Joint Pricing Strategies for Tobacco Manufacturing Equipment and Optional Value-added Service

Deng Zhenghua, Associate Professor
Hu Xiao, Lecturer

Deng Zhenghua, Associate Professor in Supply Chain Management, School of Economics and Business Administration, Chongqing University, Chongqing, China; Associate Professor in Supply Chain Management, School of Economics and Business Administration, Chongqing University of Education, Chongqing, China. Hu Xiao, Lecturer in Management Science, School of Economics and Management, Chongqing University of Posts and Telecommunications, Chongqing, China. Correspondence author: Deng Zhenghua; dengzh@cque.edu.cn

Objectives: This paper constructs an analytic model for optimal pricing in which the interrelationship between the tobacco equipment and the optional value-added service supplied by tobacco equipment manufacturers are effectively depicted, and derives the closed-form solutions of the optimal prices, which has previously been considered analytically intractable in the bundling problem of pricing two goods. The research reveals that when the marginal cost of the optional value-added service is 0 and the valuation of the service for tobacco manufacturing enterprises is relatively low, it is advisable to adopt pure bundling pricing strategy; when the marginal cost of the service is 0 but the valuation of the service is relatively high, it is advisable to adopt separate pricing strategy; when the marginal cost of the service is greater than 0, separate pricing strategy is always optimal. And it is interesting that, under separate pricing strategy, the higher valuation of the tobacco equipment leads to lower price for the service; the higher marginal cost of the service leads to higher price for the service, but lower price for the tobacco equipment. This paper also proves that there are only two basic pricing strategies for tobacco equipment manufacturers: pure bundling pricing and separate pricing of the tobacco equipment and service.

Key words: optimal price; optional value-added service; bundling; tobacco equipment manufacturer
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INTRODUCTION

With the development of informatization and automation in tobacco manufacturing industry, the demand for value-added service such as equipment maintenance and repair in tobacco manufacturing enterprises are increasing rapidly. Meanwhile, increasingly fierce competition in equipment market has prompted more and more tobacco equipment manufacturers not only to provide equipment, but also to provide value-added service based on the equipment. For example, Siemens not only sells efficient automation systems, but also provides service throughout the life cycle of the equipment. However, the relationship between equipment and optional value-added service supplied by tobacco equipment manufacturers are different from that of general consumer commodities in supermarkets, which means that their pricing analysis is different from that of two general consumer commodities. Optional value-added service depends on equipment, because tobacco manufacturing enterprises who buy service must buy equipment; but the optional value-added service provided by equipment manufacturers are not necessaries but optional for tobacco manufacturing enterprises who buy equipment, because tobacco manufacturing enterprises may repair or maintenance by themselves. Therefore, it gives rise to the following questions: whether tobacco manufacturing equipment and service should be sold as a whole or separately, and how to determine the optimal price?

The related literatures in this paper are mainly based on bundled pricing.[1-5] Early scholars have proven that bundling strategy can bring excess revenue to enterprises, and that enterprises can segment customers through mixed bundling, but they didn't derive the closed-form solutions of optimal price.[6-10] Under the assumption that the marginal costs are zero and the customer reservation prices are uniformly distributed and independent, Eckalbar (2010) derived the closed-form solutions of optimal pricing.[11] Bhargava (2013) expanded the research results of Eckalbar (2010), compared and analyzed the selection conditions of full mixed bundling and partial mixed bundling, and gave the analytical solutions of optimal pricing under different situations.[1] Prasad et al. (2010) analyzed the impact of network externalities on the bundling pricing strategy of two products.[12] In view of the phenomenon that airlines tie additional service to high-end first-class cabins while hotel chains tie additional service to low-end products, Shugan et al. (2017) analyzed that the reason why the two types of companies adopt different product line bundling strategies lies in the differentiation of core products.[13]

Nevertheless, the above studies are mainly about the bundling pricing between two products or two services. As Meyer & Shankar (2016) said, the bundling of goods or the bundling of services has been very rich in research results, but the bundling pricing problem of products and services has been less studied.[14] Cohen & Whang (1997) studied the optimal product price, service price and service quality of the manufacturer when there is competition among third-party service providers in the after-sales service market, but did not consider the full bundling problem of product and service.[15] Meyer & Shankar (2016) constructed a bundled pricing model of products and services from the perspective of retailers, and analysed the impact of factors such as quality fluctuation and scale effect of products and services on equilibrium prices in different situations.[14] However, the service studied in this paper can exist independently of the product, and they didn't derive the closed-form solutions to the optimal pricing. Taking product price and maintenance service as important means of competition, Wang, Sun, Qu and Li (2015) analysed the game equilibrium under duopoly.[16] But in this paper, service was regarded only as a competitive means, not as an important source of income, nor was it priced separately. Lee, Yoo & Kim (2016) studied the game equilibrium between a channel separately providing both goods and services and the other providing inseparable servitized goods, and analysed the influence of service dependence and channel substitutability on the equilibrium.[17]

Based on the above studies, this paper, considering the interaction of the tobacco manufacturing equipment and the optional value-added service, analyses the optimal joint pricing strategies for equipment and services supplied by tobacco equipment manufacturers when the service is an option rather than a necessary for tobacco manufacturing enterprises. Just as Levy et al. (2019) said, better understanding of the interaction between market structure and government regulation can help develop effective policies.[18-19]

In this paper, Section 2 describes the model structure and the sales analysis of different situation. Section 3 derives optimal price for the pure bundling of the tobacco manufacturing equipment and service. Section 4 develops an analytic model under conditions that the tobacco equipment manufacturer sells equipment and optional value-added service separately, and derives the closed-form solutions of optimal pricing. It also identifies when a separate selling strategy is valuable, and illustrates that separate pricing strategy is as same as the partial bundling pricing. The final section discusses the contributions and limitations, and future research directions.

MODEL

A tobacco equipment manufacturer provides not only equipment but also optional value-added service. Let $c_g \geq$ and $c_s \geq 0$ respectively denote the constant unit marginal cost of the tobacco equipment and optional value-added service. The tobacco equipment manufacturer must choose which pricing strategy to sell them. Without loss of generality, it is assumed that the potential market capacity is 1. Let U and V respectively denote the valuation of the tobacco equipment and optional value-added service for customer enterprises. Considering the heterogeneity of customer enterprises, U and V are two random variables. Assuming that their joint cumulative probability distribution function is $F(u,v)$, and the corresponding joint probability density function is $f(u,v)$.

Sales Analysis Under Situations of Pure Bundling

When the tobacco equipment manufacturer bundles the equipment and service and sells them only in pairs at a bundle price of p_B, the sales volume is $q_B = P(U+V-p_B \geq 0) = 1-\int_0^{p_B}\int_0^{p_B-u} f(u,v)dvdu.$

Sales Analysis Under Situations of Separate Sales

Obviously, if the tobacco equipment and optional value-added service are sold separately, whether or not to purchase optional value-added service does not affect the realization of basic needs of tobacco manufacturing enterprises, but can improve their total value. Therefore, whether a tobacco manufacturing enterprise ultimately purchases optional value-added service depends mainly on the value added by the optional service and the service price. On the other hand, only the tobacco manufacturing enterprises who have purchased the equipment may purchase the optional value-added service supplied by the tobacco equipment manufacturer, that is, the valuation of the optional service must depend on the tobacco equipment.

Let p_g and p_s respectively denote the price of the tobacco equipment and optional value-added service. When the tobacco equipment and service of the equipment manufacturer are sold separately, according to the relationship between the price and the valuation, tobacco manufacturing enterprises (namely potential customer enterprises) can be divided into five types, as shown in Figure 1.

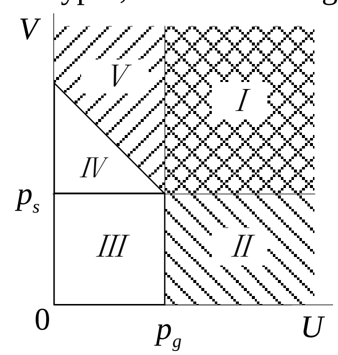


Figure 1
Classification of Tobacco Manufacturing Enterprises

Tobacco manufacturing enterprises (namely potential customer enterprises) of Type I refer to the enterprises who value the tobacco equipment and the optional value-added service more than their price, that is, $(U-p_g \geq 0) \cap (V-p_s \geq 0)$. Similarly, customer enterprises of Type II are $(U-p_g \geq 0) \cap (V-p_s < 0)$; customer enterprises of Type III are $(U-p_g < 0) \cap (V-p_s < 0)$; customer enterprises of Type IV are $(U-p_g < 0) \cap (V-p_s \geq 0) \cap (U+V-p_g-p_s < 0)$; customer enterprises of Type V are $(U-p_g < 0) \cap (V-p_s \geq 0) \cap (U+V-p_g-p_s \geq 0)$.

Therefore, the expected sales of tobacco equipment and optional value-added service are analysed as follows.

(1) Sales of tobacco equipment

As shown in Figure 1, it is obvious that the tobacco manufacturing enterprises located in Type I and Type II will definitely purchase the tobacco equipment. At the same time, the valuation of tobacco manufacturing enterprises located in Type V is lower than the price of tobacco equipment, so that the equipment alone is not worth purchasing. But considering the excess value brought by the optional service, the equipment will also be purchased. Therefore, the actual customers for the equipment will be the customers of type I, II and V in the Figure 1, and the corresponding expected sales volume is $q_B = P\left(U - p_g + (V - p_s)^+ \geq 0\right) = 1 - \int_0^{p_g}\int_0^{p_g+p_s-u} f(u,v)dvdu$.

(2) Sales of optional service

Obviously, the tobacco manufacturing enterprises who purchases the optional service must be the customer who purchases the tobacco equipment. Therefore, the expected sales volume of the optional service is $q_s = P\left(V - p_s \geq 0 \cap U - p_g + (V - p_s)^+ \geq 0\right) = 1 - F_s(p_s) - \int_0^{p_g}\int_{p_s}^{p_g+p_s-u} f(u,v)dvdu$.

Similar to the related literatures on pricing, in this paper, it is assumed that the random variables U and V are uniformly distributed on $[0, a_g] \times [0, a_s]$, where $a_g > 0$ and $a_s > 0$. Assuming $a_g > c_g$, that is, there are always some tobacco manufacturing enterprises whose valuations of equipment are greater than the manufacturing cost of the equipment manufacturer. Obviously, in general, the valuation of tobacco equipment is higher than that of service. Therefore, in this paper, it is assumed $a_g \geq a_s$, and the analysis method for $a_g < a_s$ is similar to that in this paper.

PURE BUNDLING PRICING STRATEGY

The sales function of the bundling of the tobacco equipment and service is as follows:

$$q_B = P(U + V - p_B \geq 0) = \begin{cases} 1 - \frac{{p_B}^2}{2a_g a_s}, & 0 \leq p_B < a_s; \\ 1 - \frac{p_B - a_s/2}{a_g}, & a_s \leq p_B < a_g; \\ \frac{\left(a_g + a_s - p_B\right)^2}{2a_g a_s}, & a_g \leq p_B \leq a_g + a_s. \end{cases} \quad (1)$$

It is easy to prove that the optimal bundle price must be in the interval $[0, a_g + a_s]$. Therefore, in order to simplify the formulation, the corresponding sales when $p_B < 0$ and $p_B > a_g + a_s$ are not listed in the sales function q_B. Thus, the profit function of the equipment manufacturer is: $\pi_B(p_B) = \left(p_B - c_g - c_s\right)q_B$.

Generally, the fixed costs for optional value-added service should be subtracted from the profit function, but this part of investment are sunk cost which do not affect pricing, so they are not considered in this paper.

It is easy to prove that the profit function is continuously differentiable in the interval $[0, a_g + a_s]$. By differentiating $\pi_B(p_B)$ with respect to p_B and setting the first derivative equal to 0, the optimal bundle price, sales and maximum profit can be obtained as shown in Proposition 1.

Proposition 1. If a tobacco equipment manufacturer bundles the equipment and service as a whole and does not sell the tobacco equipment and service separately, then

(1) The optimal bundle price is

$$p_B^* = \begin{cases} c/3 + \rho/3, & c < 3a_s/2 - a_g; \\ a_g/2 + a_s/4 + c/2, & 3a_s/2 - a_g \leq c < a_g - a_s/2; \\ a_g/3 + a_s/3 + 2c/3, & a_g - a_s/2 \leq c. \end{cases}$$

(2) The optimal sales are

$$q_B^* = \begin{cases} \dfrac{(\rho-2c)(\rho+c)}{9a_g a_s}, & c < 3a_s/2 - a_g; \\ \dfrac{2a_g + a_s - 2c_g - c_s}{4a_g}, & 3a_s/2 - a_g \le c < a_g - a_s/2; \\ \dfrac{2(a_g + a_s - c)^2}{9a_g a_s}, & a_g - a_s/2 \le c. \end{cases}$$

(3) The maximum profit of the equipment manufacturer is

$$\pi_B^* = \begin{cases} \dfrac{(\rho-2c)^2(\rho+c)}{27a_g a_s}, & c < 3a_s/2 - a_g; \\ \dfrac{(2a_g + a_s - 2c)^2}{16a_g}, & 3a_s/2 - a_g \le c < a_g - a_s/2; \\ \dfrac{2(a_g + a_s - c)^3}{27a_g a_s}, & a_g - a_s/2 \le c. \end{cases}$$

Where $\rho = \sqrt{6a_g a_s + (c_g + c_s)^2}$, $c = c_g + c_s$.

Proposition 1 shows that changes in tobacco equipment and service costs have the same effect on optimal bundle prices and the maximum profit. But under the strategy of pure bundling, when the marginal cost of the tobacco equipment and service is at a medium level, that is, $3a_s/2 - a_g \le c_g + c_s < a_g - a_s/2$, the impact of the change of the equipment valuation on the optimal bundle price is greater than that of service valuation. If the marginal cost of the tobacco equipment and service is higher or lower, the tobacco manufacturing enterprises' valuation of the equipment and service has the same effect on the bundle price.

SEPARATE PRICING STRATEGY

It is easy to prove that if the domain of a two-dimensional random variable (U, V) is $[0, a_g] \times [0, a_s]$, given an any price combination (p'_g, p'_s) of the tobacco equipment manufacturers, there is a corresponding price combination (p_g, p_s) in the above domain that brings the same market result (i.e., the same sales volume and profit) of the original price combination. Therefore, when deciding the optimal equipment price and optional service price, the tobacco manufacturing enterprises only need to select on the region $[0, a_g] \times [0, a_s]$, which narrows the search scope of the optimal price. Therefore, in order to simplify the formulation in the following analysis process, unless specifically stated, the pricing analysis of the tobacco equipment and optional value-added service is carried out only on the region $[0, a_g] \times [0, a_s]$.

When the tobacco equipment and optional value-added service of the equipment manufacturer are sold separately, the sales functions of the equipment and optional value-added service are as follows:

$$q_g = q_g(p_g, p_s) = \begin{cases} \frac{2a_g a_s - p_g{}^2 - 2p_s p_g}{2a_g a_s}, & p_g + p_s \le a_s; \\ \frac{2a_g a_s + a_s{}^2 - 2a_s p_g - 2a_s p_s + p_s{}^2}{2a_g a_s}, & a_s < p_g + p_s. \end{cases} \quad (2)$$

$$q_s = q_s(p_g, p_s) = \begin{cases} \frac{2a_g a_s - 2a_g p_s - p_g{}^2}{2a_g a_s}, & p_g + p_s \le a_s; \\ \frac{(a_s - p_s)(2a_g - 2p_g + a_s - p_s)}{2a_g a_s}, & a_s < p_g + p_s. \end{cases} \quad (3)$$

Marginal Cost of Optional Service $c_s = 0$

If the optional value-added service cost of the tobacco equipment manufacturer is invested beforehand, similar to Jain, N., et al. (2013), the marginal cost is 0. Thus, the total profit function is:

$$\pi(p_g, p_s) = (p_g - c_g)q_g + p_s q_s \quad (4)$$

Proposition 2. When $c_s = 0$, the optimal prices to sell tobacco equipment and optional value-added service separately are

(1) When $2a_g/3 + 2c_g/3 + c_g{}^2/(6a_g) \le a_s$,
$p_g^* = (2a_g + c_g)/3$, $p_s^* = \frac{a_s}{2} - \frac{a_g}{3} + \frac{c_g{}^2}{12a_g}$;

(2) When $(2a_g - 2c_g)/3 < a_s < 2a_g/3 + 2c_g/3 + {c_g}^2/(6a_g)$, $p_g^* = (2a_g + c_g)/3$, $p_s^* = \left(2a_s - \sqrt{2a_g a_s + {a_s}^2 - 2a_s c_g}\right)/3$

Proof. It is easy to prove that $\pi(p_g, p_s)$ is continuously differentiable in the non-negative region $(0, a_g) \times (0, a_s)$. The optimal prices under different situations are discussed respectively.

(1) When $p_g + p_s \le a_s$, $\pi(p_g, p_s) = \frac{1}{2a_g a_s}\left((2a_s p_g + (-2c_g + 2p_s)a_s - 2{p_s}^2)a_g + p_g(-{p_g}^2 + (c_g - 3p_s)p_g + 2c_g p_s)\right)$. Thus, $\frac{\partial}{\partial p_g}\pi(p_g, p_s) = \frac{1}{2a_g a_s}(-3{p_g}^2 + (2c_g - 6p_s)p_g + 2a_g a_s + 2c_g p_s)$, $\frac{\partial}{\partial p_s}\pi(p_g, p_s) = \frac{1}{2a_g a_s}\left((2a_s - 4p_s)a_g + 2p_g c_g - 3{p_g}^2\right)$

By setting both partial derivatives equal to zero and solving the simultaneous equations, we can get three sets of solutions:

(i) $p_g = (2a_g + c_g)/3$, $p_s = \frac{a_s}{2} - \frac{a_g}{3} + \frac{{c_g}^2}{12a_g}$; or (ii) $p_g = c_g/3 + \sqrt{6a_g a_s + {c_g}^2}/3$, $p_s = 0$; or (iii) $p_g = c_g/3 - \sqrt{6a_g a_s + {c_g}^2}/3$, $p_s = 0$.

The second-order partial derivatives of $\pi(p_g, p_s)$ are $\frac{\partial^2}{\partial {p_g}^2}\pi(p_g, p_s) = \frac{-3p_g + 2c_g - 6p_s}{a_g a_s}$, $\frac{\partial^2}{\partial {p_s}^2}\pi(p_g, p_s) = -\frac{2}{a_s}$, $\frac{\partial^2}{\partial p_g \partial p_s}\pi(p_g, p_s) = \frac{c_g - 3p_g}{a_g a_s}$. Let $\Delta = \frac{\partial^2}{\partial {p_g}^2}\pi(p_g, p_s) \cdot \frac{\partial^2}{\partial {p_s}^2}\pi(p_g, p_s) - [\frac{\partial^2}{\partial p_g \partial p_s}\pi(p_g, p_s)]^2$.

It is easy to prove that when $p_g + p_s \le a_s$, the Δ at the solutions (ii) and (iii) above are all less than 0, so $\pi(p_g, p_s)$ is not maximum at these points. When $2a_g/3 - {c_g}^2/(6a_g) \le a_s$, it is easy to prove that $\Delta > 0$ and $\frac{\partial^2}{\partial {p_s}^2}\pi(p_g, p_s) < 0$ at the solution (i), so $\pi(p_g, p_s)$ has a maximum at this point.

Because this solution holds for "low price" namely $p_g + p_s \le a_s$,$2a_g/3 + 2c_g/3 + {c_g}^2/(6a_g) \le a_s$.

(2) When $p_g + p_s > a_s$, $\pi(p_g, p_s) = \frac{1}{2a_g a_s}\left((p_g + p_s - c_g)(2a_g - 2p_g - 2p_s + a_s)a_s + {p_s}^2(3p_g + p_s - 2a_g - c_g)\right)$. Thus, $\frac{\partial}{\partial p_g}\pi(p_g, p_s) = \frac{1}{2a_g a_s}({a_s}^2 + (2a_g + 2c_g - 4p_g - 4p_s)a_s + 3{p_s}^2)$, $\frac{\partial}{\partial p_s}\pi(p_g, p_s) = \frac{1}{2a_g a_s}\left((a_s - 2p_s)(a_s - 2p_s + 2a_g + 2c_g - 4p_g) + p_s(2c_g - 2p_g - p_s)\right)$.

By setting both partial derivatives equal to zero and solving the simultaneous equations, we can get three sets of solutions:

(i) $p_g = \frac{(2a_g + c_g)}{3}$, $p_s = (2a_s + \sqrt{2a_g a_s + {a_s}^2 - 2a_s c_g})/3$; or (ii) $p_g = (2a_g + a_s + 2c_g)/4$, $p_s = 0$; or (iii) $p_g = (2a_g + c_g)/3$, $p_s = (2a_s - \sqrt{2a_g a_s + {a_s}^2 - 2a_s c_g})/3$.

The second-order partial derivatives of $\pi(p_g, p_s)$ are $\frac{\partial^2}{\partial {p_g}^2}\pi(p_g, p_s) = \frac{1}{2a_g a_s}({a_s}^2 + (2a_g + 2c_g - 4p_g - 4p_s)a_s + 3{p_s}^2)$, $\frac{\partial^2}{\partial {p_s}^2}\pi(p_g, p_s) = \frac{1}{a_g a_s}(3p_g + 3p_s - 2a_s - 2a_g - c_g)$, $\frac{\partial^2}{\partial p_g \partial p_s}\pi(p_g, p_s) = \frac{3p_s - 2a_s}{a_g a_s}$.

It is easy to prove that when $p_g + p_s > a_s$, the Δ at the solution (i) above is less than 0, so $\pi(p_g, p_s)$ is not maximum. When $(2a_g - 2c_g)/3 < a_s$, $\Delta < 0$ at the solution (ii), so $\pi(p_g, p_s)$ is not maximum too. When $(2a_g - 2c_g)/3 < a_s < 2a_g/3 + 2c_g/3 + {c_g}^2/(6a_g)$, it is easy to prove that $\Delta > 0$ and $\frac{\partial^2}{\partial {p_s}^2}\pi(p_g, p_s) < 0$ at the solution (iii), so $\pi(p_g, p_s)$ has a maximum at this point.

It is easy to prove that if a tobacco equipment manufacturer sells only equipment without optional value-added service, the optimal price is $(a_g+c_g)/2$. Therefore, combined with the analysis results of Proposition 1 and 2, we can see that whether the equipment and service are completely bundled or sold separately, both the optimal price of the bundle and the optimal price of the equipment considering the influence of the service are higher than $(a_g+c_g)/2$. This means that the tobacco equipment manufacturer should increase the sales price of tobacco equipment after providing optional service, instead of reducing the price of products and then earn back through service profits as some people hold.

Lemma 1. When a tobacco equipment manufacturer sells equipment and optional value-added service separately,

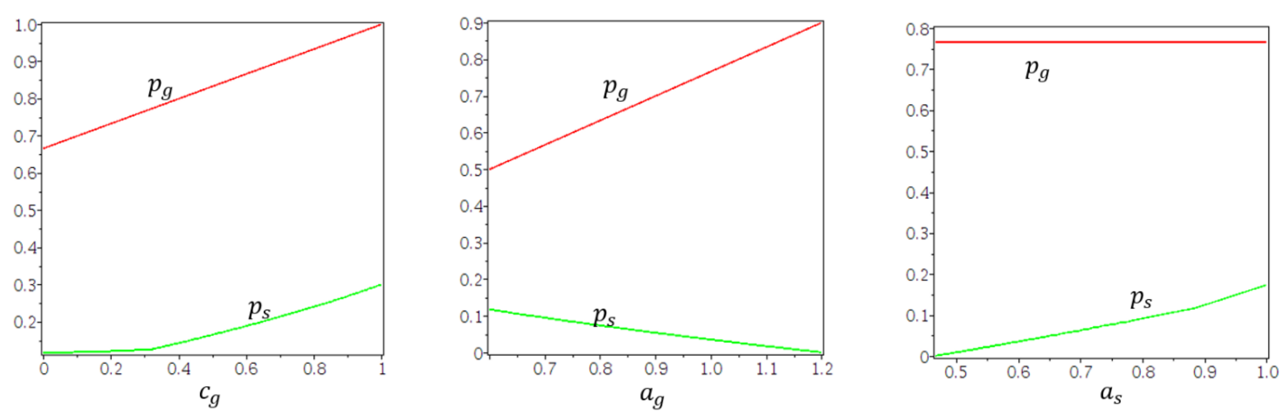
(1) $\frac{dp_g^*}{dc_g} > \frac{dp_s^*}{dc_g} > 0$;

(2) $\frac{dp_g^*}{da_g} > 0, \frac{dp_s^*}{da_g} < 0$;

(3) $\frac{dp_s^*}{da_s} > 0$, but the optimal price of the tobacco equipment is not affected by a_s.

Lemma 1 shows that, when a tobacco equipment manufacturer sells equipment and optional value-added service separately, the larger the c_g is, the higher the price of the equipment and optional value-added service will be. If the equipment cost of the tobacco equipment manufacturer rises, the price of the equipment and optional value-added service should be increased at the same time, but the increasing rate of the equipment price should be higher than the rate of optional value-added service price. When the market valuation of tobacco equipment rises, the equipment manufacturer should increase the equipment price and reduce the optional value-added service price simultaneously. It is interesting that the higher the market valuation of tobacco equipment is, the lower the optional value-added service price will be. If the valuation of optional value-added service is higher, the tobacco equipment manufacturer should increase the optional value-added service price, but keep the equipment price unchanged. The characterization of the influence of different parameters are provided in Figure 2.

Proposition 3. When tobacco equipment and optional value-added service are sold separately, the maximum profit of tobacco equipment manufacturer is



(a)Parameters: $\boldsymbol{a_g=1, a_s=0.9}$ (b) Parameters: $a_s=0.6, c_g=0.3$ (c) Parameters: $a_g=1, c_g=0.3$

Figure 2
Optimal Prices of the Tobacco Equipment and Optional Value-added Service When $c_s=0$

$$\pi_{NB}^{*} = \begin{cases} 288c_g - 144a_g - 108a_s + \frac{16a_g{}^2 - 24c_g{}^2}{a_s} - \frac{36c_g{}^2}{a_g} - \frac{16c_g{}^3}{a_g a_s} - \frac{3c_g{}^4}{a_s a_g{}^2}, \quad a_s \geq \frac{(2a_g+c_g)^2}{6a_g}; \\ \frac{\left(3a_g - 3c_g + \sqrt{a_s(2a_g - 2c_g + a_s)}\right)(2a_g - 2c_g + a_s) + a_s{}^2}{27a_g}, \quad a_s < \frac{(2a_g+c_g)^2}{6a_g}. \end{cases}$$

The above profit formula can be obtained by substituting the optimal prices in Proposition 2 in formula (4). The specific steps are omitted.

Proposition 4. Let $\theta = (a_g - c_g)/a_s$. When $c_s = 0$, the optimal pricing strategy of the tobacco equipment manufacturer is: (1) when $\theta < 3/2$, the equipment and service should be priced separately for selling; (2) when $\theta \geq 3/2$, the equipment and service should be purely bundled for pricing.

Proof. (1) When $\theta < 3/2$, namely $a_s > (2a_g - 2c_g)/3$, the discussion will be carried out in different situations.

Case 1: $c_g < 3a_s/2 - a_g$

According to Proposition 1, if the equipment and service are purely bundled, the optimal price is $p_B^* = c_g/3 + \sqrt{6a_g a_s + c_g^2}/3$. Let $p_g = p_B^* - x, p_s = x$, substitute the prices into formula (4), then profit function is $\pi(x) = \frac{1}{54a_g a_s}\left(\left(12a_g a_s + 2c_g{}^2 + 27x^2\right)\sqrt{6a_g a_s + c_g{}^2} - c_g\left(36a_g a_s - 2c_g{}^2\right) - 54x^2(a_g + x)\right)$. Obviously, $\pi(x)$ is continuously differentiable in x. It is easy to prove that when $c_g < 3a_s/2 - a_g$, $\pi(0) = \pi_B(p_B^*)$, $d\pi(x)/dx|_{x=0} = 0$, $d^2\pi(x)/dx^2|_{x=0} > 0$. Therefore, there exists $\alpha > 0$ that reducing the optimal bundle price appropriately by α as the equipment price and fixing the service price as α can increase the total profit of the equipment manufacturer. That is to say that selling the equipment and service separately is better than pure bundling.

Case 2: $3a_s/2 - a_g \leq c_g < a_g - a_s/2$ and Case 3: $a_g - a_s/2 \leq c_g$ are proved in the same way, omitted.

(2) Similar to the Proposition 2, it is proved that when $a_s \leq (2a_g - 2c_g)/3$, $p_g = (2a_g + a_s + 2c_g)/4$ and $p_s = 0$ are the unique critical point of $\pi(p_g, p_s)$. Obviously, this point is also the boundary point, so purely bundling price is better than pricing the equipment and service separately.

Proposition 4 identifies when a pure bundling strategy might be useful. When $c_s = 0$, if the market valuation of the service is low ($a_s \leq (2a_g - 2c_g)/3$), the optimal pricing strategy for a tobacco equipment manufacturer is to bundle the equipment and service as a whole and not to sell them separately; when the market valuation of the optional value-added service is relatively high ($a_s > (2a_g - 2c_g)/3$), the optimal pricing strategy for a tobacco manufacturer is to price the equipment and optional value-added service separately for sale. Specially, if the marginal cost of the equipment is zero, it can simplify the condition for the pure bundling strategy solution as $a_s/a_g \leq 2/3$. Proposition 4 extends the results proposed by Bhargava, H. K. (2013) which only compared the partial bundling strategy and the pure bundling strategy of two independent goods. The proof process of Proposition 4 also shows that if the purely bundling of the equipment and service is the optimal strategy for tobacco equipment manufacturers, the optimal bundled price must be in the range of $[a_s, a_g]$.

Marginal Cost of Optional value-added service $c_s > 0$

In the previous analysis, it is assumed that the marginal cost of optional value-added service provided by the tobacco equipment manufacturer is 0, but in many cases, the cost of service is related to sales. For example, there may be maintainers stationed in every customer enterprise. Therefore, it is necessary to analyse the situation where the marginal cost of optional value-added service $c_s > 0$. At this time, the total profit function of the tobacco equipment manufacturer is:

$$\pi(p_g, p_s) = (p_g - c_g)q_g + (p_s - c_s)q_s \quad (5)$$

Proposition 5. When $c_s > 0$, the optimal strategy for a tobacco equipment manufacturer is to price equipment and optional value-added service separately.

Proof. (1) Case 1: $p_B < a_g + c_s$.

Let p_B be the optimal bundle price, and q_B be the corresponding sales volume. Then, the profit of the tobacco equipment manufacturers is $\pi_B = (p_B - c_g - c_s)q_B$. The customer enterprises can be divided into two categories according to the relationship between V and c_s: $V \geq c_s$ and $V < c_s$. Let q_{B1} be the sales volume of the customer enterprises that $V \geq c_s$, q_{B2} be the sales volume of the customer enterprises that $V < c_s$.

Considering the pricing strategy of selling the tobacco equipment and optional value-added service separately, set the prices to be: $p_s = c_s$, $p_g = p_B - c_s$. At these prices, we can easily prove that the sales volume of customer enterprises who buy both tobacco equipment and service is q_{B1}. Let q_{g2} be the sales volume of customer enterprises who only buy tobacco equipment. It is easy to prove that $q_{g2} > q_{B2}$. Thus the profit of the tobacco equipment manufacturer is $\pi = (p_g + p_s - c_g - c_s)q_{B1} + (p_g - c_g)q_{g2} = (p_B - c_g - c_s)q_{B1} + (p_B - c_g - c_s)q_{g2}$.

Then $\pi - \pi_B = (p_B - c_g - c_s)(q_{g2} - q_{B2}) > 0$. Therefore, the optimal strategy for the tobacco equipment manufacturer is to price equipment and optional value-added service separately.

(2) Case 2: $p_B \geq a_g + c_s$.

According to Proposition 1, the optimal bundle price is $p_B = (a_g + a_s + 2c_g + 2c_s)/3$. Let $p_g = a_g - x, p_s = (-2a_g + a_s + 2c_g + 2c_s)/3 + x$. Obviously, when $x = 0$, the sales volume and profit under the separate pricing strategy at the prices (p_g, p_s) are exactly as same as those in bundling sales. By substituting the prices (p_g, p_s) above into formula (5), we can get the profit function $\pi(x)$. It is continuously differentiable, and $\mathrm{d}\pi(x)/dx|_{x=0} = \frac{2(c_s - a_g + a_s/2 + c_g)(a_g - c_g)}{3a_g a_s} > 0$. Therefore, raising the price of the tobacco equipment above properly and reducing the price of the optional value-added service can increase the total profit of the tobacco equipment enterprise, that is, the optimal strategy is to price the equipment and optional value-added service separately.

Proposition 5 shows that when the marginal cost of the optional value-added service is greater than 0, it is optimal to sell the tobacco equipment and optional value-added service separately.

Proposition 6. When $c_s > 0$, the optimal prices for a tobacco equipment manufacturer to sell equipment and optional value-added service separately are

(1) When $c_g \leq \frac{2a_g a_s - 2a_g c_s - 4a_g p_g^* + 3p_g^{*2}}{2p_g^*}$, $p_g^* = \frac{2a_g}{9} + \frac{c_g}{3} - \frac{2\tau}{9} cos\left(\frac{\pi}{3} + \frac{1}{3} arccos\left(\frac{2a_g(4a_g^2 - 54a_g a_s + 9a_g c_s - 9c_g^2 - 27c_s c_g)}{\tau^3}\right)\right)$, $p_s^* = \frac{a_s + c_s}{2} + \frac{2p_g^* c_g - 3p_g^{*2}}{4a_g}$,

(2) When $c_g > \frac{2a_g a_s - 2a_g c_s - 4a_g p_g^* + 3p_g^{*2}}{2p_g^*}$, $p_g^* = \frac{a_g + c_g}{2} + \frac{a_s^2 + 2c_s a_s - 4a_s p_s^* - 2c_s p_s^* + 3p_s^{*2}}{4a_s}$, $p_s^* = \frac{4a_s}{9} + \frac{c_s}{3} - \frac{2\omega}{9} cos\left(\frac{\pi}{3} + \frac{1}{3} arccos\left(\frac{2a_s(3\omega^2 - 27a_g c_s - 16a_s^2 + 27c_g c_s)}{\omega^3}\right)\right)$,

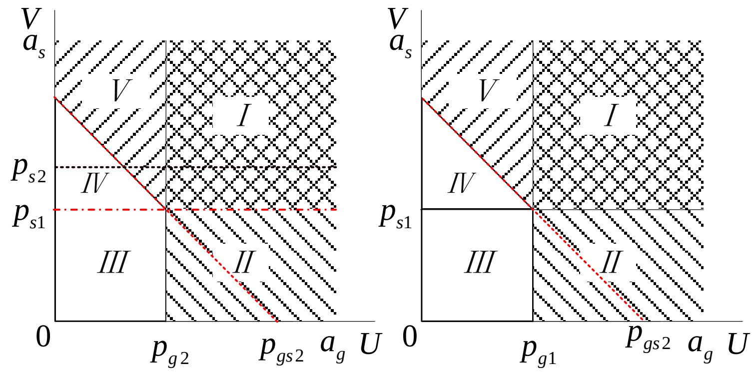
where $\tau = \sqrt{a_g(4a_g + 18a_s + 6c_s) + 3c_g^2}$, $\omega = \sqrt{6a_s(a_g - c_g - c_s) + 7a_s^2 + 3c_s^2}$.

Proof. Proposition 5 shows that, when $c_s > 0$, the points of the global maxima for $\pi(p_g, p_s)$ are not on the boundary of region $[0, a_g] \times [0, a_s]$. Since grad $\pi(p_g, p_s)$ is defined everywhere, the global maxima of $\pi(p_g, p_s)$ must be those where grad $\pi(p_g, p_s) = \vec{0}$.

(1)When $p_g + p_s \le a_s$, $\pi(p_g,p_s) = \frac{1}{2a_g a_s}\Big(\big(2p_g a_s + (-2c_g - 2c_s + 2p_s)a_s + 2p_s(-p_s + c_s)\big)a_g + p_g\big(-p_g^2 + (c_g + c_s - 3p_s)p_g + 2c_g p_s\big)\Big)$. By differentiating $\pi(p_g,p_s)$ with respect to p_s and setting the partial derivative equal to 0, we can get $p_s = \frac{2a_g a_s + 2a_g c_s + 2p_g c_g - 3p_g^2}{4a_g}$. Substitute it into $\pi(p_g,p_s)$, then $\pi(p_g) = \frac{1}{(16a_g^2 a_s)}\Big(4(a_s^2 + 4p_g a_s + (-4c_g - 2c_s)a_s + c_s^2)a_g^2 + 4p_g\big(-2p_g^2 + (-3a_s + 2c_g - c_s)p_g + 2c_g(a_s + c_s)\big)a_g + (-3p_g + 2c_g)^2 p_g^2\Big)$. Thus, $d\pi(p_g)/dp_g = \frac{1}{4a_g^2 a_s}\Big(9p_g^3 + (-6a_g - 9c_g)p_g^2 + \big((-6a_s + 4c_g - 2c_s)a_g + 2c_g^2\big)p_g + 4a_g^2 a_s + 2c_g a_g(a_s + c_s)\Big)$. Since $d\pi(p_g)/dp_g$ is a cubic function of p_g, setting it equal to 0, we can get three solutions. It is easy to prove that $\pi(p_g)$ has a maximum only at the solution $p_g^* = \frac{2a_g}{9} + \frac{c_g}{3} - \frac{2\tau}{9}\cos\Big(\frac{\pi}{3} + \frac{1}{3}\arccos\Big(\frac{2a_g(4a_g^2 - 54a_g a_s + 9a_g c_s - 9c_g^2 - 27c_s c_g)}{\tau^3}\Big)\Big)$, of which $\tau = \sqrt{4a_g^2 + 18a_g a_s + 6a_g c_s + 3c_g^2}$. Because this solution holds for "low price" namely $p_g + p_s \le a_s$, $c_g \le \frac{2a_g a_s - 2a_g c_s - 4a_g p_g^* + 3p_g^{*2}}{2p_g^*}$.

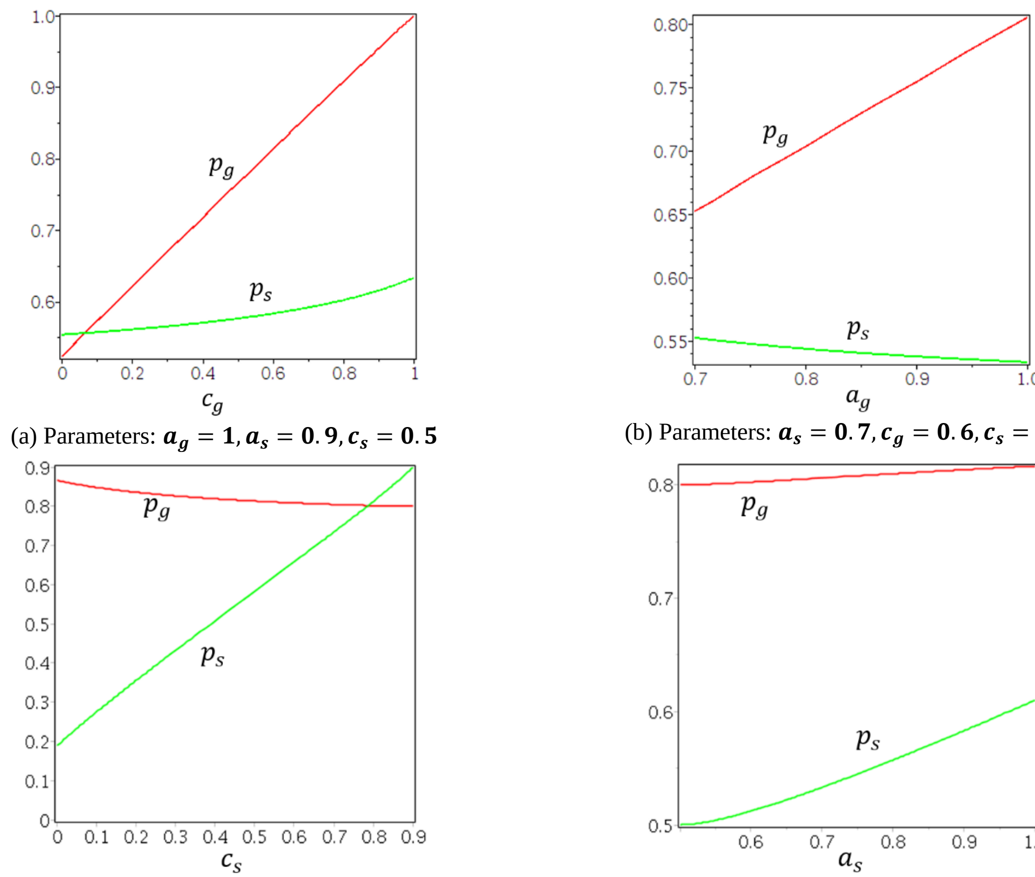
(2) When $p_g + p_s > a_s$, $\pi(p_g,p_s) = \frac{1}{2a_g a_s}\Bigg((-c_g - c_s + p_g + p_s)a_s^2 - 2(c_g + c_s - p_g - p_s)(a_g - p_g - p_s)a_s + 2p_s\Big(\frac{1}{2p_s^2} + \Big(-a_g - \frac{1}{2c_g} - \frac{1}{2c_s} + \frac{3}{2p_g}\Big)p_s + c_s(a_g - p_g)\Big)\Bigg)$. By differentiating $\pi(p_g,p_s)$ with respect to p_g and setting the partial derivative equal to 0, we can get $p_g = \frac{2a_g a_s + a_s^2 + 2c_g a_s + 2a_s c_s - 4a_s p_s - 2c_s p_s + 3p_s^2}{4a_s}$. Substituting it into $\pi(p_g,p_s)$, $\pi(p_s) = \pi(p_g(p_s),p_s)$. Thus, $d\pi(p_s)/dp_s = \frac{1}{4a_g a_s^2}\big(9p_s^3 + (-12a_s - 9c_s)p_s^2 + (3a_s^2 + (-2a_g + 2c_g + 10c_s)a_s + 2c_s^2)p_s - a_s^2 c_s + 2c_s(a_g - c_g - c_s)a_s\big)$. Since $d\pi(p_s)/dp_s$ is a cubic function of p_s, setting it equal to 0, we can get three solutions. It is easy to prove that $\pi(p_g)$ has a maximum only at the solution $p_s^* = \frac{4a_s}{9} + \frac{c_s}{3} - \frac{2\omega}{9}\cos\Big(\frac{\pi}{3} + \frac{1}{3}\arccos\Big(\frac{2a_s(-3\omega^2 - 27a_g c_s - 16a_s^2 + 27c_g c_s)}{\omega^3}\Big)\Big)$, of which $\omega = \sqrt{6a_g a_s + 7a_s^2 - 6c_g a_s - 6c_s a_s + 3c_s^2}$.

Proposition 6 gives the exact analytical solution for the general case that both the marginal cost of the tobacco equipment and optional value-added service are positive. Figure 3 illustrates the relationship between the price and the different parameters. It shows that the optimal prices of the tobacco equipment and optional value-added service will go up with the increase of marginal cost of the tobacco equipment or the increase of the customer valuation of the service provided by the tobacco equipment manufacturer. It is interesting that, if the customer valuation of the tobacco equipment increases, as shown in Figure 3(b), optimal price of optional value-added service will decline. And it is different from the common sense that, if the marginal cost of optional value-added service decreases, as shown in Figure 3(c), the optimal price of tobacco equipment will go down.



(a) (b)

Figure 4
Equivalent Conversion of Different Pricing Strategies



(a) Parameters: $\boldsymbol{a_g = 1, a_s = 0.9, c_s = 0.5}$

(b) Parameters: $\boldsymbol{a_s = 0.7, c_g = 0.6, c_s = 0.5}$

(c) Parameters: $\boldsymbol{a_g = 1, a_s = 0.9, c_g = 0.6}$

(d) Parameters: $a_g = 1, c_g = 0.6, c_s = 0.5$

Figure 3
Optimal Prices of the Product and Optional Value-added Service When $c_s > 0$

Proposition 7. The following pricing strategies are equivalent, that is, as long as the prices under either strategy are known, there always exist the corresponding prices under the other two pricing strategies, and the sales and profits under the three strategies are the same:

Strategy 1: Pure separating strategy, under which the product and service are sold separately;

Strategy 2: Mixed bundling strategy, under which the equipment, service and the bundle are sold separately;

Strategy 3: Partial bundling strategy, under which only the equipment and the bundle are sold separately.

Proof. Strategy 1 ⇔Strategy 2.

Under strategy 1, let p_{g1} and p_{s1} be the price of the equipment and service respectively, and let q_{g1} and q_{s1} be the corresponding sales volumes of the equipment and service respectively. Under strategy 2, let p_{g2}, p_{s2} and p_{B2} be the prices of the equipment, service and the bundle respectively, and let q_{g2}, q_{s2} and q_{B2} be the corresponding sales volumes of the equipment, service and the bundle respectively.

(1) Strategy 2→Strategy 1. (i) Obviously, if $p_{g2}+p_{s2}\leq p_{gs2}$, there are no customer enterprises who buy the bundle at the price p_{gs2}, so the sales and profits under the two strategies are the same when $p_{g1}=p_{g2}$ and $p_{s1}=p_{g2}$. (ii) If $p_{g2}+p_{s2}>p_{B2}$, customer enterprises of type I and V in the Figure 4(a) will definitely purchase the bundle, while customer enterprises of type II will only buy equipment, therefore, the profit under strategy 2 is $\pi_2=\left(p_{g2}-c_g\right)q_{g2}+\left(p_{B2}-c_g-c_s\right)q_{B2}$. The Figure 4(a) shows that if strategy 1 is adopted and the price of the equipment and service are $p_{g1}=p_{g2}$ and $p_{s1}=p_{B2}-p_{g2}$, respectively, the sales volumes of equipment and service are $q_{g1}=q_{g2}+q_{B2}$ and $q_{s1}=q_{B2}$, thus the profit under strategy 1 is $\pi_1=(p_{g1}-c_g)q_{g1}+(p_{s1}-c_s)q_{s1}=\left(p_{g2}-c_g\right)\left(q_{g2}+q_{B2}\right)+\left(p_{B2}-p_{g2}-c_s\right)q_{B2}\equiv\pi_2$. Therefore, given any prices under strategy 2, the corresponding prices under strategy 1 can always be found to achieve the same sales and profits.

(2) Strategy 1→Strategy 2. Given the arbitrary price combination p_{g1} and p_{s1} under strategy 1, the profit function is $\pi_1=\left(p_{g1}-c_g\right)q_{g1}+(p_{s1}-c_s)q_{s1}$. The Figure 4(b) shows that if strategy 2 is adopted at prices $p_{g2}=p_{g1}$, $p_{B2}=p_{g1}+p_{s1}$ and $p_{s2}=p_{s1}+\alpha$, of which $\alpha\geq 0$, the sales volumes of the equipment, service and the bundle are $q_{g2}=q_{g1}-q_{s1}$, $q_{s2}=0$ and $q_{B2}=q_{s1}$. Thus the profit is $\pi_2=\left(p_{g2}-c_g\right)q_{g2}+\left(p_{B2}-c_g-c_s\right)q_{gs2}=\left(p_{g1}-c_g\right)\left(q_{g1}-q_{s1}\right)+(p_{g1}+p_{s1}-c_g-c_s)q_{s1}\equiv\pi_1$. Therefore, given any prices under strategy 1, the corresponding prices under strategy 2 can always be found to achieve the same sales and profits.

Therefore, strategy 1 and strategy 2 are completely equivalent. Similarly, it can be proved that strategy 1 and strategy 3 are equivalent, so that the three strategies in the Proposition 7 are equivalent.

Proposition 7 shows that there are only two pricing strategies in essence for an equipment manufacturer: pure bundling pricing and separate pricing strategy. Therefore, a tobacco equipment manufacturer only needs to compare the two basic strategies in practice.

CONCLUSION

Nowadays, more and more tobacco equipment manufacturers provide not only the traditional tangible equipment but also optional value-added service based on the equipment. Though there are a lot of literatures that illustrated the pricing problems of product bundling, they are not suitable for the joint pricing of the equipment and optional value-added service. A joint pricing model of the tobacco equipment and optional value-added service is constructed in this paper to effectively depict the interaction between tobacco equipment and optional value-added service. Moreover, this paper derives that only the two basic strategies consisting of pure bundling and separate sale need to be considered in practical applications of pricing, and identifies the thresholds for the two basic strategies. The research results and pricing analysis model can provide reference for pricing decision analysis of tobacco equipment manufacturers.

However, in this paper, the situation of tobacco equipment manufacturers providing multiple products and/or multiple value-added service is not considered, nor is the competition among tobacco equipment manufacturers and the cooperation among supply chain members considered. Future research can be expanded in the above aspects.

Conflict of Interest Disclosure Statement

The authors have no conflicts of interest, financial or otherwise.

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