Oscillation analysis of numerical solution of Nonlinear Operator Equation

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Abstract: There is a problem of low accuracy in the analysis of the vibration of the numerical solution of the nonlinear operator equation. In this work, the vibration analysis equation is constructed by the step-by-step search method, and the vibration quadrant of the equation is divided by the dichotomy method. The vibration spectrum is determined by the iteration method, and the vibration analysis model of the numerical solution of the nonlinear operator equation is constructed. The vibration analysis of the numerical solution of the nonlinear operator equation is completed based on the solution of the model and the numerical calculation and display of the step-by-step Fourier. The experimental results show that the proposed method has higher accuracy than the traditional vibration analysis method, which meets the requirements of the vibration analysis of the numerical solution of nonlinear operator equation.

Key word: Nonlinear operator equation; Numerical solution; Vibration analysis; Step by step search; Vibrational quadrant

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1. Introduction

The vibration analysis of numerical solution of nonlinear operator equation, also known as numerical vibration calculation method, is a kind of knowledge of numerical vibration problem.This paper studies how to use the calculation tools to solve the mathematical problem. The numerical problem itself reflects the vibration relationship between two sets of data. Function calculation and equation root finding are typical examples of numerical problems. Vibration can be macro vibration and micro vibration [1]. Different atoms have different vibrational frequencies and emit different frequency spectrums, so we can find out which elements are contained in matter through spectrum analyzer. Energy source is indispensable for vibration [2]. But most of the oscillations are linear, and the numerical solutions of nonlinear operator equations often appear in practical problems, such as scientific and engineering calculation, vibration scientific prediction, etc. Therefore, studying the oscillation of numerical solutions of nonlinear operator equations is very important.

At present, the main methods to analyze the vibration of numerical solutions of linear equations are: numerical algebra method, numerical approximation method, numerical solution of ordinary differential equation and dynamic system, numerical solution of partial differential equation, optimization theory and method, error theory. Among them, numerical algebra method is the mainstream method, mainly including numerical solution of linear equation and nonlinear operator equation, eigenvalue and eigenvector However, the traditional method has low analysis accuracy when analyzing the vibration of the numerical solution of the existing nonlinear operator equation.

Aiming at the above problems, a vibration analysis method for the numerical solution of nonlinear operator equation is proposed. In this paper, the vibration analysis equation is analyzed by step search method, dichotomy method and iteration method, and then the vibration analysis model of numerical solution of nonlinear operator equation is established. When using the proposed method to analyze the vibration of the numerical solution of the existing nonlinear operator equation, the analysis accuracy is obviously higher than that of the traditional method.

2. Construction of vibration analysis model

2.1 Vibration analysis equation

All kinds of physical phenomena, including sound, light and heat, contain vibration. People's life is also inseparable from vibration. Especially in engineering technology, vibration phenomenon is also everywhere [3]. For instance, the vibration of bridges and buildings under the excitation of gusts or earthquakes and the vibraion of aircraft and ships during navigation, etc.

As we all know, the problem of finding roots of algebraic equations is an old mathematical problem. As early as in the 16th century, we have found the root formula of cubic equation and quartic equation. However, it was not until the 19th century that it was proved that the general algebraic equation of power $n \geq 5$ could not be solved by algebraic formula, and the solving process was very complicated. The general algebraic equation of power can not be solved by algebraic formula, and the solving process is very complex. Therefore, it is necessary to study the numerical method to obtain the approximate solution of the algebraic equation with a certain accuracy. In engineering and science and technology, many problems often come down to the problem of solving nonlinear operator equations, especially the problem of oscillation. For the quadratic equation $ax^2 + bx + c = 0$, we can use the familiar formula $x_{1,2} = \frac{1}{2a}\left(-b \pm \sqrt{b^2 - 4ac}\right)$ to solve it, but for the equation of more than three times, we can't solve it. In fact, the formula for finding roots of cubic and quartic equations is very complex. Generally, there is no formula for finding roots for algebraic equations above quintic, and there is no formula for finding roots for general transcendental equations. Therefore, in order to solve a nonlinear operator equation, we must rely on some numerical method to find its approximate solution, and then analyze its vibration. The analysis of vibration is directly based on the equation, gradually reducing the existence range of the root, or gradually making the approximate value of the root accurate, until meeting the accuracy requirements of the problem [4,5].

For a nonlinear operator equation, two problems must be considered to analyze the oscillation of its solution: (1) whether the equation has roots; (2) the number of square roots. First, we need to know the definition of the root of Nonlinear Operator Equation:

There is $f(x) = 0$ nonlinear operator equation a, where $f(x)$ is a nonlinear function of the real variable x.

If x^* makes $f(x^*) = 0$, then x^* is the root of the equation, or the zero point of $f(x)$.

When $f(x)$ is a polynomial, i.e.:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \text{L} + ax + a_0, (a_n \neq 0) \quad (1)$$

$f(x) = 0$ is called n -th algebraic equation. When $f(x)$ contains special functions such as exponential function or trigonometric function, $f(x) = 0$ is called special equation.

If $f(x) = (x - x^*)^m g(x)$, where $g(x^*) \neq 0$. If m is a positive integer, then x^* is the multiple root of $f(x) = 0$. When $m = 1$, x^* is called a single root of $f(x) = 0$.

Secondly, the existence of root is closely related to the zero point theorem. The significance of the zero point theorem is: if $f(x)$ is continuous on $[a,b]$, and $f(a) \cdot f(b) < 0$, then $x^* \in (a,b)$ exists, making $f(x^*) = 0$, that is, $f(x)$ is zero in (a,b) [6].

For the equation $f(x) = 0$, $x \in [a,b]$, in order to make the process more concise, Let $f(a) < 0$, $f(b) > 0$, start from the left end of the interval $x_0 = a$, according to a predetermined step h (if $h = \frac{b-a}{N}$, N is a positive integer), step by step to the right, each step to search the root. That is, check the sign of function value $f(x_k)$ on node $x_k = a + kh$, if $f(x_k) = 0$, then x_k is the solution of the equation; if $f(x_k) > 0$, then the root of the equation is in the interval $[x_{k-1}, x_k]$, and its width is h. For example, consider the root of equation $f(x) = x^3 - x - 1 = 0$. Note that if $f(0) = -1 < 0, f(2) = 5 > 0$, then $f(x)$ has at least one root in $(0, 2)$. Let $x = 0$ from the start, and search the root to the right in steps of $h = 0.5$. It can be seen that there must be one equation in $[1.0, 1.5]$. It can be seen that the key to the application of this method lies in the selection of step h. As long as the step h is small enough, the root of any precision can be obtained by using this method. However, if the step h is reduced, the number of search steps will be increased, so the amount of calculation will be increased. Therefore, the vibration analysis equation will be constructed, so that the vibration analysis of the numerical solution of the nonlinear operator equation can be carried out quickly.

2.2 Division of vibration quadrant by dichotomy

According to the vibration analysis equation constructed by the step-by-step search method, the dichotomy method is used to quadrant partition the vibration of the numerical solution of the nonlinear operator equation. Nonlinear operators, also known as nonlinear mapping, are operators that do not satisfy linear conditions. The research object of functional analysis is mainly linear operators and linear functional in special cases. However, most problems in nature and engineering technology are nonlinear. In fact, some linear equations in mathematical physics are approximate under certain conditions. In order to study these nonlinear problems, the operators (maps) involved cannot be limited to linear operators[7-10].

Linear relation is an independent relation, while nonlinear relation is an interaction relation. It is this interaction relation that makes the whole not simply equal to the sum of parts, but may produce gain or loss different from "linear superposition". Up to now, there is no clear and complete understanding of the concept and nature of non-linearity, and its philosophical significance has not been fully explored. Linearity is defined from two interrelated angles. The functional relationship between superposition principle and physical variables is a straight line, and the rate of change between variables is a constant. For the nonlinear operator equation $f(x) = 0$, where $f(x)$ is continuous on $[a,b]$ and $f(a) \cdot f(b) < 0$, it is better to assume that $f(x)$ has only one zero point in $[a,b]$, which is used for phenomenon division, that is, dichotomy is used to divide vibrational quadrants. The process of finding the real root $x*$

of the equation is the process of dividing quadrants by dichotomy, dividing $[a,b]$ into half parts step by step, and checking the changes of function value symbols, so as to determine the sufficient cells containing roots [11].

The steps of dividing vibration quadrant by dichotomy are as follows: record $a_1 = a$, $b_1 = b$.

First, $(k=1)$ is calculated in half, that is, $(k=1)$ is divided in half. Calculate the middle points $x_1 = \frac{a_1 + b_1}{2}$ and $f(x_1)$. If $f(a_1)\cdot f(x_1) < 0$, the root must be in $[a_1, x_1]$, otherwise it must be in $[x_1, b_1]$ (if $f(x_1) = 0$, then $x^* = x_1$), then we get half the length of the root interval $[a_2, b_2]$, that is, $f(a_2) f(b_2) < 0$, and $b_2 - a_2 = \frac{1}{2}(b_1 - a_1)$.

Then, the above process is repeated in half. Suppose that step 1, L, step $k-1$ has been completed, the root-containing interval $[a_1,b_1] \supset [a_2,b_2] \supset \text{L} \supset [a_k, b_k]$ is obtained in half calculation, and $f(a_k) f(b_k) < 0$ is satisfied, that is, $x^* \in [a_k, b_k]$, $b_k - a_k = \frac{1}{2^{k-1}}(b-a)$, then k Calculate the number of steps in half: $x_k = \frac{a_k + b_k}{2}$, and have [12,13]:

$$|x^* - x_k| \le \frac{b_k - a_k}{2} = \frac{1}{2^k}(b-a) \tag{2}$$

Determine the new rooted interval $[a_{k+1}, b_{k+1}]$, that is, if $f(a_k) f(x_k) < 0$, the root must be in $[a_k, x_k]$, otherwise it must be in $[a_{k+1}, b_{k+1}]$ and there is: $b_{k+1} - a_{k+1} = \frac{1}{2^k}(b-a)$. In short, the sequence $\{x_k\}$ is obtained from the above dichotomy, and by formula (2): $\lim_{k\to\infty} x_k = x^*$ [14].

The approximation of the real root x^* of the equation $f(x) = 0$ can be taken to an arbitrary specified precision by using the dichotomy method because:

Let $\varepsilon > 0$ be a given accuracy requirement, then from $|x^* - x_k| \le \frac{b-a}{2^k} < \varepsilon$, the number of scorable half calculations k should satisfy:

$$k > \frac{\left(\ln(b-a) - \ln\varepsilon\right)}{\ln 2} \tag{3}$$

The advantage of dichotomizing the vibration quadrant is that the method is simple and only requires $f(x)$ to be continuous. All real roots of $f(x) = 0$ in $[a,b]$ can be obtained by the dichotomy method, but the dichotomy method cannot find complex roots and even multiple roots, and it has slower convergence and more times of function value calculation. If the dichotomy method is adopted to find a real root of $f(x)=x^6-x-1$ in $[1,2]$, and to divide the quadrant of vibration (that is $|x^*-x_k|<\frac{1}{2}\times 10^{-3}$), then substitute $\varepsilon=0.5\times 10^{-3}$ into formula (3), where $(a=1, b=2)$, it can be determined that the required number of half times is $k=11$, and the calculated vibration quadrant results are shown in Table 1 (obviously $f(1)=-1<0, f(2)>0$) [15].

Table 1 Vibration quadrant results

k	ak	bk	xk	f(xk)
8	1.132813	1.140625	1.136719	0.020619
9	1.132813	1.136719	1.134766	0.4268415
10	1.132813	1.134766	1.133789	-0.00959799
11	1.133789	1.134766	1.134277	-0.0045915

2.3 Determining the vibration spectrum

In order to realize the vibration analysis of the numerical solution of the nonlinear operator equation, the vibration spectrum is the key data of the vibration. Therefore, the iterative method is used to determine the vibration spectrum. The iterative method is a successive approximation method, which can solve the algebraic equation, transcendental equation and equation system, but there are problems of convergence and convergence speed. In order to solve the approximate root of $f(x)=0$ by iterative method, we first need to transform this equation into an equivalent equation: $x=g(x)$, but and transform $f(x)=0$ into an equivalent equation to determine the vibrational spectrum.

The steps to determine the vibration spectrum by iterative method: set the equation as $x=g(x)$:

Take an initial approximate value x_0 of the root of the equation, and construct an approximate solution sequence according to the following successive substitution method:

$$x_1=g(x_0), x_2=g(x_1), \ldots x_{k+1}=g(x_k) \tag{4}$$

This method is called iterative method (or single point iterative method), and $g(x)$ is called iterative function.

If there is a limit in the sequence $\{x_k\}$ generated by the iterative method, that is $\lim_{k\to\infty} x_k = x^*$, $\{x_k\}$ is called convergence or convergence of the formula (4) of the iterative process, otherwise it is called non convergence of the iterative method. If $g(x)$ is continuous and $\lim_{k\to\infty} x_k = x^*$, then [16]:

$$x^* = \lim_{k\to\infty} x_{k+1} = \lim_{k\to\infty} g(x_k) = g\left(\lim_{k\to\infty} x_k\right) = g(x^*) \tag{5}$$

That is to say, x^* is the solution of the equation (x^* is the fixed point of the function $g(x)$). Obviously, when the equation $f(x)=0$ is transformed into the equivalent equation $x=g(x)$, different iterative functions $g(x)$ will be selected, and different sequences $\{x_k\}$ (even if the initial value x_0 is the same) will be generated, and the convergence of these sequences is not necessarily the same.

As shown in the figure below, solving equation $f(x)=0$ can be transformed into solving equation $x=g(x)$.

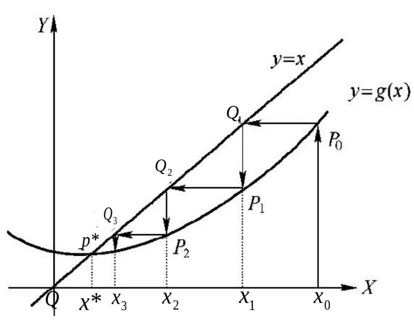
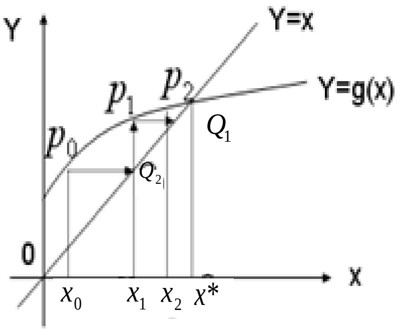
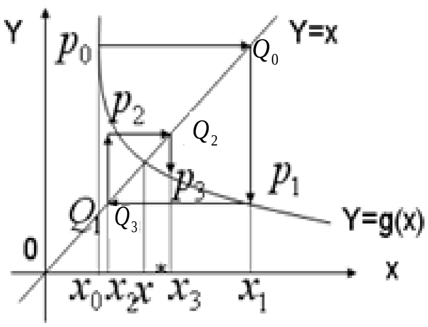


Fig. 1 Schematic diagram of iterative method for finding root of equation

In geometry, the coordinate x^* of the intersection p^* of the curve $y=x$ and $y=g(x)$ is equivalent. Starting from the point x_0 in the graph, $y=p_0$ is obtained from the function $y=g(x_0)$, Q_1 is obtained by substituting $y=x$ in the function, and then p_1 is obtained by substituting the x coordinate x_1 of Q_1 into the equation $y=g(x)$. Thus, a series of points p_0, p_1,..., p_k, the x coordinate of these points is the iterative sequence x_1, x_2,..., x_k, it tends to be the root x^* of the equation. The elements of the sequence are the approximate values of the root of the equation.

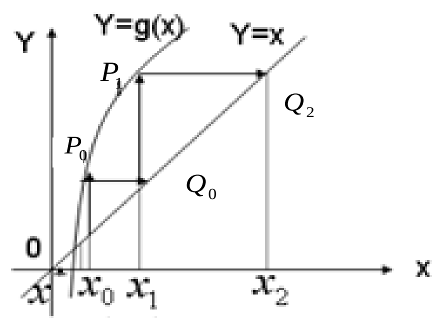
The convergence of the sequence is equivalent to that the curve $y=x$ and $y=(x)$ can intersect at a point. As for the convergence, it is known from geometry that $\{x_k\}$ converges to x^* in (1) and (2), and $\{x_k\}$ does not converge to x^* in (3) and (4).

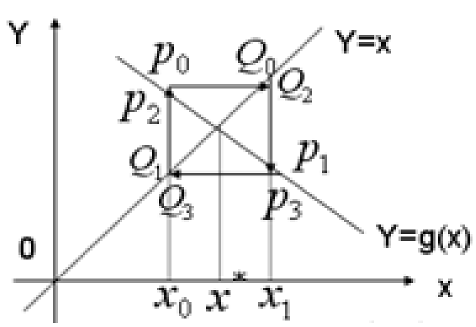




(1) $0 < g'(x^*) < 1$

(2) $0 < g'(x^*) < 1$





(3) $g'(x^*) > 1$

(4) $g(x)$=1-x $\quad g'(x^*) = -1$

Fig. 2 Convergence and geometric significance

The convergence theorem of the iterative method is set with equation $x = g(x)$, and $g(x)$ has the first derivative on $[a,b]$. When $x \in [a,b]$, there is $g(x) \in [a,b]$, $g'(x)$ meets the condition: $|g'(x)| \le L < 1, \forall x \in [a,b]$, we have:

$x = g(x)$ has a unique solution $x*$ on $[a,b]$.

For any initial value $x \in [a,b]$, the iterative process $x_{k+1} = g(x_k), k = 0,1,...,N$ converges to $\lim x_k = x*$ [17].

$$|x*-x_k| \le \frac{1}{1-L}|x_{k+1} - x_k|.$$

Error estimation formula: $|x*-x_k| \le \frac{1}{1-L}|x_1 - x_0|,(k = 1,2,...,N)$.

The selection of $g(x)$ must satisfy:

(1) the same solution of the two equations;
(2) the iteration sequence converges to its root. If the vibration spectrum of Nonlinear Operator Equation $f(x) = x - \sin x - 0.5 = 0$ is determined, it can be transformed into equivalent equation by different methods:

$$x = \sin x + 0.5 = g_1(x) \qquad (2)$$

$x = \sin^{-1}(x - 0.5) = g_2(x)$

It is transformed into the iterative function as below:

$(a) x_{k+1} = \sin x_k + 0.5, k = 1, 2, ..., N$

$(b) x_{k+1} = \sin^{-1}(x_k - 0.5), k = 1, 2, ..., N$

It can be seen from the calculation that the convergence cases of the two selected functions $g_1(x), g_2(x)$ are different in constructing sequence $\{x_k\}$ respectively (the initial value is taken as 1). In (a), sequence $\{x_k\}$ converges and $x^* \approx 1.497300$, and in (b), $\sin^{-1}(x_4 - 0.5) = \sin^{-1}(-1.987761)$ is calculated without definition. The vibration spectrum is shown in Table 2:

Table 2 Vibrational spectrum

k	（a）xk	（b）xk
0	1.0	1.0
1	1.341471	0.523599
2	1.473820	0.023601
3	1.049530	-0.496555
4	1.497152	-1.487761
5	1.497289	0.063049
6	1.497300	0.012703
7	1.497300	-0.042132

According to the step-by-step search method, the vibration analysis equation is constructed, the vibration quadrant is divided by the dichotomy method, and the vibration spectrum is determined by the iteration method.

3. Model solution and numerical analysis

3.1. Solution of the model

The following nonlinear operator equations are often required in scientific calculation:

$$\begin{cases} f_1(x_1, x_2, \cdots, x_n) = 0 \\ f_2(x_1, x_2, \cdots, x_n) = 0 \\ \cdots\cdots\cdots\cdots \\ f_n(x_1, x_2, \cdots, x_n) = 0 \end{cases} \tag{6}$$

In the formula, $fi(i = 1, 2, \cdots, n)$ is a real valued function given in the region D of n-dimensional Euclidean space R^n. In this paper, vector symbols are introduced:

$$F(x) = \begin{Bmatrix} f_1(x) \\ f_2(x) \\ \cdots \\ f_n(x) \end{Bmatrix}, x = \begin{Bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{Bmatrix}, 0 = \begin{Bmatrix} 0 \\ 0 \\ \cdots \\ 0 \end{Bmatrix} \tag{7}$$

Here, F represents the nonlinear image defined on $D \subset R^n$ and valued at R^n, which is abbreviated as:

$$F : D \subset R^0 \to R^n \tag{8}$$

If $x^* \in D$ exists and $F(x^*) = 0$, then x^* is the oscillatory solution of the numerical solution of the nonlinear operator equation.

The oscillation of value solutions of nonlinear operator equations can be divided into two categories: one is polynomial equation, which can be defined as: $f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n = 0$, $n \in N, a_0, a_1, \cdots, a_n \in C$. The other is non polynomial equation. Let $f(x)$ be the real value function of real variable x, that is, the solution of oscillation equation

$x_0 = (a+b)/2$, or the root of $f(x)$. Generally, it is divided into two steps: first, the distribution area of its root is determined by mathematical analysis tools, and then the root is obtained by the more refined method -- step-by-step approach in the area, and the vibration is further obtained. Let x^* be the root of $f(x)=0$, we have:

$$f(x^*)=0 \tag{9}$$

If there is a positive integer m , $f(x)=(x-x^*)g(x)$, and $0<|g(x)|<\infty$, then x^* is called its multiple root of m. When $m=1$, it is called a single root.

The following intermediate value theorem is the simplest way to determine the existence interval of roots:

Theorem 1: If $f(x)$ is continuous on $f(x)$, and $f(a).f(b)<0$, then $f(x)$ has at least one root on $[a,b]$.

Theorem 2: In the neighborhood $S(x_0)=\{x:|x-x_0|\le\beta|f(x_0)|\}$ of x_0 , $f(x)$ is continuously differentiable and $|f^{'}(x)|^{-1}\le\beta$, then there is a unique solution x^* of equation $f(x)=0$ on $S(x_0)$, i.e. the oscillation of the numerical solution of nonlinear operator equation. However, it is numerical oscillation, so the step-by-step Fourier numerical calculation is used to display the numerical value, so as to realize the vibration analysis of the solution of the visualized nonlinear operator equation.

3.2 Step fourier numerical calculation and numerical display

In the numerical display of the numerical oscillation of the numerical solution of nonlinear operator equation, the step-by-step Fourier numerical calculation, matlab program and matlab program are introduced.

After Fourier transform, the nonlinear operator equation to be solved is as follows:

$$\frac{\partial A}{\partial z}+\frac{\alpha}{2}A-\frac{\beta i}{4}\frac{\partial^2 A}{\partial T^2}-\gamma i|A|^2 A=0 \tag{10}$$

In the formula, $A(z,T)$ represents the slowly varying complex amplitude, and z is the distance of the pulse propagating along the optical fiber; $T=t-\beta_1 z$, $\beta_1=1/v_g$, v_g is the group velocity; $\beta(ps/km)$ is the dispersion coefficient; $\gamma(1/w\cdot km)$ represents the nonlinear coefficient; $\alpha(1/km)$ represents the fiber loss coefficient, and its relationship with the loss coefficient $\alpha_{dB}(dB/km)$ expressed in decibels is: $\alpha_{dB}=4.343\alpha$.

First, equation (10) can be normalized: $U=A(z,T)/\sqrt{P_0}$, P_0 is the peak power of the incident pulse, at this time equation (10) can be rewritten as:

$$\frac{\partial U}{\partial z}=-\frac{\alpha}{2}U+\frac{\beta i}{4}\frac{\partial^2 U}{\partial T^2}+\gamma P_0 i|U|^2 U \tag{11}$$

In order to solve equation (11) using the step-by-step Fourier method, write equation (11) in the following form:

$$\frac{\partial U}{\partial z}=(\hat{D}+\hat{N})U$$

Further, the following equation (12) can be obtained:

$$\hat{D}=\frac{-\alpha+\frac{\beta i}{2}\frac{\partial^2 U}{\partial T^2}}{2}$$
$$\hat{N}=\gamma P_0 i|U|^2 \tag{12}$$

Then, the linear operator and the nonlinear operator of equation (12) are calculated successively according to step 1 and step 2. Finally, in step 3, the matlab program of step 1 and step 2 is run to get the exact numerical solution and simulation curve of linear and nonlinear operators.

Step 1: solve the linear operator equation. The equation of the linear operator is as follows:

$$\frac{\partial U}{\partial z}=\frac{-\alpha+\frac{\beta i}{2}\frac{\partial^2 U}{\partial T^2}}{2}U \tag{13}$$

An ordinary differential equation (14) is obtained by Fourier transform

$$\frac{\partial \tilde{U}}{\partial z}=-\frac{\alpha}{2}\tilde{U}-\frac{i(i\omega)^2\beta}{4}\tilde{U} \tag{14}$$

The solution of equation (14) shows that:

$$\tilde{U}(z,\omega)=\tilde{U}(0,\omega)\exp[\frac{i\beta\omega^2-2\alpha}{4}z] \tag{15}$$

Where $\tilde{U}(0,\omega)$ is the Fourier transform of the initial value $\tilde{U}(0,\omega)$, and the inverse Fourier transform of $\tilde{U}(z,\omega)$ is used to obtain $U(z,T)$. The solution formula of equation (X) is:

$$U(z,T)=\tilde{F}\{\exp[(\frac{i}{2}\beta\omega^2-\alpha)\frac{z}{2}]\cdot F[U(0,T)]\} \tag{16}$$

Where F and $\tilde{F}$ represent Fourier transform operationand inverse Fourier transform operation, respectively.

Step 2 solution of Nonlinear Operator Equation:

The equations for the nonlinear part are as follows:

$$\frac{\partial U}{\partial z}=\gamma P_0 i|U|^2 U \tag{17}$$

In the same way as step1, solve equation (17), and obtain:

$$\tilde{U}(z,\omega)=\tilde{U}(0,\omega)\exp[\gamma P_0 i|U(0,T)|^2 z] \tag{18}$$

Where $\tilde{U}(0,\omega)$ is the Fourier transform of the initial value $U(0,T)$, and the inverse Fourier transform of $\tilde{U}(z,\omega)$ gives $U(z,T)$ .The solution formula of equation (18) is as follows:

$$U(z,T)=\tilde{F}\{\exp[\gamma P_0 i|U(0,T)|^2 z]\cdot F[U(0,T)]\} \tag{19}$$

Where F and $\tilde{F}$ represent

Fourier transform operationand inverse Fourier transform operation, respectively.

Step 3 algorithm in MATLAB.In MATLAB, the length of a time-limited sequence $x(n)$ is $N(1 \le n \le N)$, which corresponds to a frequency-domain finite sequence $X(k)(1 \le k \le N)$ of length N, and the angular frequency $\omega(k) = \frac{2\pi k}{NT}(1 \le k \le N)$ of $X(k)$, where T is the sampling interval of the sequence $x(n)$. The relationship between the positive and negative DFT is as follows:

$$X(k) = DFT[x(n)] = \sum_{j=1}^{N} x(n)\exp(-j \cdot \frac{2\pi}{N} \cdot k \cdot n) \quad (1 \le k \le N)$$

$$x(n) = IDFT[X(k)] = \frac{1}{N}\sum_{j=1}^{N} X(k)\exp(j \cdot \frac{2\pi}{N} \cdot k \cdot n) \quad (1 \le n \le N) \tag{20}$$

Then, the DFT function FFT and IDFT function IFFT in MATLAB are used to realize the Fourier and anti Fourier operations in the equation. Furthermore, the numerical solution and simulation curve are obtained. Finally, by testing a set of parameters, the MATLAB results of equation (10) under the algorithm are obtained. The total time of MATLAB is 34.26s, and the result curve is shown in the figure below. The oscillation analysis of numerical solution of nonlinear operator equation is completed.

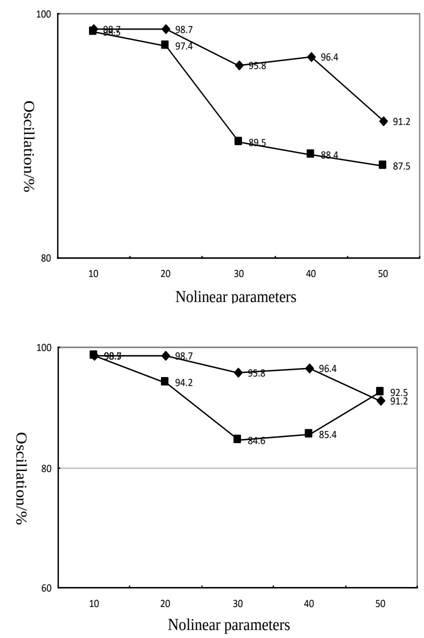


Fig. 3 Vibration analysis curve

4. Experimental verification

4.1 Experimental equipment and materials

During the experiment, prepare the vibration source, vibration frequency measurement system, vibration quadrant measurement system, polarization measurement instrument calibration kit, conventional analysis method (numerical algebra method), and the proposed vibration analysis method. The actual instrument parameters are shown in Table 3:

Table 3 Experimental instrument parameters

Project	Production unit	Remarks
Vibration frequency measurement system	Hinds instruments, Inc	Minimum detection intensity of mid far infrared (130nm to 18um): Leather tile
Vibration quadrant	Meadowlark options, Inc	Ultra wide band (450-

measurement system		1100nm, measurable power of 1 μ w light wave polarization state
Polarimeter calibration kit	Meadowlark options, Inc	Polarization tracking

4.2 Experimental process design

During the experiment, the specified vibration source which conforms to the nonlinear operator equation is used to stabilize the data in the laboratory at 25℃ for 2 hours, and the temperature of the vibration source which conforms to the nonlinear operator equation is ensured to be constant through measurement. The vibration frequency measuring system and the ultra bandwidth polarization measuring instrument are calibrated by using the polarization measuring instrument calibration kit. In the same experimental environment, the two vibration analysis methods and measurement system (equipment) are used to carry out the verification test with different frequencies in turn. The proposed method, the partial differential equation analysis method and the eigenvector numerical analysis method are used respectively, and the vibration output amplitude and the accuracy of vibration analysis are taken as the experimental indicators for the comparative experiment.

4.3 Result analysis

4.3.1 Stability of vibration output amplitude

The stability of vibration output amplitude can directly reflect the analysis performance of the method. The higher the stability result, the more effective the vibration analysis result of the method. The comparison results of vibration output amplitude stability of the three methods are shown in Figure 4.

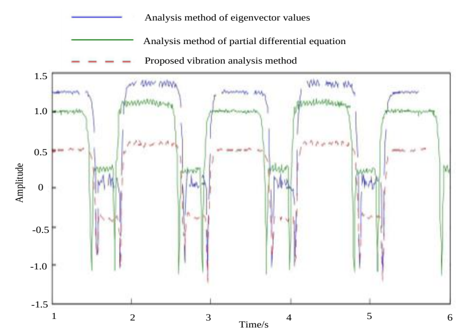


Fig. 4 Amplitude stability comparison results

It can be seen from Fig. 4 that compared with the two comparison methods, the vibration output amplitude of the proposed method is relatively stable with an amplitude of about 0.5, while that of PDE method and eigenvector numerical method reach 1.0 and 1.4 respectively. Therefore, the proposed method has high stability of vibration output amplitude.

4.3.2 Accuracy of vibration analysis

According to the field records of the laboratory, draw the polarization comparison experimental results curves of the three methods, as shown in Figure 5.

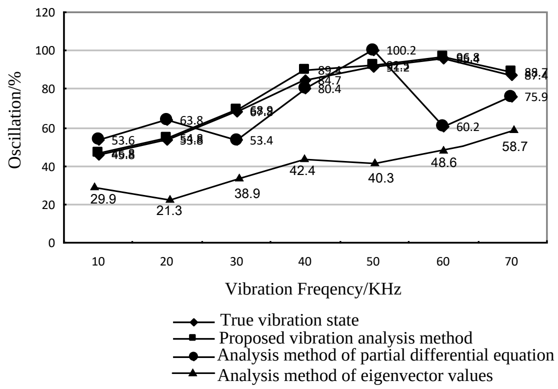


Fig. 5 Accuracy comparison curve of vibration analysis

By analyzing the experimental results, the proposed vibration analysis method has a high degree of coincidence with the real value, and the fluctuation is stable. However, the two traditional analysis methods will produce certain fluctuations, and their numerical accuracy has a certain gap with the real value. Among them, the vibration analysis method of eigenvector values has relatively small fluctuation, but its numerical accuracy is very low in accordance with the actual value, the highest analysis accuracy is only 29.9%, while the vibration analysis method of partial differential equation has the highest numerical accuracy close to the actual value, but the fluctuation is too large. Compared with traditional method, it can be seen that the proposed method has higher accuracy in vibration analysis of the numerical solution of the nonlinear operator equation.

5. Conclusion

The vibration analysis of the numerical solution of the nonlinear operator equation is proposed by constructing the vibration analysis model of the numerical solution of the nonlinear operator equation. The experimental results show that the proposed method can accurately analyze the vibration of the numerical solution of the nonlinear operator equation. This study provides theoretical basis for the vibration analysis of the numerical solution of the nonlinear operator equation. On the basis of this study, further research is needed to address the following issues:

(1) For the bifurcation phenomenon of nonlinear vibration system, how to combine homotopy analysis method, renormalization group method, increment harmonic balance method and wavelet method to solve more complex strong nonlinear vibration problem is worth further study.

(2) The periodic solutions of the frequency response curve and the existence of the step phenomenon of the oscillator under the parameters are discussed.

(3) The periodic solution, double periodic solution and quasi periodic solution of oscillator convergence in a long time range.

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