Solving Fuzzy Error Matrix Equation based on Runge Kutta Method

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Abstract: Fuzzy error logic represents the object in the real world with (u,x) as $\{[U,S(t),\vec{p},T(t),L(t)],[x(t)=f(u(t),p),Gu(t)]\}$, Fuzzy error transformation matrix can be used to express six transformation methods, such as decomposition, similarity, addition, replacement, destruction and unit transformation. Based on solving equation XA=B and decomposition of p, this paper studies the solution of error matrix equation based on Runge Kutta method, in order to explore the law of error transformation from the perspective of solving matrix equation.

Keywords: Runge—Kutta method; Fuzzy error matrix; Matrix equation solution; Decomposition; Similarity
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1 Introduction

In order to study the occurrence mechanism and transformation mode and law of errors in economy and management, this paper studies a mathematical tool, fuzzy error matrix equation, which quantitatively describes these errors and their laws, and explores the method of using fuzzy error matrix equation to express the logic transformation of errors [1, 2]. In this paper, the fuzzy error matrix equation is used to represent the error logic transformation, and the solution of the fuzzy error matrix equation is explored to obtain a transformation method based on eliminating the current state A error and transforming to the expected state B [3, 4].

According to the error logic, we can describe the object in the real world, that is, the concrete object in the real world is expressed as A, and the element (u,x) in A can be expressed as $\{[U,S(t),\vec{p},T(t),L(t)],[x(t)=f(u(t),p),Gu(t)]\}$. U is the universe, $S(t)$ is the thing described, p is the current space, $T(t)$

is the feature, $L(t)$ is the quantity, $x(t)=f(u(t),p),Gu(t)$ and $x(t)$ are error functions, $Gu(t)$ is the rule [5, 6]. The error transformation matrix can be used to express six transformation methods: decomposition, similarity, addition, replacement, destruction and unit transformation, so that we can realize reasoning with the help of error transformation matrix. In other words, all parameters of u are reasoned with six transformation methods. Such as the decomposition and transformation of things, the decomposition and transformation of space similarity transformation, the decomposition and transformation of rules and the similarity transformation of rules. The error matrix can be divided into two types, one is general matrix multiplication, the other is superior, the other is inferior, the other is and. In this paper, we discuss the general matrix multiplication equation [7] Based on space decomposition transformation.

2 Definition of fuzzy error matrix

2.1 Definition

Hypothesis

$$A=\begin{bmatrix} (u_{111},u_{112},...,u_{11k}),x_{11} \\ (u_{211},u_{212},...,u_{21k}),x_{21} \\ ... \quad ... \quad ... \quad ... \quad ...| \\ (u_{m11},u_{m12},...,u_{m1k}),x_{m1} \end{bmatrix} \tag{1}$$

When the range h of error function is $R_{an}(f)=[0,1]$, A is called k-ary fuzzy error matrix of order $m\times n$.

When we only study the unitary object in A, each element is (u,x), u is called object, including five parameters, namely U、$S(t)$、p、$T(t)$、$L(t)$; x contains two parameters, $x(t)$ and $G_U(t)$, so the element (u,x) in A can be represented as $\{[U,S(t),\vec{p},T(t),L(t)],[x(t)=f(u(t),p),Gu(t)]\}$. U is the universe, $S(t)$ is the thing described, p is the current space, $T(t)$ is the feature, $L(t)$ is the quantity, $[x(t)=f(u(t),p),Gu(t)]$ is the fuzzy error function, $Gu(t)$ is the rule [8]. The formula is as follows.

$$A=\begin{bmatrix} u_{10}\ x_{10} \\ u_{11}\ x_{11} \\ ...\ ... \\ u_{1t}\ u_{1t} \end{bmatrix}=\begin{bmatrix} u_{10} & s_{10}(t) & p_{10} & T_{10}(t) & L_{10}(t) & x_{10}(t) & G_{U10}(t) \\ u_{11} & s_{11}(t) & p_{11} & T_{11}(t) & L_{11}(t) & x_{11}(t) & G_{U11}(t) \\ ... & ... & ... & ... & ... & ... & ... \\ u_{1t} & s_{10}(t) & p_{1t} & T_{1t}(t) & L_{1t}(t) & x_{1t}(t) & G_{U1t}(t) \end{bmatrix} \tag{2}$$

It is called $(t+1)\times 7$ -fuzzy error matrix.

Based on this, we can describe the current state with errors as u, and describe the expected state as u_1. by studying the form of equation $T(u)=u_1$, we can get T, T which can define six logical transformation modes: decomposition, similarity, increase, replacement, destruction and unit transformation. After solving the equation, the transition law or method from current u to expected state u_1 can be obtained. The form of the equation is given below [9].

2.2 Fuzzy error matrix equation

According to $T(u)=u_1$, after knowing the fuzzy error matrix described by the current state and the fuzzy error matrix of the expected state, the equation can be determined and solved to obtain T, that is, the transformation scheme, and T is also expressed by the error matrix [10]. The error matrix equation is described in two categories based on the form of five operators, as shown in Table 1.

Table 1 Types of fuzzy error matrix equations

	1	2	3	5	6
Class A	AX=B	A*X=B	A×X=B	A∨X=B	A∧X=B
Class B	XA=B	X*A=B	X×A=B	X∨A=B	X∧A=B
Operator meaning	General matrix multiplication	Excellent	Inferior	Or	And

There are three limitations in solving matrix equation (which should be the final limit after the solution is finished)

(1) The limit of objective conditions is kg;

(2) Artificial restriction RW;

(3) The limitation of demand XQ. In general, the elements in the matrix take values on a certain set; the three constraints can also be expressed by a certain set. When solving the matrix equation, the solution is generally on a certain set [11, 12]. Therefore, the solution satisfying the three restrictions of matrix equation can be obtained by intersection of the result set of matrix equation and the set of three restrictions of matrix equation [13, 14].

3 Solving fuzzy error matrix equation based on Runge Kutta method

3.1 Theorem

Assuming that μ is a non multiple measure, relevant scholars have obtained a conclusion that $\left[L^p, L^q\right]$ of $\left[b, I_\alpha\right]$ is bounded through a series of studies. Therefore, this conclusion can be extended to the following fuzzy error matrix equation.

Theorem 1

Suppose a is μ measure, which satisfies the $\|\mu\|=\infty$ condition, and set $0<\alpha<2n$、$b_j \grave{o} RBMO\left(R^n\right)$、$j=1,2$. Thus, it is determined that the bounded operators from $L^{q1}\times L^{q2}$ to L^q can be represented by $\left[b_1, b_2, I_{\alpha,2}\right]$, where $1/q=1/q_1+1/q_2-\alpha/2n>0$, and $1<q_{1,}q_2<\infty$.

Theorem 2

Suppose $m\grave{o}N$, μ denote the measure. Under the condition of $\|\mu\|=\infty$, set $0<\alpha<mn$ 、 $b_j \in RBMO\left(R^n\right)$ 、 $j=1,2,\ldots,m$. The inequality relation described in formula (1) is obtained.

$$\left\|\left[\vec{b}, I_{\alpha,m}\right]\left(\vec{f}\right)\right\|_{L^q(\mu)} \leq C \tag{3}$$

In the inequality described by formula (3), $1/q=1/q_1+1/q_2+\ldots+1/q_m-\alpha/mn>0$, and $1<q_j<\infty, j=1,2,\ldots,m$.

3.2 Theorem proving

Since theorem 1 and theorem 2 can be verified by the same method, only the verification process of Theorem 1 needs to be described in detail in the following theorem verification process, and theorem 2 can be verified by the same method without detailed description [15].

Theorem 3

Suppose $1\leq p<\infty$, and $1<\rho<\infty$, we can obtain $b \in RBMO(\mu)$. For all cube $Q\subset R^n$ and all double cube $Q\subset R$, where Q and R represent arbitrary cube and double cube respectively, the inequality described in formula (4) and formula (5) is obtained

$$\frac{1}{\mu(\rho Q)}\int_Q\left|b(x)-m_{\tilde{Q}}(b)\right|^p d\mu(x)\leq C\|b\|_*^p \tag{4}$$

$$\left|m_Q(b)-m_R(b)\right|\leq CK_{Q,R}\|b\| \tag{5}$$

Lemma 1

Suppose $f\in L^1_{loc}(\mu)$, $\int fd\mu=0$, if $\|\mu\|<\infty$ get $1<p<\infty$, if $\inf(1,Nf)\in L^p(\mu)$, get $0\leq\beta<n$, based on the above description, we get the inequality described in formula (6).

$$\|Nf\|_{L^p(\mu)}\leq C\left\|M^{\#,(\beta)}f\right\|_{L^p(\mu)} \tag{6}$$

Lemma 2

Suppose that $p<r<n/\alpha$, and $1/q=1/r-\alpha/n$, the inequality described in formula (7) is obtained.

$$\left\|M^{(\alpha)}_{p,(\eta)}f\right\|_{L^p(\mu)}\leq C\|f\|_{L^r(\mu)} \tag{7}$$

In formula (7), $\eta>1$, $0\leq\alpha<\eta/p$.

Lemma 3

Let μ denote the measure and let $m \in N$, $1/s=1/r_1+\ldots+1/r_m-\alpha/n>0$, $0<\alpha<mn$, $1\leq r_j\leq\infty$. The following two cases are obtained.

(1) When all r_j are greater than 1, the inequality described in formula (8) is generated.

$$\left\|I_{\alpha,m}\left(f_1,\ldots,f_m\right)\right\|_{L^s(\mu)}\leq C\prod_{j=1}^m\|f_j\|_{L^{r_j}(\mu)} \tag{8}$$

(2) If there is a certain j where r_j is equal to 1, the inequality described in formula (9) is generated.

$$\left\|I_{\alpha,m}\left(f_1,\ldots,f_m\right)\right\|_{L^{s,\infty}(\mu)}\leq C\prod_{j=1}^m\|f_j\|_{L^{r_j}(\mu)} \tag{9}$$

Lemma 4

In $\left[b_1, b_2, I_{\alpha,2}\right]$, if $0<\alpha<2n$ 、 $\tau>1$ 、 b_1 and b_2 are all $\grave{o}RBMO(\mu)$, then there is a constant $C>0$. For all $x\grave{o}\mathbf{N}$ and $f_1\grave{o}L^{q1}(\mu)$ 、 $f_2\grave{o}L^{q2}(\mu)$, there is an inequality relationship described in formula (10).

$$M^{\#,(\alpha)}=\left(\left[b_1, b_2, I_{\alpha,2}\right](f_1, f_2)\right)(x) \tag{10}$$

In the inequality relationship described by the above formula:

$$\left[b_1, I_{\alpha,2}\right](f_1, f_2)(x)=b_1(x) I_{\alpha,2}(f_1, f_2)(x)-I_{\alpha,2}(b_1 f_1, f_2)(x) \tag{11}$$

$$\left[b_2, I_{\alpha,2}\right](f_1, f_2)(x)=b_2(x) I_{\alpha,2}(f_1, f_2)(x)-I_{\alpha,2}(f_1, b_2 f_2)(x) \tag{12}$$

Verification: according to the definition, in order to determine lemma 4, it is only necessary to confirm that all $x\grave{o}R^n$ and the cube $Q \ni x$ conform to formula (13).

$$\begin{aligned}\frac{1}{\mu\left(\frac{3}{2}Q\right)}\int_Q\left|\left[b_1, b_2, I_{\alpha,2}\right](f)(z)-h_Q\right| d\mu(z) \le C&\Big[\|b_1\|_*\|b_2\|_* M_{T(3/2)}\left(I_{\alpha,2}(f_1, f_2)\right)(x)\\
&+\|b_1\|_* M_{T(3/2)}\left(\left[b_2, I_{\alpha,2}\right](f_1, f_2)\right)(x)\\
&+\|b_2\|_* M_{T(3/2)}\left(\left[b_1, I_{\alpha,2}\right](f_1, f_2)\right)(x)\\
&+\|b_1\|_*\|b_2\|_* M_{p_1(9/8)}^{(\alpha)} f_1(x) M_{p_2(9/8)}^{(\alpha)} f_2(x)\Big]\end{aligned} \tag{13}$$

At the same time, for any cube Q, there is an inequality relationship described by formula (14).

$$\begin{aligned}\left|h_Q-h_R\right| \le C K_{Q,R}^2 K_{Q,R}^{(\alpha)}&\Big[\|b_1\|_*\|b_2\|_* M_{T(3/2)}\left(I_{\alpha,2}(f_1, f_2)\right)(x)\\
&+\|b_1\|_* M_{T(3/2)}\left(\left[b_2, I_{\alpha,2}\right](f_1, f_2)\right)(x)\\
&+\|b_2\|_* M_{T(3/2)}\left(\left[b_1, I_{\alpha,2}\right](f_1, f_2)\right)(x)\\
&+\|b_1\|_*\|b_2\|_* M_{p_1(9/8)}^{(\alpha)} f_1(x) M_{p_2(9/8)}^{(\alpha)} f_2(x)\Big]\end{aligned} \tag{14}$$

Formula (14) describes the inequality relationship.

$$h_Q=m_Q\left(I_{\alpha,2}\left(\left(m_{\tilde{Q}}(b_1)-b_1\right) f_1\chi_{R^n \setminus \frac{4}{3}Q},\left(m_{\tilde{Q}}(b_2)-b_2\right) f_2\chi_{R^n \setminus \frac{4}{3}Q}\right)\right) \tag{15}$$

$$h_R=m_R\left(I_{\alpha,2}\left(\left(m_{\tilde{R}}(b_1)-b_1\right) f_1\chi_{R^n \setminus \frac{4}{3}R},\left(m_{\tilde{R}}(b_2)-b_2\right) f_2\chi_{R^n \setminus \frac{4}{3}R}\right)\right) \tag{16}$$

Based on the above formula (15) and formula (16), the inequality described in formula (17) can be obtained.

According to Herder's inequality and lemma 2, I value, II value and III value, the value of D can be defined as:

$$\left|IV(z)\right|=IV_1(z)+IV_2(z)+IV_3(z)+IV_4(z) \tag{17}$$

According to Herder's inequality and lemma 4, $IV_1(z)$ value can be determined; $IV_2(z)$ value can be determined based on lemma 2; $IV_3(z)$ value and $IV_4(z)$ value can be determined based on $IV_1(z)$ value and $IV_2(z)$ value calculation method.

Based on formula (17) and the calculation process of I value, II value,

III value and IV value, for any cube Q, At the same time, $x \in Q$, where Q and R denote any cube and double cube respectively, N is used to replace $N_{Q,R+1}$ to simplify the calculation process.

$$\left|h_Q - h_R\right| = \left|m_Q\left[I_{\alpha,2}\left(b_1 - m_Q b_1\right) f_1^{\infty}, \left(b_2 - m_Q b_2\right) f_2^{\infty}\right] - m_R\left[I_{\alpha,2}\left(b_1 - m_R b_1\right) f_1^{\infty}, \left(b_2 - m_R b_2\right) f_2^{\infty}\right]\right| = \sum_{i=1}^{6} A_i \tag{18}$$

Similar to the calculation process of IV_4 value, the inequality described in formula (19) can be determined by calculation.

$$A_1 \le C\left[K_{Q,R}\right]^2 \left\|b_1\right\|_* \left\|b_2\right\|_* M^{(\alpha)}_{p_1,\left(\frac{9}{8}\right)} f_2(x) \tag{19}$$

After A1 value is calculated by formula (19), it is changed in order to calculate A_2 value, thus the equation relationship shown in formula (20) is obtained.

$$I_{\alpha,2} = \left(\left(b_1 - m_R b_1\right) f_1 \chi_{\frac{R^n}{2^N Q}}, \left(b_2 - m_R\left(b_2\right)\right) f_2 \chi_{\frac{R^n}{2^N Q}}\right)(z) \tag{20}$$

According to the change of equation in formula (20), the calculation formula of A_2 value can be obtained, as shown in formula (21).

$$\begin{aligned} A_2 &\le \left|m_R\left(b_2\right) - m_Q\left(b_2\right)\right| \times \left|\frac{1}{\mu(R)} \int_R I_{\alpha,2}\left(\left(b_1 - m_R\left(b_1\right)\right) f_1 \chi_{\frac{R^n}{2^N Q}}, f_2 \chi_{\frac{R^n}{2^N Q}}\right)(z)\, d\mu(z)\right| \\ &= A_{21} + A_{22} + A_{23} \end{aligned} \tag{21}$$

In formula (21), A_{23} can be described by the inequality in formula (22).

$$A_{23} \le CK_{Q,R}^2 \left\|b_1\right\|_* \left\|b_2\right\|_* M_{\tau,\left(\frac{3}{2}\right)}\left(I_{\alpha,2}\left(f_1, f_2\right)\right)(x) \tag{22}$$

In order to obtain the value of A_{21}, the equation relationship in formula (23) is obtained by transforming it.

$$I_{\alpha,2}\left(\left(b_1 - m_R b_1\right) f_1 \chi_{\frac{R^n}{2^N Q}}, f_2 \chi_{\frac{R^n}{2^N Q}}\right)(z) = \sum_{j=1}^{7} B_j(z) \tag{23}$$

In order to solve $B_1(z)$, it can be decomposed, and the inequality in formula (24) is obtained

$$\left|I_{\alpha,2}\left(\left(b_1 - m_R b_1\right) f_1, f_2\right)(z)\right| \le \left|I_{\alpha,2}\left(\left(b_1 - b_1(z)\right) f_1, f_2(z)\right)\right| + \left|I_{\alpha,2}\left(\left(b_1(z) - m_R b_1\right) f_1, f_2(z)\right)\right| \tag{24}$$

According to the principle of Herder's inequality and the basic fact that R represents a double cube, two conclusions can be drawn, which are described by the inequality relationship, which are described by formula (25) and formula (26), respectively.

$$\frac{1}{\mu(R)} \int_R I_{\alpha,2}\left(\left(b_1 - m_R b_1\right) f_1, f_2\right)(z)\, d\mu(z) \le C\left\|b_1\right\|_* M_{\tau,\left(\frac{3}{2}\right)}\left(I_{\alpha,2}\left(f_1, f_2\right)\right)(x) \tag{25}$$

$$\frac{1}{\mu(R)} \int_R I_{\alpha,2}\left(\left(b_1 - b_1(z)\right) f_1, f_2\right)(z)\, d\mu(z) \le CM_{\tau,\left(\frac{3}{2}\right)}\left(\left[b_1, I_{\alpha,2}\right]\left(f_1, f_2\right)\right)(x) \tag{26}$$

Based on the inequality described in formula (25) and formula (26), the inequality relationship in formula (27) is obtained.

$$\left|m_R B_1\right| \le C\left(\left\|b_1\right\|_* M_{\tau,\left(\frac{3}{2}\right)}\left(I_{\alpha,2}\left(f_1, f_2\right)\right)(x) + M_{\tau,\left(\frac{3}{2}\right)}\left(\left[b_1, I_{\alpha,2}\right]\left(f_1, f_2\right)\right)(x)\right)$$

(27)

In order to calculate the value of $B_2(z)$, s_1 and s_2 are equal to $\sqrt{p_1}$ and p_2, $1/v=1/s_1+1/s_2-\alpha/n$, respectively. Under this condition, based on Herder's inequality principle and lemma 4, the inequality relation described in formula (28) is obtained.

$$\frac{1}{\mu(R)}=\int_R I_{\alpha,2}B_2 d\mu(z) \tag{28}$$

According to formula (29), the inequality described in formula (29) is obtained.

$$|m_R B_2| \le C\|b_1\|_* M^{(n)}_{p_1,\left(\frac{9}{8}\right)} f_1(x) M^{(n)}_{p_1,\left(\frac{9}{8}\right)} f_2(x) \tag{29}$$

Based on the same principle, the inequality relations described in formula (30) and formula (31) are obtained.

$$|m_R B_3| \le C\|b_1\|_* M^{(\alpha)}_{p_1,\left(\frac{9}{8}\right)} f_1(x) M^{(\alpha)}_{p_2,\left(\frac{9}{8}\right)} f_2(x) \tag{30}$$

$$|m_R B_4| \le C\|b_1\|_* M^{(\alpha)}_{p_1,\left(\frac{9}{8}\right)} f_1(x) M^{(\alpha)}_{p_2,\left(\frac{9}{8}\right)} f_2(x) \tag{31}$$

In the process of solving $B_5(z)$, due to $z\grave{o}R$, the inequality described in formula (32) is obtained.

$$|B_5(z)| \le \int_{2^N Q}\int_{R^N \setminus \frac{4}{3}Q} \frac{|(b_1-m_R(b_1))f_1||f_2(y_2)|}{|(z-y_1,z-y_2)|^{2n-\alpha}} d\mu(y_1) \tag{32}$$

The average value of z is calculated by formula (33) on R.

$$|m_R B_5| \le CK^{(\alpha)}_{Q,R}\|b_1\|_* M^{(\alpha)}_{p_1,\left(\frac{9}{8}\right)} f_1(x) M^{(\alpha)}_{p_2,\left(\frac{9}{8}\right)} f_2(x) \tag{33}$$

Based on the same principle, the inequality relations described in formula (34) and formula (35) are obtained.

$$|m_R B_6| \le CK^{(\alpha)}_{Q,R}\|b_1\|_* M^{(\alpha)}_{p_1,\left(\frac{9}{8}\right)} f_1(x) M^{(\alpha)}_{p_2,\left(\frac{9}{8}\right)} f_2(x) \tag{34}$$

$$|m_R B_7| \le CK^{(\alpha)}_{Q,R}\|b_1\|_* M^{(\alpha)}_{p_1,\left(\frac{9}{8}\right)} f_1(x) M^{(\alpha)}_{p_2,\left(\frac{9}{8}\right)} f_2(x) \tag{35}$$

A_{21} value is obtained by combining formula (25) — formula (35), as shown by inequality in formula (36).

$$A_{21} \le CK_{Q,R}K^{(\alpha)}_{Q,R}\left[\|b_1\|_*\|b_2\|_* M_{\tau,\left(\frac{3}{2}\right)}\left(I_{\alpha,2}(f_1,f_2)\right)(x)\right. \tag{36}$$

According to the same calculation principle, the value of A_{22} is determined, as shown in the inequality in formula (37).

$$\begin{aligned} A_{22} \le CK_{Q,R}K^{(\alpha)}_{Q,R}\Big[&\|b_1\|*\|b_2\|*M_{\tau,\left(\frac{3}{2}\right)}\left(I_{\alpha,2}(f_1,f_2)\right)(x) \\ &+\|b_1\|*M_{\tau,\left(\frac{3}{2}\right)}\left(\left[b_2,I_{\alpha,2}\right](f_1,f_2)\right)(x) \\ &+\|b_1\|*\|b_2\|*M_{p_1,\left(\frac{9}{8}\right)} f_1(x) M_{p_2,\left(\frac{9}{8}\right)} f_2(x)\Big] \end{aligned} \tag{37}$$

According to the inequality in formula (36) and formula (37), A_2 value as shown in formula (38) can be obtained.

$$A_2 \le CK^2_{Q,R}K^{(\alpha)}_{Q,R}\left[\|b_1\|_*\|b_2\|_* M_{\tau,\left(\frac{3}{2}\right)}\left(I_{\alpha,2}(f_1,f_2)\right)(x)\right. \tag{38}$$

Based on the demonstration process of $B_5(z)$ in formula (32), the inequality relationship in formula (39) is obtained.

$$A_3 + A_4 + A_5 + A_6 \le CK_{Q,R}^2 K_{Q,R}^{(\alpha)} \|b_1\|_* \|b_2\|_* M_{p_1,\left(\frac{9}{8}\right)}^{(\alpha)} f_1(x) M_{p_2,\left(\frac{9}{8}\right)}^{(\alpha)} f_2(x) \tag{39}$$

Based on formula (39), the inequality shown in formula (40) is obtained.

$$\left|h_Q - h_R\right| \le CK_{Q,R}^2 \left[\|b_1\|_* \|b_2\|_* M_{\tau,\left(\frac{3}{2}\right)} \left(I_{\alpha,2}(f_1, f_2)\right)(x) \right. \tag{40}$$

After the derivation of the above formula, assuming that Q is a double cube and $x \grave{o} Q$ exists, the inequality relationship in formula (41) can be obtained.

$$\left| m_Q = \left(\left[b_{1,}b_2, I_{\alpha,2}\right](f_1, f_2)\right) - h_Q \right| \tag{41}$$

Similarly, for any cube Q and the existence of $x \grave{o} Q$, $K_{Q,\tilde{Q}} \le C$ and $K_{Q,\tilde{Q}} \le C$, the inequality described in formula (42) is obtained.

$$\frac{1}{\mu\left(\frac{3}{2}Q\right)} = \int_Q \left| \left[b_1, b_2, I_{\alpha,2}\right](f_1, f_2)(z) - m_{\tilde{Q}} \left[b_1, b_2, I_{\alpha,2}\right](f_1, f_2) \right| d\mu(z) \tag{42}$$

From another point of view, for all the dipoles $Q \subset R$, the inequality described in formula (43) is obtained under the condition that $x \grave{o} Q$ exists at the same time and satisfies $K_{Q,R}^{(\alpha)} \le P'_\alpha$ (where P'_α is the self defined constant).

$$\begin{aligned}\left|h_Q - h_R\right| \le CK_{Q,R}^2 P'_\alpha \Bigg[& \|b_1\|_* \|b_2\|_* M_{\tau,\left(\frac{3}{2}\right)} \left(I_{\alpha,2}(f_1, f_2)\right)(x) \\ & + \|b_1\|_* M_{\tau,\left(\frac{3}{2}\right)} \left(\left[b_2, I_{\alpha,2}\right](f_1, f_2)\right)(x) \\ & + \|b_2\|_* M_{\tau,\left(\frac{3}{2}\right)} \left(\left[b_1, I_{\alpha,2}\right](f_1, f_2)\right)(x) \\ & + \|b_1\|_* \|b_2\|_* M_{p_1,\left(\frac{9}{8}\right)} f_1(x) M_{p_2,\left(\frac{9}{8}\right)} f_2(x) \Bigg]\end{aligned} \tag{43}$$

Moreover, the inequality described in formula (44) can be obtained.

$$\left|h_Q - h_R\right| = CK_{Q,R}^{(\alpha)} \left[\|b_1\|_* \|b_2\|_* M_{\tau,\left(\frac{3}{2}\right)} \left(I_{\alpha,2}(f_1, f_2)\right)(x) \right. \tag{44}$$

For all the dipoles s and $Q \subset R$, according to the inequality described in formula (42), formula (45) is obtained.

$$\begin{aligned}&\left| m_Q \left(\left[b_{1,}b_2, I_{\alpha,2}\right](f_1, f_2)\right) - m_R \left(\left[b_{1,}b_2, I_{\alpha,2}\right](f_1, f_2)\right) \right| \\ &\le \left| m_Q \left(\left[b_{1,}b_2, I_{\alpha,2}\right](f_1, f_2)\right) - h_Q \right|\end{aligned} \tag{45}$$

Through the above description and sharp maximum function theory, the argument of Lemma 3 is realized. The results of lemma 4 and lemma 5 can also be obtained in the same way. Due to the influence of the length of the paper and the research time, the process of the round argument of lemma 4 and lemma 5 is not detailed.

3.3 Main conclusions

Theorem 3

Under the conditions of $w \grave{o} A_p(\mu)$, $b \grave{o} RBMO(\mu)$, $f \grave{o} L_{loc}^1(\mu)$, $\|\mu\| < \infty$, make $1 < p < n/\alpha$, the inequality described in formula (46) is obtained.

$$\int_{R^d}\left|[b,I_\alpha]f\right|^p w(x)d\mu(x)\le C\int_{R^d}\left|f(x)\right|^p w(x)d\mu(x)$$

(46)

Verification of Theorem 3

The Sharp calculation of fractional integral operator is carried out, and the inequality described in formula (47) is obtained.

$$M^{\#,(\alpha)}\left(I_{\alpha,b}f\right)(x)\le C\|b\|*\left[M_{r,\left(\frac{3}{2}\right)}\left(I_\alpha f\right)(x)+M^{(\alpha)}_{r,\left(\frac{3}{2}\right)}f(x)+I_\alpha\left(|f|\right)(x)\right]$$

(47)

In the process of calculating formula (47), it is necessary to clarify the inequality relationship described by formula (54) between all $x\grave{o}R^d$ and all cubes Q covering x.

$$\frac{1}{\mu\left(\frac{3}{2}Q\right)}\int_Q\left|I_{\alpha,b}f(y)-h_Q\right|d\mu(y)\le C\|b\|*\left[M_{r,\left(\frac{3}{2}\right)}\left(I_\alpha f\right)(x)+M_{r,\left(\frac{9}{8}\right)}f(x)\right]$$

(48)

At the same time, in the process of calculating formula (48), it is necessary to have the inequality relationship described in formula (49) for any cube $Q\subset R$ (where Q and R are arbitrary cube and double cube respectively).

$$\left|h_Q-h_R\right|\le C\|b\|_*\left(M^{(\alpha)}_{r,\left(\frac{9}{8}\right)}f(x)+I_\alpha\left(|f|\right)(x)\right)K_{Q,R}K^{\alpha}_{Q,R}$$

(49)

In formula (49),

$$h_Q=m_Q\left(I_\alpha\left[b-\left(m_Q(b)\right)\int\chi_{\frac{R^d}{\frac{4}{3}Q}}\right]\right)$$

、

$$h_R=m_R\left(I_\alpha\left[b-\left(m_R(b)\right)\int\chi_{\frac{R^d}{\frac{4}{3}Q}}\right]\right)$$

. Formula (49) is used to describe $I_{\alpha,b}f(y)$ by $I_\alpha f(y)$, and $[b,I_\alpha]$ is described by the equation relationship in formula (50) for the square Q under the condition of $x\grave{o}Q$.

$$[b,I_\alpha]f=\left(b-m_Q(b)\right)I_\alpha f-I_\alpha\left(\left(b-m_Q(b)\right)f_1\right)-I_\alpha\left(\left(b-m_Q(b)\right)f_2\right)$$

(50)

Based on formula (50), the inequality described in formula (51) is obtained.

$$\frac{1}{\mu\left(\frac{3}{2}Q\right)}\int_Q\left|I_{\alpha,b}f(y)-h_Q\right|d\mu(y)\le\frac{1}{\mu\left(\frac{3}{2}Q\right)}\int_Q\left|\left(\left[b-m_Q(b)\right]I_\alpha f\right)(y)\right|d\mu(y)=I_1+I_2+I_3$$

(51)

Through the (p,q) property of I_α and Hurd's inequality, the inequality relation in formula (52) is obtained.

$$I_1\le C\|b\|_*M_{r,\left(\frac{9}{8}\right)}\left(I_\alpha f\right)(x)$$

$$I_2\le C\|b\|_*M^{(\alpha)}_{r,\left(\frac{9}{8}\right)}f(x)$$

(52)

Using

$$\left|m_Q(b)-m_{2^k\left(\frac{4}{3}\right)Q}(b)\right|\le 2K_{Q,2^k\left(\frac{4}{3}\right)Q}\|b\|_*\le ck\|b\|_*$$

and $\int_{\frac{4}{3}Q}\left|b(y)-m_Q(b)\right|^{ss'}d\mu(y)\le C\|b\|_*^{ss'}\mu\left(\frac{3}{2}Q\right)$, we can determine:

$$\frac{1}{\mu\left(\frac{3}{2}Q\right)}\int_Q\left|I_\alpha\left(\left[b-m_Q(b)\right]f_2\right)(y)\right|d\mu(y)\le C\|b\|*M^{(\alpha)}_{r,\left(\frac{9}{8}\right)}f(x)$$

(53)

Formula (53) indicates

$I_3 \le c\|b\| * M^{(\alpha)}_{r,\left(\frac{9}{8}\right)} f(x)$.

According to the above description, the formula is proved.

For any cube, there is an equation described in formula (54).

$$\left| m_Q\left[I_\alpha\left(\left[b - m_Q(b) \right] f \chi_{R^d \setminus \frac{4}{3}Q} \right) \right] - m_R\left[I_\alpha\left(\left[b - m_R(b) \right] f \chi_{R^d \setminus \frac{4}{3}R} \right) \right] \right| = A_1 + A_2 + A_3 + A_4 + A_5 \tag{54}$$

According to the calculation process of I_3, the formula (55) is obtained.

$$A_1 \le C\|b\| * M^{(\alpha)}_{r,\left(\frac{9}{8}\right)} f(x) \tag{55}$$

Based on formula (55), formula (56) and formula (57) are obtained

$$A_4 \le CK_{Q,R}\|b\| * \left[I_\alpha(|f|)(x) + M^{(\alpha)}_{r,\left(\frac{9}{8}\right)} f(x) \right] \tag{56}$$

$$A_4 \le C\|b\| * M^{(\alpha)}_{r,\left(\frac{9}{8}\right)} f(x) \tag{57}$$

For A_5, formula (58) can be used to calculate:

$$\left| I_\alpha\left(\left(b - m_R(b) \int \chi_{R^d \setminus 2^N Q} \right) \right)(y) - I_\alpha\left(\left(b - m_R(b) \right) \int \chi_{R^d \setminus 2^N Q} \right)(z) \right| \le C\|b\|_* M^{(\alpha)}_{r,\left(\frac{9}{8}\right)} f(x) \tag{58}$$

Taking all the cube Q with y and R with z, we obtain the inequality relationship shown in formula (59).

$$A_5 \le C\|b\| * M^{(\alpha)}_{r,\left(\frac{9}{8}\right)} f(x) \tag{59}$$

In the calculation of A_3, $y \in Q$ is considered.

$$\begin{aligned} &\left| I_\alpha\left(\left[b - m_R(b) \right] f \chi_{2^N Q \setminus 2Q} \right)(y) \right| \\ &\le C \sum_{i=1}^{N-1} \frac{1}{l(2^i Q)^{n-\alpha}} \int_{2^{i+1}Q \setminus 2^i Q} \left| (b - m_R(b))(z) \right| \left| f(z) \right| d\mu(z) \\ &\le C \sum_{i=1}^{N-1} \frac{1}{l(2^i Q)^{n-\alpha}} \left\{ \int_{2^{i+1}Q} \left| (b - m_R(b))(z) \right|^{r'} d\mu(z) \right\}^{\frac{1}{r'}} \left\{ \int_{2^{i+1}Q} \left| f(z) \right|^{r} d\mu(z) \right\}^{\frac{1}{r}} \end{aligned} \tag{60}$$

Among

$$\begin{aligned} &\int_{2^{i+1}Q} \left| b - m_R(b) \right|^{\frac{1}{r'}} \\ &\le \left(\int_{2^{i+1}Q} \left| b - m_{2^{i+1}Q}(b) \right|^{r'} d\mu \right)^{\frac{1}{r'}} + \mu\left(2^{i+1}Q\right)^{\frac{1}{r'}} \left| m_{2^{i+1}Q}(b) - m_R(b) \right| \\ &\le cK_{Q,R} \|b\|_* \, \mu\left(2^{i+1}Q\right)^{\frac{1}{r'}} \end{aligned}$$

, So there is:

$$\left(I_\alpha \right)\left(\left(b - m_R(b) \right) f \chi_{2^N Q \setminus 2Q} \right)(y) \le cK_{Q,R} K^{(\alpha)}_{Q,R} \|b\|_* M^{(\alpha)}_{r,\left(\frac{9}{8}\right)} f(x) \tag{61}$$

From this we can get $A_3 \le CK_{Q,R} K^{(\alpha)}_{Q,R} \|b\| * M^{(\alpha)}_{r,\left(\frac{9}{8}\right)} f(x)$.

After the above verification, theorem 3 is verified by the following formula.

Considering $w \in A_p$ and $b \in RBMO(\mu)$, the formula (62) is obtained by using Lebesgue differential theory.

$$\begin{aligned} &\int_{R^d} \left| \left([b, I_\alpha] f \right)(x) \right|^p w(x) d\mu(x) \\ &\le C \int_{R^d} \left| N\left([b, I_\alpha] f \right)(x) \right|^p w(x) d\mu(x) \\ &\le C \int_{R^d} \left(M^{\#,(\alpha)}\left([b, I_\alpha] f(x) \right) \right)^p w(x) d\mu(x) \end{aligned} \tag{62}$$

In the second inequality in formula (62), I_α is bounded, so Theorem 3 holds.

By analyzing the solution of the error

matrix equation $XA = B$ of type $1(t+1\times 7)$, we can know that $p(t) \supseteq p_{20}(t) \cup p_{21}(t) \cup ... p_{27}(t)$ has the final solution.

$$X_0 = \begin{cases} p_{100x}(t) = p_{20}(t) \\ p_{110x}(t) = p_{21}(t) \\ \quad ... \qquad ... \\ p_{170x}(t) = p_{27}(t) \end{cases} \tag{63}$$

4 Conclusion

Using $T(u) = u_1$, we can know u and u_1 to find T. the parameters of object u include U is the universe, $S(t)$ is the thing described, p is the current space, $T(t)$ is the feature, $L(t)$ is the quantity, $x(t) = f(u(t), p), Gu(t)$ is the error function, etc, Combined with one of the six transformations of T: decomposition, similarity, addition, replacement, destruction and unit transformation, the solution of error logic transformation based on p-space decomposition is studied. In this paper, a method for solving the second kind of $1(t+1\times 7)$-fuzzy error matrix equation is proposed. The fuzzy error matrix is used to solve the transformation from the current state to the desired state in the case of spatial decomposition.

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