1,1)-GARCH(1,1) model reflects dynamic characteristics of conditional expectation and conditional variance of multivariate financial assets return. It solves the problem that restricts the in-depth research on high order dynamic portfolio optimization, which is the estimation of conditional coskewness matrix and conditional cokurtosis matrix. By constructing a multi-dimensional fluctuation model with biased t distribution, conditional asymmetric parameter and conditional free degree parameter, the distribution of multivariate financial assets return is researched. Experimental results show that the proposed model can reasonably reflect the time-varying characteristics of the multivariate stock return distribution in China's stock market.**

**Key words: Mathematical model; multivariate financial assets; income distribution; biased thick tail; time varying; high order dynamics.**
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## 1 Introduction

The return on financial assets is a very important concept in financial economics. The correct description of the distribution of the return rate is directly related to the correctness of the portfolio selection, the effectiveness of the risk management, and the rationality of the option pricing. In the classical econometric model for the description of stock price behavior, stock market return is usually assumed to obey normal distribution. Metrological financiers have made a lot of theoretical and empirical analysis of this classic hypothesis (Lutzenberger Gleich and Mayer 2017). The results show that the majority of stock market returns in financial markets do not obey normal distribution, but are characterized by peak, thick tail and asymmetry. At present, for the research on the distribution characteristics of multivariate financial assets, most of them focus on the time variation of the second-order moment. The research progress of risk time variability considering higher order moments is very slow (Rombouts Stentoft and Violante 2014). The reason is mainly that the conditional cokurtosis matrix and conditional coskewness matrix is difficult to estimate. To solve this problem, a reasonable multivariate financial assets return distribution model based on mathematical model is built in this paper. It can describe the time-varying characteristics of the distribution of multivariate financial assets return, and facilitate the estimation of the parameters of the model and the estimation of the conditional coskewness matrix and the conditional cokurtosis matrix.

First, through the Taylor series expansion of the utility function, the method of solving the dynamic portfolio considering the high-order moment risk is obtained, and the difficulties encountered in the process are analyzed. Then, based on the multivariate biased t distribution of Bauwens and Laurent (2005), the time variation of the biased thick tail property is further considered (Bonfiglioli 2016). Combined with DCC-GARCH model proposed by Engle and Sheppard (2001), a dynamic model of multivariate financial asset return distribution based on mathematical model is proposed in this paper. The estimation of the parameters of the model and the estimation of the conditional coskewness matrix and the conditional cokurtosis matrix are given. Finally, the proposed model is used to make an empirical analysis.

## 2 Dynamic model of multivariate financial assets return distribution based on mathematical model

In this paper, a dynamic model of multivariate financial assets return distribution based on mathematical model is built. This model consists of two parts. The first part is the

$$AR(1)-DCC(1,1)-GARCH(1,1)$$

model, which is used to describe the time varying characteristics of conditional expectation and conditional variance for multivariate financial assets return (Shi and Valdez 2014). Through this part of the proposed model, the conditional expectation vector and conditional covariance matrix can be estimated. The second part is the multivariate conditional biased distribution model, which is used to describe the time variation of the biased thick tail characteristics of multivariate financial assets return distribution (Vitiello and Rebelo 2015). Through this model, the conditional coskewness matrix $S_t$ and conditional cokurtosis matrix $K_t$ can be reasonably estimated.

### 2.1 $AR(1)-DCC(1,1)-GARCH(1,1)$ model

$$AR(1)-DCC(1,1)-GARCH(1,1)$$

model is built to reflect dynamic characteristics of conditional expectation and conditional variance of multivariate financial assets return distribution. $AR(1)$ model reflects conditional expectation autocorrelation, $DCC(1,1)-GARCH(1,1)$ model reflects conditional variance aggregation.

Assume $\varepsilon_t=(\varepsilon_{1,t},...,\varepsilon_{n,t})=R_t-\mu_t$ is the error vector of n financial assets returns in the t period, $R_t$ is the vector of the financial assets return in the t period, $\mu_t$ is the conditional expectation vector of financial assets return, which can be set in autoregressive form, such as introducing exogenous variable (David and Gallo 2014). In this paper, $AR(1)$ equation is used. $\sigma_{ii,t}(i=1,2,...,\mathrm{n})$ is the conditional variance of $R_{i,t}$, $\sigma_{ij,t}(i=1,2,...n;\ j=1,2,...,n)$ is the conditional covariance of $R_{i,t}$ and $R_{j,t}$.

$AR(1)-DCC(1,1)-GARCH(1,1)$ model is expressed as

$$R_t=\mu+\varphi R_{t-1}+\varepsilon_t \quad (1)$$

$$\varepsilon_t=\sum\nolimits_t^{1/2} z_t \quad (2)$$

$$\sum\nolimits_t = D_t\Gamma_t D_t \quad (3)$$

$$D_t=diag(\sqrt{\sigma_{11,t}},...,\sqrt{\sigma_{nn,1}}) \quad (4)$$

$$\sigma_{ii.t}=\omega_i+\beta_i\sigma_{ii,t-1}+\gamma_i\varepsilon_{i,t-1}^2 \quad i=1,...,n$$

(5)

$$\Gamma_t=(diag(Q_t))^{-1}Q_t(diag(Q_t))^{-1} \quad (6)$$

$$Q_t=(1-\delta_1-\delta_2)\bar{Q}+\delta(u_{t-1}u'_{t-1})+\delta_2 Q_{t-1}$$

(7)

$$\bar{Q}=\begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{12} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \rho_{n-1,n} \\ \rho_{1n} & \cdots & \rho_{n-1,n} & 1 \end{pmatrix} \quad (8)$$

$$u_t=D_t^{-1}\varepsilon_t \quad (9)$$

where $\varphi$ is the n*n-dimensional diagonal matrix, $z_t$ is the residual vector of error vector $\varepsilon_t$ after standardization. In order to reflect the biased thick tail characteristics of the distribution of financial assets return and the time variability (Evans and Hnatkovska 2014), it is assumed that $z_t$ has multivariate conditional biased t distribution. The conditional covariance matrix $\sum_t$ can be regarded as a function consisting of diagonal matrix $D_t$ and conditional correlation matrix $\Gamma_t$ composed of standard deviation $\sqrt{\sigma_{ii,t}}$ .Conditional variance $\sigma_{ii,t}$ obeys

$GARCH(1,1)$ model while conditional correlation coefficient matrix $\Gamma_t$ also has time variation. $\bar{Q}$ is the unconditional covariance matrix of the vector $u_t$. In order to ensure the smoothness of the variance process and the positive definiteness of the conditional correlation coefficient matrix (Thomson Schonert-Reichl and Oberle 2015), the restrictive condition is added, which is $\beta_i + \gamma_i < 1, i = 1, 2, ..., n; 0 \le \delta_1, \delta_2 \le 1$ and $\delta + \delta_2 \le 1$

## 2.2 Multivariate conditional biased t distribution model

Although $AR(1) - DCC(1,1) - GARCH(1,1)$ model can describe the dynamic characteristics of the conditional expectation and the conditional variance of multivariate financial assets return, it is still necessary to model the biased thick tail characteristics and time variability of multivariate financial assets return distribution. Bauwens and Laurent (2005) introduced asymmetric parameter in multivariate t distribution (Scheufele 2015) and proposed a multivariate biased t distribution model to describe the biased thick tail characteristics of the distribution of multivariate financial assets return. Although the parameters of the model are easy to estimate and have good practicability, it does not consider the time variation of the biased thick tail characteristics. Based on the results of Bauwens and Laurent (2005), the time variation of the biased thick tail characteristics is further considered to propose multivariate conditional biased t distribution model. The built multivariate conditional biased t distribution model is divided into two parts. The first part is the multidimensional biased t distribution, the second part is the fluctuation model of conditional freedom parameter and condition non-multi-parameter.

(1) Multidimensional biased t distribution. Assume standardized residual vector $z_t = (z_{1t}, ..., z_t)'$ obeys multivariate biased t distribution and the joint density function is given by

$$t\left(z_t \middle| v_{1,t}, ..., v_{n,t}, \lambda_{1,t}, ..., \lambda_{n,t}\right) = \prod_{i=1}^{n} b_i c_i \left(1 + \frac{\zeta_{i,t}^2}{v_{i,t} - 2}\right)^{-\frac{v_{i,t}+1}{2}} \quad (10)$$

where

$$\zeta_{i,t} = \begin{cases} (b_i z_{i,t} + a_i) / (1 - \lambda_{i,t}) & \text{when } z_{i,t} < -a_i / b_i \\ (b_i z_{i,t} + a_i) / (1 + \lambda_{i,t}) & \text{when } z_{i,t} \ge -a_i / b_i \end{cases}$$

and

$$c_i = \frac{\Gamma(\frac{v_{i,t}+1}{2})}{\sqrt{\pi(v_{i,t}-1)}\Gamma(\frac{v_{i,t}}{2})}, a_i = 4\lambda_{i,t} c_i \frac{v_{i,t}-2}{v_{i,t}-1}, b_i^2 = 1 + 3\lambda_{i,1}^2 - a_i^2$$

$$-1 < \lambda_{i,t} < 1, 2 < \mathrm{v}_{i,t} < \infty$$

where $\Gamma(g)$ is the gamma function, $\lambda_{i,t}$ is the asymmetrical parameter of the edge distribution of $z_{i,t}$, which is to describe the biased characteristics of distribution. When $\lambda_{i,t} = 0$, $z_{i,t}$ is the standard t distribution. When $\lambda_{i,t} < 0$, the distribution of $z_{i,t}$ is left-biased. When $\lambda_{i,t} > 0$, the distribution of $z_{i,t}$ is rights-biased. a, b, and c are the constants. $v_{i,t}$ is the degree of freedom $z_{i,t}$ obeys, which is to describe thick tail characteristics of the distribution. The smaller $v_{i,t}$, The more obvious the thick tail characteristics of the distribution. When $v_{i,t} \to +\infty$, $z_{i,t}$ has no thick tail characteristics. In order to ensure the existence of the second-order matrix of edge distribution, the restrictive condition $2 < v_{i,t} < \infty$ is added.

(2) Fluctuation model of conditional asymmetric parameter and conditional degree of freedom parameter

Ang and Bekaert (2002) pointed out that the bull market and bear market are persistent (Wurthmann 2015). Positive fluctuation is usually concentrated at certain periods, while negative fluctuation is concentrated in other periods, which indicates that asymmetric parameters have a certain time aggregation. Das and Uppal (2004) pointed out that large fluctuation usually does not have continuity. After the earlier fluctuation, the thick tail characteristics of the return distribution obviously weakened, that is, the larger fluctuation in the early stage tends to follow the larger degree of freedom parameter (Grisse and Nitschka 2015). Based on the above analysis, the following conditional degree of freedom parameter fluctuation model and conditional asymmetric parameter fluctuation model are built.

$$\frac{1}{v_{i,t}} = \frac{1}{v_i} + \sum_{j=1}^{p} b_{ij} \left|\varepsilon_{t-j}\right| \quad i = 1,2,\ldots,n$$

(11)

$$\lambda_{i,t} = \lambda_i + \sum_{k=1}^{q} c_{ik}\varepsilon_{t-k} \quad i = 1,2,\ldots,n$$
$$-1 < \lambda_{i,t} < 1, 2 < v_{i,t} < \infty$$

(12)

It should be explained that the return distribution tends to normal distribution due to the infinite degree of freedom parameter. In order to express convenience, $\frac{1}{v_{it}}$ is selected to be modeled. After the short term events, they tend to follow larger degrees of freedom (Frédéric Teulon Khaled Guesmi and Selim Mankai 2014). So $v_i$ can be set to a larger positive number to reflect the proximity to normal distribution. The use of the absolute value of the error term in the fluctuation model of the degree of freedom parameter $v_{i,t}$ is because the change of the degree of freedom parameter $v_{i,t}$ depends mainly on the magnitude of the previous fluctuation rather than the direction. The use of an error term with symbol in the fluctuation model of asymmetric parameters is because the change of asymmetric parameters depends on the size of the previous fluctuation and the direction of the previous fluctuation.

(3) Estimation of model parameters

$$\theta = (\mu_i, \varphi_i, \omega_i, \beta_i, \gamma_i (i = 1, 2, ..., n), \delta_1, \delta_2, \rho_{jk} (1 \le j < k \le n))$$

is the parameter vector to be estimated of $AR(1) - DCC(1,1) - GARCH(1,1)$ model. $\delta_1$, $\delta_2$, and $\delta_3$ are the nonlinear function of $w_t$.

$$\zeta = (\lambda_1, ...\lambda_n, b_{ij} (i = 1, 2, ...n; j = 1, 2, ..., p), c_{ik} (i = 1, 2, ..., n; k = 1, 2, ..., q))$$

is the parameter vector to be estimated of conditional degree of freedom parameter fluctuation model and conditional asymmetric parameter fluctuation model.

The parameter vectors $\theta$ and $\zeta$ are estimated. First, the conditional log likelihood function of the sample is obtained.

$$\ln L(R_1, ..., R_T \left| \theta, \zeta \right.) = \sum_{t=1}^{T} \ln \left[ t \textstyle\sum_t (\theta)^{-1/2} (R_t - \mu_t(\theta)) \left| \zeta \right. \right] = \sum_{t=1}^{T} \ln \left[ t(z_t \left| \zeta \right.) \right] - \frac{1}{2} \sum_{t=1}^{T} \ln \left| \textstyle\sum_t (\theta) \right|$$

(13)

where $t(\sum_t (\theta)^{-1/2} (R_t - \mu_t(\theta)) \left| \zeta \right.)$ and $t(z_t \left| \zeta \right.)$ are the density function of the multivariate biased distribution in Eq. (10). By maximizing Eq. (13), the maximum likelihood estimation of the parameters to be estimated is obtained (Guo and 2014). The estimation of parameter vectors $\theta$ and $\xi$ can also be realized by two-stage maximum likelihood estimation.

(4) Estimation of conditional coskewness matrix $S_t$ and conditional cokurtosis matrix $K_t$.

Under the multivariate conditional biased t distribution, the conditional covariance matrix $S_t$ and the conditional cokurtosis matrix $K_t$ can be estimated by matrix computation (Harris Hartzmark and Solomon 2015). The following is a description of the estimation process. The intermediate derivation process is omitted in this paper.

According to Eq. (12) and Eq. (13), $R_t - \mu_t = \varepsilon_t = \sum_t^{1/2} z_t$ can be obtained, where $\sum_t^{1/2} = (\omega_{ij,t})_{n \times n} (i, j = 1, 2, ..., n)$ is the choleski decomposition of conditional covariance matrix. For conditional coskewness matrix, there is the following relationship.

$$S_t = \left[ s_{ijk,t} \right]_{n \times n^2} = E_{t-1} \left[ (R_t - \mu_t)(R_t - \mu_t)' \otimes (R_t - \mu_t)' \right] = E_{t-1} \left[ (\textstyle\sum_t^{1/2} z_t)(\sum_t^{1/2} z_t)' \otimes (\sum_t^{1/2} z_t)' \right]$$

(14)

where $\sum_t^{1/2} z_t$ is $n \times 1$ -dimensional vector, $(\sum_t^{1/2} z_t)(\sum_t^{1/2} z_t)' \otimes (\sum_t^{1/2} z_t)'$ is $n \times n^2$ -order matrix, $E_{t-1} \left[ \sum_t^{1/2} z_t)(\sum_t^{1/2} z_t)' \otimes (\sum_t^{1/2} z_t)' \right]$ represents conditional expectation for each component of the matrix, $\otimes$ is the Kronecker product. Through expansion and conditional expectation of

$$K_t = \left[k_{ijkl,t}\right]_{n\times n^3}$$
$$= E_{t-1}§R_t - \mu_t)(R_t - \mu_t)' \otimes (R_t - \mu_t)' \otimes (R_t - \mu_t)¨ = E_{t-1}§\sum\nolimits_t^{1/2} z_t)(\sum\nolimits_t^{1/2} z_t)' \otimes (\sum\nolimits_t^{1/2} z_t)¨ \Rightarrow k_{ijkl,t}$$
$$= \sum_{r=1}^{n} \omega_{ir,t}\omega_{jr,t}\omega_{kr,t}\omega_{lr,t}E_{t-1}(z_{r,t}^4) + \sum_{r=1}^{n}\sum_{\substack{s=1\\ s\neq r}}^{n}(\omega_{ir,t}\omega_{jr,t}\omega_{ks,t}\omega_{ls,t} + \omega_{ir,t}\omega_{js,t}\omega_{kr,t}\omega_{ls,t} + \omega_{ir,t}\omega_{js,t}\omega_{ks,t}\omega_{lr,t}), i, j, k, l = 1, 2, ..., n$$

, it can be obtained that

$$s_{ijk,t} = \sum_{r=1}^{n} \omega_{ir,t}\omega_{jr,t}\omega_{kr,t}E_{t-1}(z_{r,t}^3), i, j, k = 1, 2, ..., n$$

(15)

Similarly,

$$K_t = \left[k_{ijkl,t}\right]_{n\times n^3} = E_{t-1}§R_t - \mu_t)(R_t - \mu_t)' \otimes (R_t - \mu_t)' \otimes (R_t - \mu_t)¨ = E_{t-1}§\sum\nolimits_t^{1/2} z_t)(\sum\nolimits_t^{1/2} z_t)' \otimes (\sum\nolimits_t^{1/2} z_t)¨ \Rightarrow k_{ijkl,t}$$
$$= \sum_{r=1}^{n} \omega_{ir,t}\omega_{jr,t}\omega_{kr,t}\omega_{lr,t}E_{t-1}(z_{r,t}^4) + \sum_{r=1}^{n}\sum_{\substack{s=1\\ s\neq r}}^{n}(\omega_{ir,t}\omega_{jr,t}\omega_{ks,t}\omega_{ls,t} + \omega_{ir,t}\omega_{js,t}\omega_{kr,t}\omega_{ls,t} + \omega_{ir,t}\omega_{js,t}\omega_{ks,t}\omega_{lr,t}), i, j, k, l = 1, 2, ..., n$$

(16)

where $\omega_{ij,t}(i, j = 1, 2, ..., n)$ can be determined by Choleski decomposition of conditional covariance matrix. $E_{t-1}(z_{i,t}^3)(i = 1, 2, ..., n)$ and $E_{t-1}(z_{i,t}^4)(i = 1, 2, ..., n)$ are given by

$$E_{t-1}(z_{i,t}^3) = \frac{M_{i,3} - 3a_i M_{i,2} + 2a_i^3}{b_i^3} (i = 1, 2, ..., n, \text{when } v_{i,t} > 3)$$

(17)

$$E_{t-1}(z_{i,t}^4) = \frac{M_{i,4} - 4a_i M_{i,3} + 6a_i^2 M_{i,2} - 3a_i^4}{b_i^4} (i = 1, 2, ..., n, \text{when } v_{i,t} > 4)$$

(18)

where $M_{i,2} = 1 + 3\lambda_{i,t}^2 = b_i^2 + a_i^2$,

$$M_{i,3} = 16c_{1,3}\lambda_{i,t}(1 + \lambda_{i,t}^2)\frac{(v_{i,t} - 2)^2}{(v_{i,t} - 1)(v_{i,t} - 3)} (\text{when } v_{i,t} > 3)$$

,

$$M_{i,4} = 3\frac{(v_{i,t} - 2)}{(v_{i,t} - 4)}(1 + 10\lambda_{i,t}^2 + 5\lambda_{i,t}^4)(\text{when } v_{i,t} > 4).$$

**3 Experimental analysis**

**3.1 Data selection and basic statistical characteristics**

4 stocks of the Shanghai Stock Exchange are randomly selected: Dongan power (600178), Ling Steel shares (600231), Capital Tourist shares (600258), Yili energy (600277). A week is taken as an investment cycle and weekly closing data is taken as the research object (Paul Sullivan and Ted To 2014). The sample interval is from January 1, 2012 to December 31, 2016. Excluding the holidays and asynchronous trading days, the sample size is N=231. Data is from the Tai'an CSMAR database (Jiayu 2014). The stock yield is calculated from the first-order difference of the natural logarithm of stock week closing price. From Table 1, it can be seen that, the 4 stocks significantly reject the assumption that the distribution of return is subject to normal distribution. Therefore, the influence of higher moments should be considered in portfolio optimization.

Table 1 The basic statistical characteristics of the yield sequence

| Stock name | Average value | Standard deviation | Skewness | Kurtosis | J-B test |
|---|---|---|---|---|---|
| Dongan power | -0.002941 | 0.048160 | -0.138925 | 5.032973 | 39.83825* |

| | | | | | |
|---|---|---|---|---|---|
| Linggang shares | -0.002574 | 0.055216 | 0.091024 | 7.959601 | 237.0713* |
| First brigade shares | 0.001353 | 0.050435 | 0.268827 | 5.828865 | 79.80618* |
| Yili energy | -0.003946 | 0.052070 | -0.925895 | 10.27976 | 543.0809* |

Note: * represents it is significant under the significance level of 1%.

**3.2 Estimation of model parameters**

The dynamic characteristics of the 4 stock returns distribution are built with the proposed dynamic model of multivariate financial assets return distribution. $v_i = 100(i = 1, 2, 3, 4)$ is set. For the sake of simplification, the conditional asymmetry parameter fluctuation model of conditional degree of freedom parameter fluctuation model is set to 1st-order lag (Song Li and Wang 2015). Using Eviews5.0 and Matlab software, the two-stage maximum likelihood estimation method is used to estimate the parameters of the model. The estimation results are shown in Table 2 and Table 3.

Table 2 Parameter estimation of one dimensional GARCH model

| Stock name | Dongan power | Linggang shares | Capital Tourist shares | Yili energy |
|---|---|---|---|---|
| $\mu$ | -0.00301（-6.1067） | -0.00284（-4.6617） | 0.001244（-2.6826） | -0.00481（-7.5783） |
| $\varphi$ | -0.023002（-3.4836） | -0.101515（-5.4687） | 0.082438 -8.0546 | -0.21932（-3.4054） |
| $\omega$ | 0.0009（7.2199） | 0.0022（7.1144） | 0.0009（4.3958） | 0.0008（6.4135） |
| $\beta$ | 0.1801（18.7604） | 0.2904（1.5376） | 0.1773（7.0357） | 0.2849（0.7376） |
| $\gamma$ | 0.4453（6.7367） | 0.2001（6.1008） | 0.4983（17.6077） | 0.4296（5.8863） |

Table 3 Parameter estimation of DCC model, degree of freedom parameter and asymmetric parameter wave model

| | Stock name | Dongan power | Linggang shares | Capital Tourist shares | Yili energy |
|---|---|---|---|---|---|
| Parameter fluctuation model of conditional freedom degree | $b_1$ | -4.09405 | -1.1262 | -2.67258 | -2.17928 |
| | | -10.7966 | -11.8782 | -6.3425 | -3.2441 |
| Conditional asymmetric parameter fluctuation model | $\lambda_i$ | -0.011818 | 0.010054 | 0.73989 | -0.09121 |
| | | -6.1517 | -12.2035 | -5.2984 | -3.9934 |
| | $c_1$ | 0.91693 | 2.92702 | 8.67778 | 5.1022 |
| | | -4.1495 | -3.5627 | -6.7284 | -3.0446 |
| DCCModule | $\delta_1$ | 0.0089 | | | |
| | | -2.2546 | | | |
| | $\delta_2$ | 0.9486 | | | |
| | | -7.1239 | | | |

The data in parentheses after parameter estimation in Tables 2 and 3 are statistical values of regression coefficient t test. $t_{0.025}(230-2)\approx 1.960$ is known. Besides the parameter $\beta$ in the conditional variance equation of Linggang shares and Yili energy, the other parameters are passed significance test.

**3.3 Solving and comparison of dynamic portfolio optimization problem**

By using estimated value of the dynamic model of multivariate financial asset return distribution, it can be calculated by Matlab software that

$$\sum\nolimits_t^{1/2}=(\omega_{ij,t})_{4\times 4}=\begin{pmatrix} 0.0441 & 0 & 0 & 0 \\ 0.0166 & 0.0528 & 0 & 0 \\ 0.0269 & 0.0152 & 0.0642 & 0 \\ 0.0103 & 0.0135 & 0.0019 & 0.0467 \end{pmatrix} \quad (19)$$

$$(\mathrm{E}_{t-1}(z_{1,t}^3),E_{t-1}(z_{2,t}^3),E_{t-1}(z_{3,t}^3),E_{t-1}(z_{4,t}^3))=(0.2107,0.0575,0.8655,0.5117)$$
$$(\mathrm{E}_{t-1}(z_{1,t}^4),E_{t-1}(z_{2,t}^4),E_{t-1}(z_{3,t}^4),E_{t-1}(z_{4,t}^4))=(3.0324,3.0030,3.8725,3.1932) \quad (20)$$

Based on this, conditional expectation vector, conditional covariance matrix, conditional coskewness matrix and conditional kurtosis matrix for the next investment period can be estimated. The CRRA utility function is selected to describe the utility function of the investor, and the genetic algorithm is used to solve the dynamic portfolio problem with high order moment risk in the Matlab software.

In order to analyze the effect of dynamic portfolio, the optimization results of dynamic portfolio and static portfolio with higher order moment risk are compared (Lou Hu and Hu 2015). The used reference index is deterministic equivalent loss (CEL%), which is given by

$$CEL\% = \frac{U^{-1}(V^*(w^*)) - U^{-1}(V'(w'))}{U^{-1}(V'(w))} \times 100\%$$

. $V^*(\cdot)$ and $V'(\cdot)$ are the maximum approximate expected utility of dynamic portfolio and static portfolio, respectively. $w^*$ and $w'$ are the optimal solution of dynamic portfolio and static portfolio. The larger the certainty equivalent loss is, the better the dynamic portfolio compared with the static portfolio.

Table 4 Comparison of optimal static and dynamic portfolio

| | | $\mu_{p,t}$ | $\sigma^2_{p,t}$ | $s^3_{p,t}(\times 10^{-5})$ | $k^4_{p,t}(\times 10^{-6})$ | Utility function value | CEL % |
|---|---|---|---|---|---|---|---|
| $\lambda = 2$ | Dynamic investment | 0.0090 | 0.0027 | -1.9363 | 9.9128 | -0.9936 | 0.82% |
| | Static investment | 0.0086 | 0.0029 | -1.9421 | 10.0750 | -1.0018 | |
| $\lambda = 5$ | Dynamic investment | 0.0072 | 0.0022 | -1.8845 | 8.9641 | -0.2483 | 2.67% |
| | Static investment | 0.0064 | 0.0026 | -1.8930 | 9.4803 | -0.2581 | |
| $\lambda = 10$ | Dynamic investment | 0.0067 | 0.0020 | -1.7549 | 6.3893 | -0.1145 | 5.35% |
| | Static investment | 0.0051 | 0.0025 | -1.7645 | 7.2105 | -0.127 | |
| $\lambda = 15$ | Dynamic investment | 0.0045 | 0.0017 | -1.5624 | 3.5435 | -0.0804 | 17.34 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Static investment | 0.0024 | 0.0022 | -1.5921 | 4.6957 | -0.0972 | |
| $\lambda = 20$ | Dynamic investment | 0.0038 | 0.0012 | -1.2934 | 1.5351 | -0.0669 | 26.48% |
| | Static investment | 0.0016 | 0.0019 | -1.3552 | 3.1389 | -0.0910 | |

Table 4 shows the first four moments, utility function values and CEL% values of dynamic and static optimal portfolios with different risk aversion coefficients. From Table 4, it can be seen that, whether the static investment portfolio or the dynamic portfolio, the expected return, variance and kurtosis of the optimal portfolio are smaller as the risk aversion coefficient becomes larger, and the value of the bias and the utility function become larger. As the risk aversion coefficient becomes larger, the investor pays more attention to the risk of the higher moment, and is willing to reduce the high moment risk by reducing the expected return (Rausch and Schwarz 2016). For example, in a dynamic optimal portfolio with the risk aversion coefficient of 15, investors are willing to reduce the expected return at the cost of 0.0022, in exchange for the increase of $0.1925 \times 10^{-5}$ in bias, that is, $(-1.5624 \times 10^{-5}) - (-1.7549 \times 10^{-5})$ and the decrease of $2.8458 \times 10^{-6}$ in the kurtosis, that is, $(3.3893 \times 10^{-6} - 3.5435 \times 10^{-6})$ .From an empirical point of view, it illustrates the significance of the resrach on portfolio investment with higher order moment risk.

From Table 4, it can be also seen that, under the same risk aversion coefficient, the utility function value of the dynamic optimal portfolio is higher than that of the static optimal portfolio, and CEL% is positive. It shows that the optimization effect of dynamic portfolio is better than that of static portfolio. This is because dynamic portfolio can better characterize and avoid higher order moments than static portfolios. The greater the risk aversion coefficient, the higher the utility function value of the dynamic optimal portfolio, the greater the CEL% value. When the risk aversion coefficient is 2, CEL% is only 0.82%, while CEL% is 26.48% when the risk aversion coefficient is 20. This is because, although the dynamic portfolio method can describe and avoid the high order moment risk in spite of the static investment portfolio, the investor is not sensitive to the higher risk in the case of low risk aversion. Even though dynamic combination can better depict the risk of higher moments, its role is not outstanding. With the increase of risk aversion coefficient, investors are paying more and more attention to high order risk, and the advantage of dynamic portfolio also appears.

In addition, from the first four moments of the optimal portfolio, the expected returns and skewness of the dynamic optimal portfolio are higher than the static optimal portfolios under the same risk aversion parameters, and the variance and kurtosis are lower than those of the static optimal portfolio. It also shows that the performance of dynamic optimal portfolio is better than that of static portfolio. With the increase of risk aversion coefficient, this advantage is more obvious. The effectiveness of the proposed model in the application of multivariate financial asset return distribution characteristics is verified from another aspect.

**3.4 Logarithmic return**

As the Shanghai and Shenzhen 300 index shares come from the Shanghai and Shenzhen two cities, and account for about 60% of the total market value of the two cities, the Shanghai and Shenzhen 300 index is selected as the research object. The data is from January 1, 2015 to December 31, 2016. According to the closing price of the day, the logarithmic return per day was calculated, 1701 sample data were collected, and 2 models were selected, including AR (1) -GARCH (1,1) -Normal (AEPD, SKST, AST, ALD) and the proposed model.

Descriptive statistics of selected data samples are given. The statistics are shown in Table 5.

Table 5 Descriptive statistics of the logarithmic yield of the Shanghai Shenzhen 300 index

| mean value | standard deviation | skewness | Excess Kurtosis | JB statistics | Q(16) | No ARCH | Number of observations |
|---|---|---|---|---|---|---|---|
| 5.01E-04 | 0.0198 | -0.3857 | 2.47 | 474.92*** | 34.69*** | 26.35*** | 17.1 |

Note: *** represents it is significant under the significance level of 1%. The JB statistics are Jarque-Berate statistics. Q () is a Ljunj-boxQ statistic. No, ARCH is the test result of LM statistics.

From Table 5, it can be seen that, the bias of the logarithmic yield of Shanghai and Shenzhen 300 index is less than zero, showing a certain degree of left deviation. The excess kurtosis is greater than zero, indicating that the distribution of the daily logarithmic yield of Shanghai and Shenzhen 300 index has “high peak and heavy tail” phenomenon. From the JB statistics, it can be seen that the daily logarithmic yield of the Shanghai Shenzhen 300 index has rejected the normal distribution hypothesis at a significance level of 1%. The daily logarithmic yield series of Shanghai and Shenzhen 300 index for autocorrelation and heteroscedasticity test can be obtained. The results all reject the hypothesis that there is no autocorrelation and no ARCH effect at the significance level of 1%, indicating the existence of autocorrelation and heteroscedasticity in the diurnal logarithmic return sequence of the Shanghai and Shenzhen 300 index.

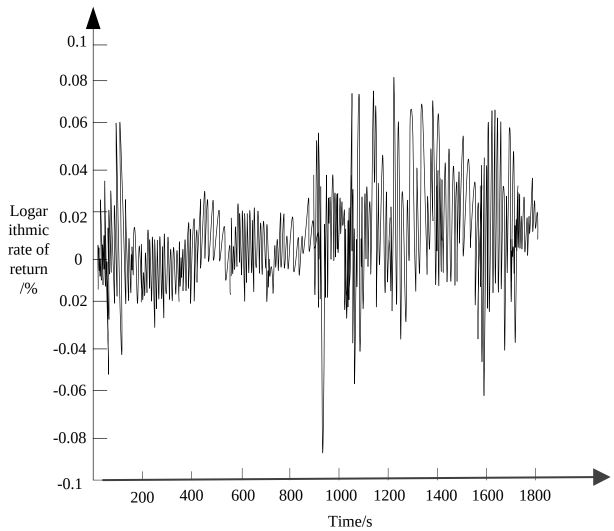


Fig.1 The time series chart of the daily logarithmic return of the Shanghai and Shenzhen 300 index

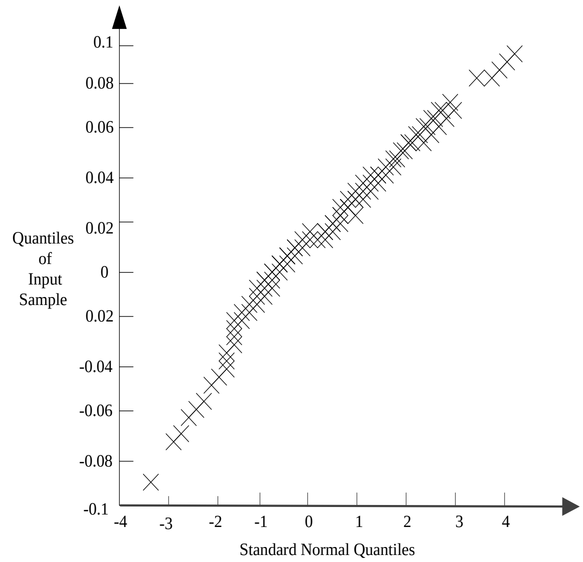


Fig.2 QQ diagram of the daily logarithmic return of the Shanghai and Shenzhen 300 index

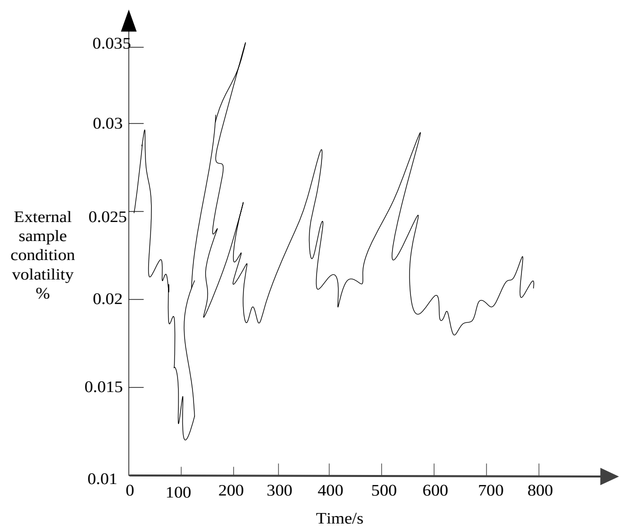


Fig.3 Out of sample conditional volatility series for Shanghai and Shenzhen 300 index returns

Fig. 1 and Fig. 2 are the time series chart and QQ diagram of daily logarithmic yield of Shanghai and Shenzhen 300 index. From Fig. 1, it can be seen that, there is fluctuation clustering in the daily logarithmic yield of the Shanghai Shenzhen 300 index. Fig. 2 shows that the daily logarithmic yield series of the Shanghai Shenzhen 300 index do not obey normal distribution. Figure 3 is the conditional fluctuation sequence diagram for the estimation of the logarithmic yield of the Shanghai and Shenzhen 300 index by using the proposed model. It is found that there exists obvious fluctuation clustering in the daily logarithmic yield series of the Shanghai Shenzhen 300 index. Compared with Fig. 1 and Fig. 3, it is found that when the fluctuation of return in Fig. 1 is large, the conditional fluctuation of return in Fig. 3 is also larger. When the fluctuation of return is small, the conditional fluctuation of return is also small. It shows that the proposed model can depict the fluctuation characteristics of the daily logarithmic yield of the Shanghai Shenzhen 300 index. Using the same analysis method to apply the proposed model to multivariate financial asset return, it can also effectively depict the distribution characteristics of returns.

## 4 Conclusions

In this paper, the characteristics of return distribution in dynamic portfolio with higher order moment risk are discussed. Taylor series expansion based on utility function is applied to solve the dynamic portfolio problem with higher order moment risk. A dynamic model of multivariate financial asset yield distribution based on mathematical model is proposed. The estimation method of model parameters and the estimation method of conditional coskewness matrix and conditional cokurtosis matrix are given. The empirical analysis shows that the proposed model can reasonably reflect the time-varying characteristics of the multivariate stock return distribution in China's stock market. Compared with the static portfolio, the dynamic portfolio with higher moments risk is better.

This study not only enriches the empirical research results of the non-symmetry test of financial assets return, but also provides new and powerful evidence for the exploration of the typical statistical characteristics of the international exchange rate market. At present, the research on asymmetry of financial assets return is still in the initial stage in the world. The theoretical circles have reached a basic agreement on the importance and influence of asymmetric asset returns in many theoretical issues. But there is still a lot of controversy on "whether asymmetry exists in the income of financial assets" and "what is the micro-mechanism of asymmetric formation". More empirical results and theoretical models are needed for improvement and interpretation. Therefore, how to explain the main empirical results in the existing research, that is, to explore why the return distribution of some financial assets will appear to be distinctly different from other assets, and what is the micro-mechanism of its asymmetric characteristics, will be the important direction of our further research.

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