Research on Splitting Isomorphism of Leibniz Algebra and Non-Abelian expansion

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Abstract: In this study, Leibniz algebras and the derivations and properties of Leibniz algebras were given, respectively. The stable automorphism group of explicit splitting extension was calculated via the stable automorphism group of Abelian extension of finite group splitting. Based on the stable automorphism group of the splitting extension studied, the non-Abelian extension and the second order non-Abelian co-homology group of Leibniz algebra were investigated in detail according to the stable automorphism group of the splitting extension.

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1 Introduction

In order to study continuous transformation groups, the concept of Lie algebra is introduced by Marius sophuslie for the first time. After that, the related higher-order theory has been widely concerned (Cao et al. 2017). Leibniz algebra, as a non-commutative deformation of Lie algebra, has been studied by many researchers (Zhang et al. 2018) and achieved rich results, thus promoting the development of other branches of mathematics.

Second order co-homology groups of Lie algebras classify Abelian extensions of Lie Algebras (Yang et al. 2017). The derivations of Lie algebras can linearly map non-Abelian extensions of Lie algebras.

Recently, Leibniz algebra has important applications in many fields. Liu, Weinstein and Xu studied the double of lie bialgebroid germs and introduced Courant algebroids(Hua et al. 2019), which played an important role in classical field theory and generalized complex geometry. Courant algebras can be equivalently described as a Leibniz algebra satisfying some compatibility conditions (Schweitzer 2017). Nambu Poisson structure was found and its definition was given when Nambu studied generalized Hamiltonian system. Subsequently, many researchers have carried out a lot of research on Nambu

Poisson structure. It is reported that the cotangent bundle of Nambu Poisson manifolds is studied, and it is found that there is a Leibniz algebraic structure on it that satisfies some compatibility conditions (Lin et al. 2018). Based on this, a deeper analysis of Nambu Poisson manifolds is given(Kutas 2019).

2Leibniz Algebra and Its Properties

2.1 Leibniz Algebra

Leibniz algebra is a non-commutative popularization of Lie algebra, which is defined as follows.

Definition 1

Suppose q is a vector space (Gnedbaye 2018). q contains the bilinear mapping $[\cdot,\cdot]_g : q \otimes q \to q$, which meets the following conditions:

$$\left[x,[y,z]\right] = \left[[x,y]_q, z\right]_q + \left[y,[x,z]_q\right]_q \quad (1)$$

$\left(q,[\cdot,\cdot]_q\right)$ is a Leibniz algebra.

Definition 2

Suppose q is a Leibniz algebra:

$$L(q) = \left\{x \in \left[x,[y,z]\right]\right\} \quad (2)$$

$L(q)$ is called the left center of algebra q.

Definition 3

Suppose q is a Leibniz algebra and V is a vector space (Lewis et al. 2018), l and r sum is a linear mapping from q to $gl(V)$. The following conditions are met:

$$l_{[x,y]_g} = \left[l_x, l_y\right]; \quad r_y o l_x = -r_y o r_x; \quad (3)$$

The triples (V,l,r) is called a representation of q (Ozaksoy 2018).

2.2 Derivations of Leibniz Algebras and Their properties

Definition 4

Let q be a Leibniz algebra, D^L and D^R are the linear mapping from q.

$$D^R[x,y]_q = \left[x, D^R y\right]_q + \left[y, D^R x\right]_q \quad (4)$$

D^L is called the left derivation of Leibniz algebra q (Song et al. 2018), and D^R is the right derivation of q. All sets of left derivations and right derivations are denoted as $\mathrm{Der}^L(q)$ and $\mathrm{Der}^R(q)$, respectively.

Definition 5

Proof: through direct calculation, we can get:

$$\left[D^L x, y\right] = D^L[x,y] - \left[x, D^L y\right] \quad (5)$$

Lemma is True.

[Q.E.D]

Definition 2.2.3.

If $x \in q, D^L, D_1^L, D_2^L \in Der^L(q), \mathrm{D}^R \in Der^R(q)$, we can infer:

(1) $\left[D_1^L, D_2^L\right] \in Der^L(q)$;

(2) $\left[D^L, D^R\right] \in Der^R(q)$;

Proof: according to the definition of commutator, left derivation and right derivation(Ara et al. 2017), we can see that:

$$\begin{aligned}\left[D_1^L, D_2^L\right][x,y]_q &= D_1^L D_2^L [x,y]_q - D_2^L D_1^L [x,y]_q \\ &= D_1^L\left(\left[D_2^L x, y\right]_q + \left[x, D_2^L y\right]_q\right) - D_2^L\left(\left[D_1^L x, y\right]_q + \left[x, D_1^L y\right]_q\right) \\ &= \left[x, \left[D_1^L, D_2^L\right] y\right]_q + \left[\left[D_1^L, D_2^L\right] x, y\right]_q\end{aligned}$$

(6)

This shows that the inference (1) is valid.

Randomly:

$$\begin{aligned}&\left[D^L, ad_x^L\right](y) \\ &= D^L ad_x^L y - ad_x^L D^L y \\ &= \left[D^L x, y\right]_q = ad_{D^L x}^L(y)\end{aligned} \quad (7)$$

We can know $\left[D^L, ad_x^L\right] = ad_{D^L x}^L$, and:

$$\begin{aligned}\left[D^L, ad_x^R\right](y) &= D^L ad_x^R y - ad_x^R D^L y \\ &= D^L [y,x]_q - \left[D^L y, x\right]_q \\ &= ad_{D^L x}^R(y)\end{aligned}$$

(8)

We can get $\left[D^L, ad_x^R\right] = ad_{D^L x}^R$, and then,

$$\begin{aligned}\left[ad_x^L, D^R\right](y) &= ad_x^L D^R y - D^R ad_x^L y \\ &= \left[x, D^R y\right]_q - [x,y]_q \\ &= ad_{D^R x}^R(y)\end{aligned}$$

(9)

We can know that $\left[ad_x^L, D^R\right] = ad_{D^R x}^R$.

Definition 6

$Der^L(q)$ forms a Leibniz algebra under commutator brackets. ρ is defined as:

$$\rho\left(D^L\right)\left(D^R\right) = \left[D^L, D^R\right], \quad D^R \in Der^R(q)$$

(10)

$\left(D(q), [\cdot,\cdot]_s\right)$ is the Leibniz algebra of semidirect product of $Der^L(q)$ under the representation of $\left(Der^R(q), \rho, 0\right)$. Where, $D(q) = Der^L(q) \oplus Der^R(q)$.

3 Stable Automorphism Group of Splitting Extension

Suppose H represents a finite group, A is an H-module. A is an Abel group with operation "+", and it contains group homomorphism $\theta: H \to AutA$. $AutA$ is denoted as the automorphism group of A. Another equivalent expression: θ defines a mapping $H \times A \to A, (h,a) \to {}^h a$, where ${}^h a = \theta(h)(a)$. It satisfies ${}^h(a+a') = {}^h a + {}^h a'$, ${}^{(hh')}a = {}^h\left({}^{h'}a\right)$ and ${}^I a = a$. According to definition, the extension of group H through H-module A is the exact sequence $0 \to A \xrightarrow{i} G \xrightarrow{\pi} H \to 1$ of a group, so that the effect of H induced by it on A is θ. The extension is called splitting extension. If there is a group homomorphism $s: H \to G$, which makes

$\pi s = id_H$. Here, id_H denotes the identical endomorphism of *H*.

Let's assume that the above extension is a splitting extension. For simplicity, *A* is equal to $i(A) \le G$, and H is equal to $s(H) \le G$ (i and s can be regarded as the inclusion mapping). Thus, the element in G is uniquely written as $a \cdot h$, where, $a \in A$ and $h \in H$. The action of $h \in H$ on A is the conjugate action: ${}^h a = hah^{-1}$, $a \in A$. In this case, the group *G* is also called the semidirect product of *H* and *A*, which is denoted as $G = AQH$.

If $a \in AutG$, let $a(A) = A$. It is the following structure.

$$\begin{array}{ccccccccc} 0 & \to & A & \to & G & \to & H & \to & 1 \\ & & \downarrow & & a\downarrow & & \downarrow & & \\ 0 & \to & A & \to & G & \to & H & \to & 1 \end{array}$$

Then, *a* is called a stable automorphism of *G*. Obviously, the set of all stable automorphisms of *G* constitutes a group, which is denoted as

$$Aut(G)_A = \{a \in AutG \mid a(A) = A\} \quad (11)$$

If H-module *A* is known under the addition of mapping, there are the following Abel groups:

$$Der(H, A) = \{\delta : H \to A \mid \delta \text{ is the derivation}\}$$

(12)

This mapping is called derivations, and it satisfies the following conditions:

$$\delta(hh') = \delta(h) + {}^h\delta(h') \quad (13)$$

For any $h, h' \in H$, let $\beta \in AutH$, and the composite homomorphism $\beta\theta : H \xrightarrow{\beta} H \xrightarrow{\theta} \mathrm{Aut}A$. *A* is also called an *H*-module, which is denoted as A_β. It is used to distinguish from the original *HH*-module *A*. Let $\gamma : A \to A_\beta$ be an *H*-module isomorphism, that is to say, for any $a \in A$ and $h \in H$, it satisfies $\gamma({}^h a) = {}^{\beta(h)}\gamma(a)$.

Obviously, if $\gamma : A \to A_\beta$ is *H*-module isomorphism (Chen 2018), for any $\beta' \in AutH$ $\gamma : A_{\beta'} \to A_{\beta\beta'}$ is also the *H*-module isomorphism. Considering all ordered pairs (β, γ), let:

$$Pair(H, A) = \{(\beta, \gamma) \mid \beta \in AutH, \gamma : A \to A_\beta 1 \text{ is } H\text{-module isomorphism}\}$$

(14)

The $Pair(H, A)$ operation is defined as follows:

$$(\beta, \gamma)(\beta', \gamma') = (\beta\beta', \ \gamma\gamma') \quad (15)$$

For any $(\beta, \gamma), (\beta', \gamma') \in \mathrm{Pair}(H, A)$, it is easy to prove that $\gamma\gamma' : A \to A_{\beta\beta'}$ is *H*-module isomorphism and $\mathrm{Pair}(H, A)$ is a group.

The mapping is defined as follows:

$$\text{Pair}(H,A)\times Der(H,A)\to Der(H,A),((\beta,\gamma),\delta)\to\gamma\delta\beta^{-1}$$
(16)

Definition 7

We define the action of group $\text{Pair}(H,A)$ on an Abel group $Der(H,A)$ and make it a $\text{Pair}(H,A)$ -module.Meanwhile, there is group isomorphism:

$$Aut(G)_A\cong Der(H,A)\times \text{Pair}(H,A)$$
(17)

Let $a\in Aut(G)_A$. Based on the restriction, *a* clearly induces an automorphism γ_a of *A*, that is, for any $a\in A$, $a(a)=\gamma_a(a)$. On the other hand, *a* induces the automorphism $\beta_A\in Aut(H)$ of quotient group $H=G/A$. For any $h\in H$, $b\in A$ is true, and $a(h)=b\cdot\beta_a(h)$. Obviously, $a\to\beta_a$ is a group homomorphism from $Aut(G)_A$ to $AutH$. Then,

$$\gamma_a({}^ha)=a(hah^{-1})=a(h)\cdot a(a)\cdot a(h^{-1})$$
$$=(b\cdot\beta_a(h))\cdot\gamma_a(a)\cdot(\beta_a(h)^{-1}\cdot(-b))$$
$$={}^{\beta_a(h)}\gamma_a(a)$$
(18)

$\gamma_a:A\to A_{\beta_a}$ is an *H*-module isomorphism.

The following mapping is a group homomorphism

$$p:Aut(G)_A\to Pair(H,A),a\to(\beta_a,\gamma_a)$$
(19)

For any $(\beta,\gamma)\in Pair(H,A)$, the mapping $a_{\beta,\gamma}:G\to G$ is defined as $b\cdot\beta_a(h)$.

For any $a\in A,h\in H$, we can obtain $\gamma({}^ha)={}^{\beta(h)}\gamma(a)$. It is easy to verify that $a_{\beta,\gamma}$ is an automorphism of G and $\gamma:Pair(H,\text{A})\to Aut(G)_A,(\beta,\gamma)\to a_{\beta,\gamma}$ is a group homomorphism. And

$$p\lambda=id_{Pair(H,A)}\ (20)$$

Moreover, we calculate the Homomorphic kernel *Kerp* of the group homomorphism *p* (Figure eroa-O'farrell 2018). Obviously, if and only if $a\in Kerp$ and $\beta_a=ad_H$, this kind of *a* is called the self-equivalence of splitting extension *G* (i.e. semi direct product).

A derivation $\delta\in Der(H,A)$ exactly determines a self-equivalence a_δ. For any $a\in A,h\in H$, there is a mapping:

$$\iota:Der(H,A)\to Aut(G)_A,\delta\to A_\delta$$
(21)

An isomorphism in normal subgroup

formed by self-equivalence from derivation subgroup $Der(H,A)$ to $Aut(G)_A$ is given.

According to the above analysis, the exact sequence of a group can be obtained

$$0 \to Der(H,A) \xrightarrow{\iota} Pair(H,A) \to 1$$

(22)

It is known that Formula (25) is a split exact sequence.

Let the action of group $Pair(H,A)$ derived from exact sequences on Abel group $Der(H,A)$ be a group homomorphism.

$$\eta : Pair(H,A) \to Aut(Der(H,A))$$

(23)

Let's suppose that $\delta \in Der(H,A)$ and $(\beta,\gamma) \in Pair(H,A)$. For $a \in A$ and $h \in H$:

$$\begin{aligned}\iota(\eta(\beta,\gamma)(\delta))(a\cdot h) &= \gamma(\beta,\gamma)\iota(a)\lambda(\beta,\lambda)^{-1}(a\cdot h)\\ &= a_{\beta,\gamma}a_{\delta}a_{\beta,\gamma}^{-1}(a\cdot h)\\ &= \gamma\left(\left(\gamma^{-1}(a)+\delta\left(\beta^{-1}(h)\right)\right)\cdot\beta\left(\beta^{-1}(h)\right)\right)\\ &= \left(a+\left(\gamma\delta\beta^{-1}\right)(h)\right)\cdot h\end{aligned}$$

(24)

For any $h, h' \in H$, then:

$$\begin{aligned}\gamma\delta\beta^{-1}(hh') &= \gamma\left(\delta\left(\beta^{-1}(h)\beta^{-1}(h')\right)\right) = \gamma\left(\delta\beta^{-1}(h) + {}^{\beta-1}\delta\beta^{-1}(h')\right)\\ &= \gamma\left(\delta\beta^{-1}(h)\right) + {}^{\beta\beta^{-1}(h)}\gamma\left(\delta\beta^{-1}(h')\right) = \gamma\delta\beta^{-1}(h) + {}^{h}\gamma\delta\beta^{-1}(h')\end{aligned}$$

(25)

Therefore, $\gamma\delta\beta^{-1} \in Der(H,A)$. This shows that the above process completes the definition mapping, andthe above calculation results can be rewritten as follows:

$$\iota(\eta(\beta,\gamma)(\delta))(a\cdot h) = \iota\left(\gamma\delta\beta^{-1}\right)(a\cdot h)$$

(26)

i.e. $\eta(\beta,\gamma)(\delta) = \gamma\delta\beta^{-1}$. Therefore, the above process is the action η of group $Pair(H,A)$ on Abel group $Der(H,A)$ given by group extension a group and gives.

According to the above process, the semidirect product $Der(H,A) \times Pair(H,A)$ is constructed. The isomorphic mapping of the following groups is derived by splitting the exact sequence

$$Der(H,A) \times Pair(H,A) \xrightarrow{\cong} \mathrm{Aut}(G)_A, (\delta(\beta,\gamma)) \to a_\delta \cdot a_{\beta,\gamma}$$

(27)

[Q.E.D]

4 Non-AbelianCohomology Groups of Leibniz Algebras

Let $\dot{q}$ be a non-Abelian extension of q through h. σ is a split of $\dot{q}$. There is isomorphism $\dot{q} \cong q \oplus h$ in vector space. Considering the Leibniz algebraic structure on $q \oplus h$ (Maekawa& Miura 2018), we define linear mappingas follows:

$$w(x,y)=[\sigma(x),\sigma(y)]_q-\sigma[x,y]_q$$

(28)

For any $x,y\in g,\alpha,\beta\in h$, the bilinear mapping $[\cdot,\cdot]_{(l,r,w)}$ on $q\oplus h$ is defined as:

$$[x+\alpha,y+\beta]_{(l,r,w)}=w(x,y)_q+l_x\beta+r_y\alpha+[\alpha,\beta]_h$$

(29)

The following proposition transfers the structure on Leibniz algebra $\dot{q}$ into $q\oplus h$.

Definition 8

$(q\oplus h,[\cdot,\cdot]_{(l,r,w)})$ is a Leibniz algebra when l,r,w meet the conditions below:

$$l_x[\alpha,\beta]_h=[l_x\alpha,\beta]+[\alpha,l_x\beta]_h \quad (30)$$

$$r_x[\alpha,\beta]_h=[\alpha,l_x\beta]_h-[\beta,r_x\alpha]_h \quad (31)$$

$$[l_x\alpha+r_x\alpha,\beta]_h=0 \quad (32)$$

Suppose $(q\oplus h,[\cdot,\cdot]_{(l,r,w)})$ is a Leibniz algebra (Fleet et al. 2017).For any $x,y,z\in g,\alpha,\beta\in h$, according to:

$$\left[x,[\alpha,\beta]_{(l,r,w)}\right]=\left[\alpha,[x,\beta]_{(l,r,w)}\right]+\left[[x,\alpha]_{(l,r,w)},\beta\right]$$

(33)

$$l_xr_y-r_{[x,y]_q}+r_yr_x=ad^R_{w(x,y)} \quad (34)$$

Combined with Formula(34), we can see that Formula(34)is workable.

On the other hand, if Formula (30) - Formula (34) are true, $(q\oplus h,[\cdot,\cdot]_{(l,r,w)})$ is a Leibniz algebra.

Note: Formula (33) shows that $l_x\in Der^L(h)$. Formula (34) shows that $r_x\in Der^R(h)$.

Definition 9

(1) If a linear mapping φ form q to h satisfies:

$$l^1_x-l^2_x=ad^L_{\varphi(x)} \quad (35)$$

$$r^1_x-r^2_x=ad^R_{\varphi(x)} \quad (36)$$

$$w^1(x,y)-w^2(x,y)=l^2_x\varphi(y)+r^2_y\varphi(x)+[\varphi(x),\varphi(y)]_h-\varphi[x,y]_q$$

(37)

Then, (l^1,r^1,w^1) and (l^2,r^2,w^2) are equivalent.

(2) The quotient set of 2-closed chain $Z^2(g,h)$ under above equivalence relations is called the second-order non-Abeliancohomology group where q takes value form h. It is denoted as $H^2(g,h)$.

5 Non-Abelian Extension of Leibniz Algebra and Second-Order Non-AbelianCohomology Group

Definition 10

Proof: Suppose $\dot{q}$ is a non-Abelian extension of q through h. $\sigma_1:q\to\dot{q}$ is a split of $\dot{q}$. Then, we can get a 2-closed chain$\left(l^1, r^1, w^1\right)$. Firstly, we prove that the cohomology class that takes $\left(l^1, r^1, w^1\right)$ as the representative does not depend on the splitting selection. In fact, we can take two different splits, σ_1 and σ_2, and then define $\varphi : g \to h$ as $\varphi(x) := \sigma_1(x) - \sigma_2(x)$. According to $l_x^j(\beta) = \left[\sigma_j(x), \beta\right]_{\dot{q}}$, $r_y^j(\alpha) = \left[\alpha, \sigma_j(y)\right]_{\dot{q}}$ and $j = 1, 2$, we can get the following results:

$$\begin{aligned} w^1(x,y) &= \left[\sigma_1(x), \sigma_1(y)\right]_{\dot{q}} - \sigma_1[x,y]_q \\ &= \left[\sigma_2(x) + \varphi(x), \sigma_2(y) + \varphi(y)\right]_{\dot{q}} - \sigma_2[x,y]_q \\ &= l_x^2\varphi(y) + r_y^2\varphi(x) + \left[\varphi(x), \varphi(y)\right]_h \end{aligned}$$

(38)

This shows that $\left(l^1, r^1, w^1\right)$ and $\left(l^2, r^2, w^2\right)$ are in the same homology class.

Next, we can prove that isomorphic extension corresponds to the same element in $H^2(g,h)$. Let's assume that $\dot{q}_1$ and $\dot{q}_2$ are a non-Abelian extension of isomorphism of q through h. $\theta : \dot{q}_2 \to \dot{q}_1$ is a Leibniz algebraic homomorphism. If $\sigma_2 : q \to \dot{q}_2$ and $\sigma_1 : q \to \dot{q}_1$ are two splits, we can define $\sigma_2' : q \to \dot{q}_2$ as $\sigma_2' = \theta^{-1} \circ \sigma_1$. $P_1 \circ \theta = P_2$, so:

$$P_2 \circ \sigma_2'(x) = \left(P_1 \circ \theta\right) \circ \theta^{-1} \circ \sigma_1(x) = x$$

(39)

Therefore, σ_2' is a split of $\dot{q}_2$. $\varphi_\theta : q \to h$ is defined as $\varphi_\theta(x) = \sigma_2'(x) - \sigma_2(x)$. For any $x \in q, \alpha \in h$, $\theta^{-1} : q_1 \to q_2$ is the homomorphism of Leibniz algebras, so:

$$\begin{aligned} &\theta^{-1}\left[\sigma_1(x,\alpha)\right]_{\dot{q}_1} \\ &= \left[\theta^{-1}\sigma_1(x), \alpha\right]_{\dot{q}_2} \quad (40) \\ &= l_x^2\alpha + ad^L_{\varphi_\theta(x)}\alpha \end{aligned}$$

On the other hand, $\theta|_h = id$, so:

$$\theta^{-1}\left[\sigma_1(x), \alpha\right]_{\dot{q}_1} = l_x^1\alpha \quad (41)$$

Therefore:

$$l_x^1 - l_x^2 = ad^L_{\varphi\theta(x)} \quad (42)$$

Similarly, we can get:

$$r_x^1 - r_x^2 = ad^R_{\varphi\theta(x)} \quad (43)$$

For all $x, y \in q$, on the one hand:

$$\begin{aligned} &\theta^{-1}\left[\sigma_1(x), \sigma_1(y)\right]_{\dot{q}_1} \\ &= \left[\theta^{-1}\sigma_1(x), \theta^{-1}\sigma_1(y)\right]_{\dot{q}_2} \\ &= \left[\varphi_\theta(x), \varphi_\theta(y)\right]_h + l_x^2\varphi_\theta(y) + r_y^2\varphi_\theta(x) + \sigma_2[x,y]_q + w^2 \end{aligned}$$

(44)

On the other hand:

$$\begin{aligned} \theta^{-1}\left[\sigma_1(x), \sigma_1(y)\right]_{\dot{q}_1} &= \theta^{-1}\left(\sigma_1[x,y]_q + w^1(x,y)\right) \\ &= \varphi_\theta[x,y]_q + \sigma_2[x,y]_q + w^1(x,y) \end{aligned}$$

(45)

Therefore:

$$\left(w^1 - w^2\right)\left(x, y\right) = l_x^2 \varphi_\theta\left(y\right) + r_y^2 \varphi_\theta\left(x\right)$$

(46)

If $\varphi : q \to h$, Formula (40) – Formula (46) are workable(Zhang et al. 2017). Next, we need to prove that the expansion $\left(q \oplus h, [\cdot,\cdot]_{\left(l^1, r^1, w^1\right)}\right)$ and expansion $\left(q \oplus h, [\cdot,\cdot]_{\left(l^2, r^2, w^2\right)}\right)$ are isomorphic. $\theta : q \oplus h \to \dot{q} \oplus h$ is defined as:

$$\theta\left(x + \alpha\right) = x - \varphi\left(x\right) + \alpha \quad (47)$$

θ has the following exchange forms.

$$\begin{array}{ccccccccc} 0 & \to & h & \overset{i_2}{\to} & q \oplus h_{\left(l^2, r^2, w^2\right)} & \overset{p_2}{\to} & q & \to & 0 \\ & & \| & & \theta \downarrow & & \| & & \\ 0 & \to & h & \overset{i_1}{\to} & q \oplus h_{\left(l^1, r^1, w^1\right)} & \overset{P_1}{\to} & q & \to & 0 \end{array}$$

(48)

Q.E.D]

6 Conclusions

We verify the isomorphism relationship of Lie 2-algebras formed by Leibniz 2-algebra $\left(h, \Pi\left(h\right), \left(ad^L, ad^R\right), l_2\right)$ and Lie algebra derivations viareducing h to Lie algebra, which indicates that it is a natural extension of Lie 2-algebras formed by derivation of Lie algebras.The non-Abelian extensions of Leibniz algebras whose centers meet certain conditions can be described by limiting the central condition.A differential graded Lie algebra $\left(L, [\cdot,\cdot]_C, \bar{\partial}\right)$ is constructed, and the equivalence between Maurer Cartan element of $\left(L, [\cdot,\cdot]_C, \bar{\partial}\right)$ and q is proved.

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