Control of Flexible Half Car Model Suspension System

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Abstract

The suspension system in vehicles plays a vital role in improving driving comfort and safety. Many researchers regarded the structure as rigid when investigating the behavior of the car's suspension and chassis with various driving conditions and unregulated road conditions. They have faced certain challenges to: ensure safety, driving comfort, and contact of the wheels with the road all the time, especially when turning at different road conditions. The purpose of this paper is to consider the flexibility of the structure during the design stage of the vehicle. especially when converting the suspension system from a passive suspension system to an active suspension system. Using finite element technique and MATLAB to obtain mass and stiffness matrices, which are to define the "state space". The optimal system can be designed using the Linear Quadratic regulator technique (LQR). The results show that case for converting the system from its passive state to an active state, So, it is necessary to consider the chassis flexibility during design, especially for cars that have long wheelbases.

Keywords (1) Finite element method (2) Beam equation (3) Half vehiclemodel (4) Simply supported beam

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1.Introduction

Passive and active suspension systems for automobiles have been extensively studied, while a few predictions of the capabilities of dissipative and slow active systems have been considered. Most of the work surveyed here is limited to passive, active, semi-active, etc. The optimal control theory for the suspension of passenger cars does not include chassis flexibility; this means they relate to compact automobiles, but for long-wheelbase vehicles, chassis flexibility will affect suspension system performance. Just a few searchers consider this effect, but for a simple model, they represent the vehicle chassis with a beam [1]. Considering the conventional two-Dof model as a rigid chassis, which developed into a flexible chassis, the comparison between both cases found that the flexible chassis was impacting the system [2]. Also, D. A. Crolla Proved by the comparison of the responses of the rigid and flexible body models, it has been found that the framing flexibility strongly affects the accelerations of both the driver and the truck body. Therefore, frame flexibility effects must be taken into account in the design of the cab and engine suspension systems [3]. Then they analyzed and designed a Hydropneumatics limited bandwidth active suspension system [4]. The effect of the vehicle chassis flexibility on the dynamic response is analyzed and evaluated based on the maximum car center acceleration [5]. It is concluded that the increase in flexural stiffness of the car body can lead to a notable decrease in its maximum acceleration [6]. Finally, the main goal of this work is to study and modify the passive system to the active suspension system for vehicles, taking into account chassis flexibility using a half-vehicle dynamic analysis system package in conjunction with a finite element program. In this paper, the linear quadratic regulator LQR is a special type of optimal control used in active suspension [7 and 8]. The Riccati equation is used to optimize linear LQR tuning parameters to achieve the desired output response. The robustness of the control system will be tested within a simulation in MATLAB software. Also, the present report covers half the vehicle models described by their first vibration modes, which are achieved by the finite element model. Additionally, the solution is obtained by evaluating the dynamic equation of motion in a state variable, additionally with the application of linear optimal control theory.

2. A half model of a vehicle and the governing equation

Figure (1) shows the model of a half vehicle that subjected the irregular excitation from a road surface at the front/rear wheels. We consider the excitation acts at the chassis tip. where m is the mass for the vehicle body, I_{cg} is the mass moment of inertia for the vehicle body, $m_1$and m_2 are the masses of the front/rear wheels respectively, C_{sf} and C_{sr} are the damping coefficientsof front /rear suspensions respectively, k_{sf} and k_{sr} are the spring stiffness of front/rear suspensionsrespectively, k_{tf} and k_{tr} are the stiffness of front/reartires respectively. x_{cg} is the vertical displacementof the vehicle body at the center of gravity, φ is the rotary angle of the vehicle body at the center ofgravity, x_{tf} and x_{tr} are the vertical displacements of the front/rear wheels, y_f and y_r are theirregular excitations from the road surface, $a_1$and $a_2$are the distances of the front/rear suspension locations, regarding the Centre of gravity of the vehicle body and $a_1 + a_2 = L$

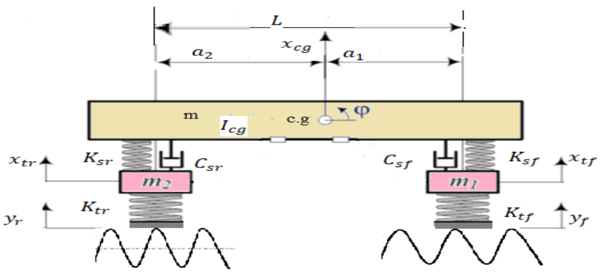

Figure (1) Half-vehiclemodel

The beam's transverse deflection u_d(x) is governed by the Euler-Bernoulli beam theory [9, 10] as a fourth-order differential equation.

$$\frac{d^2}{d(x_d)^2}\left[y_d(x_d)\frac{d^2u_d}{d(x_d)^2}\right] = f(x_d, u_d), \qquad 0 \le x_d \le L(1)$$

Subject to boundary conditions.

$$u_d(0) = (u_d)_0,\quad \frac{d^2 u_d(0)}{d(x_d)^2} = \varphi_0,\quad u_d(L) = (u_d)_L,\quad \frac{d^2 u_d(L)}{d(x_d)^2} = \varphi_L\ . \qquad (2)$$

Assume the function $y_d(x_d) = I * E$is constant, where $(I, E)$are the beam's moment of inertia and modulus of elasticity, respectively. In the linear case, the transversely distributed load is $f(x_d, u_d)$, which equals$q_d(x_d) * u_d(x_d) + p_d(x_d)$, where$u_d(x_d)$is the beam deflection, $q_d(x_d)$ is the coefficient of ground elasticity, and $p_d(x_d)$ is the uniform load applied to the beam. Substituting the value of equation (1), we get the following.

$$\frac{d^2}{(dx_d)^2}\left[EI\frac{d^2 u_d}{(dx_d)^2}\right] = q_d(x_d)u_d(x_d) + p_d(x_d),\ \ 0 \le x_d \le L, \qquad (3)$$

When the beam is fixed at ends and$u_d(0) = 0$, the solution of $u_d(x_d)$describes the deflection of the beam under the load$p_d(x_d)$. In this case, the governing equations become as following equation.

$$\frac{d^2}{(dx_d)^2}\left[EI\frac{d^2 u_d}{(dx_d)^2}\right] = p_d(x_d),\ \ 0 \le x_d \le L, \qquad (4)$$

We will utilize the Galerkin Finite Element Method (FEM) to solve equation (4) [11].

3.GalerkinFinite Element Method

Figure 2 shows the system after divided into 6 elements that have 2 nodes. For simplicity, we assume chassis as two elements only.

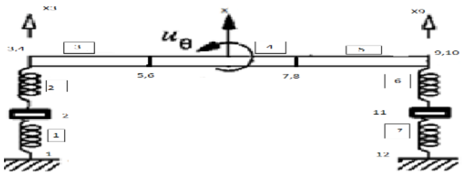

Figure 2 system divided into elements

Figure (3) shows a beam (chassis) element that has two degrees of freedom per node,u_i,φ_i represents displacement and deflection of node (i) respectively, u_{i+1} and φ_{i+1} represents displacement and deflection ofnode $(i+1)^{th}$respectively. The length of the element is l_e[2].

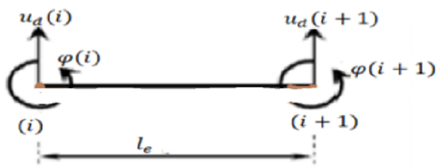

Figure (3) beam element

Obtain the weak form from equation (4). Multiply the residual of a governing equation (4) by a weight function $N_i(x_d)$ and integrate it by parts to evenly distribute the order of differentiation on u_d and $N_i(x_d)$toget the equation as follows:

$$\int_0^{l_e}\left[EI\frac{d^4u_d}{d(x_d)^4}-p_d(x_d)\right]N_i dx_d=\left[EI\frac{d^3u_d}{d(x_d)^3}N_i\right]_0^{l_e}-\left[EI\frac{d^2u_d}{d(x_d)^2}\frac{dN_i}{dx_d}\right]_0^{l_e}$$

$$+\int_0^{l_e}\left[EI\;\frac{d^2N_i}{d(x_d)^2}\frac{d^2u}{d(x_d)^2}-p_dN_i\right]dx_d=0(5)$$

We can rewrite equation (5) in the following form

$$\int_0^{l_e}\left[EI\frac{d^4u_d}{d(x_d)^4}-p_d(x_d)\right]N_i dx_d=[EIu_{xxx}N_i]_0^{l_e}-[EI(x_d)_{xx}(N_i)_x]_0^{l_e}$$

$$+\int_0^{l_e}EIN_{i,x}u_{xx}\;dx_d-\int_0^{l}p_dN_idx_d=0(6)$$

After obtaining the weak form, we proceed to choose the suitable elements approximating functions. It can be noted that the highest order of the derivative on (x) in the weak form (6) is three; therefore, we choose an approximating function that is thrice differentiable. This request is satisfied by the cubic interpolation polynomial where these cubic interpolation functions are known as Hermit cubic interpolation functions.

$$N_1=\frac{1}{(l_e)^3}(2x^3-3x^2l_e+(l_e)^3),$$

$$N_2=\frac{1}{(l_e)^3}(x^3l_e-2x^2(l_e)^2+x(l_e)^3),$$

$$N_3=\frac{1}{(l_e)^3}(-2x^3-3x^2l_e),$$

$$N_4=\frac{1}{(l_e)^3}(x^3l_e-x^2(l_e)^2)(7)$$

Where $N_1,N_2,N_3,\&N_4$Are called the shape functions for a beam element. For the beam element, $N_1=1$ when evaluated at node 1 and $N_1=0$ when evaluated at node 2. From equation (6) the stiffness matrix and force vector are given as follows

$$K_{i,j}^{(e)}=EI\int_0^{l_e}\frac{d^2N_i}{dx^2}\frac{d^2N_j}{dx^2}\;dx\;\;,$$

Where: $K_{i,j}^{(e)}$ stiffness element

And the force vector is $f_i=\int_0^L p_dN_i\,dx$ for the first element

$K_{11}^{(e)} = \int_0^{l_e} \frac{d^2N_1}{dx^2}\frac{d^2N_1}{dx^2}\,dx = \int_0^{l_e} \frac{1}{(l_e)^3}(12x-6l_e)\frac{1}{(l_e)^3}(12x-6l_e)dx = \frac{1}{(l_e)^6}\int_0^{l_e} 144x^2 - 144\,xl_e + 36\,(l_e)^2dx = \frac{1}{(l_e)^6}\int_0^{l_e}[48\,x^3 - 72xl_e + 36xl_e]_0^{l_e} = \frac{1}{(l_e)^6}[12(l_e)^3] = \frac{12}{(l_e)^3}$(8)The remaining elements are found similarly. The stiffness matrixfor any element becomes as follows

$$K_{i,j}^{(e)} = \frac{EI}{(l_e)^3}\begin{bmatrix} 12 & 6(l_e) & -12 & 6(l_e) \\ 6(l_e) & 4(l_e)^2 & -6(l_e) & 2(l_e)^2 \\ -12 & -6(l_e) & 12 & -6(l_e) \\ 6(l_e) & 2(l_e)^2 & -6(l_e) & 4(l_e)^2 \end{bmatrix} \quad (9)$$

Similarly, we can obtain the force vector matrix. The first value in the force vector is

Evaluate below.

$$\int_0^{l_e} p_d\left(1-\frac{3x^2}{(l_e)^2}+\frac{2x^3}{(l_e)^3}\right)dx = p_d\left[x-\frac{x^3}{(l_e)^2}+\frac{x^4}{2(l_e)^3}\right]_0^{l_e} = p_d\left((l_e)-\frac{(l_e)^3}{(l_e)^2}+\frac{(l_e)^4}{2(l_e)^3}\right) = p_d\left(\frac{l_e}{2}\right) \quad (10)$$

The remaining values are obtained in a similar manner using their corresponding shape functions. The resulting force vector ($f^{(e)}$) is given as

$$f^{(e)} = \frac{p_d l_e}{2}\begin{bmatrix} 1 \\ 6l_e \\ 1 \\ -6l_e \end{bmatrix} \quad (11)$$

The corresponding beam equation from equation (9), (11) can be represented as

$$\{f^{(e)}\} = \left[K_{i,j}^{(e)}\right] * \{u_d\}$$

$$\frac{p_d l_e}{2}\begin{bmatrix} 1 \\ 6l_e \\ 1 \\ -6l_e \end{bmatrix} = \frac{EI}{(l_e)^3}\begin{bmatrix} 12 & 6(l_e) & -12 & 6(l_e) \\ 6(l_e) & 4(l_e)^2 & -6(l_e) & 2(l_e)^2 \\ -12 & -6(l_e) & 12 & -6(l_e) \\ 6(l_e) & 2(l_e)^2 & -6(l_e) & 4(l_e)^2 \end{bmatrix}\begin{bmatrix} u_{d\,1} \\ u_{d\,2} \\ u_{d\,3} \\ u_{d\,4} \end{bmatrix} \quad (12)$$

The system of equations is solved by using MATLAB software. The element displacements were solved under different conditions prescribed [13, 14].

3.1Stiffness matrix for a spring element:

The relation between the nodal force(f_i & f_j) and nodal displacement (u_i&u_j) of typical spring element k_{se}at nodal spring$i$ *and* j respectively as shown in Figure (4) [13].

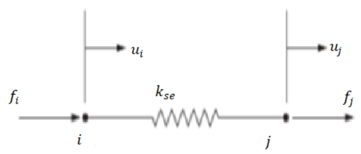

Figure (4) spring element

Thus, the force at nodals i and j can be expected as

$$f_i = k_{se}\left(u_i - u_j\right) \qquad (13)$$

$$f_j = k_{se}\left(u_j - u_i\right) (14)$$

Or Equations 13,14 may be arranged as follows.

$$\begin{Bmatrix} f_i \\ f_j \end{Bmatrix} = \begin{bmatrix} k_{se} & -k_{se} \\ -k_{se} & k_{se} \end{bmatrix} \begin{Bmatrix} u_i \\ u_j \end{Bmatrix} \qquad (15)$$

3.2 The global stiffness matrix for our case

From equations (12, 15) for the beam element and a spring element respectively (for simplicity assume the beam is one element) we can get the global stiffness matrix(K_G) for the system as follows

$$K_G = \frac{EI}{l^3}\begin{bmatrix} \frac{k_{se}l^3}{EI} & -\frac{k_{se}l^3}{EI} & 0 & 0 & 0 & 0 \\ -\frac{k_{se}l^3}{EI} & 12+\frac{k_{se}l^3}{EI} & 6l & -12 & 6l & 0 \\ 0 & 6l & 4l^2 & -6l & 2l^2 & 0 \\ 0 & -12 & -6l & 12+\frac{k_{se}l^3}{EI} & -6l & -\frac{k_{se}l^3}{EI} \\ 0 & 6l & 2l^2 & -6l & 4l^2 & 0 \\ 0 & 0 & 0 & -\frac{k_{se}l^3}{EI} & 0 & \frac{k_{se}l^3}{EI} \end{bmatrix} (16)$$

Use boundary condition in equation (16). Then a global stiffness matrix becomes as follows:

$$K_{sys} = \frac{EI}{l^3}\begin{bmatrix} 12+\frac{k_{se}l^3}{EI} & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12+\frac{k_{se}l^3}{EI} & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} (17)$$

3.3 The element mass matrix for the beam

$$M = \int_0^L N^T m N \, dx$$

Where: $m = \rho AL$, N=$[N_1 \quad N_2 \quad N_3 \quad N_4]$, we can get the value of (N) from equation (7). Now, we can get the value of $(N^T N)$ as follows

$$N^T N = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} [N_1 \quad N_2 \quad N_3 \quad N_4] \qquad \text{Or} N^T N = \begin{bmatrix} N_1N_1 & N_1N_2 & N_1N_3 & N_1N_4 \\ N_2N_1 & N_2N_2 & N_2N_3 & N_2N_4 \\ N_3N_1 & N_3N_2 & N_3N_3 & N_3N_4 \\ N_4N_1 & N_4N_2 & N_4N_3 & N_4N_4 \end{bmatrix} (18)$$

$$N_1N_1 = \left\{\frac{1}{l^3}(2(x_d)^3 - 3(x_d)^2 l + l^3)\right\}^2 = \frac{1}{l^6}(2(x_d)^3 - 3(x_d)^2 l + l^3)^2 \dots\dots\dots$$

$$= 4(x_d)^6 - 12(x_d)^5 l + 9(x_d)^4 l^2 + 4(x_d)^3 l^3 - 6x^2 l^4 + l^6$$

Now,

$$\frac{1}{l^6}\int_0^L N_1N_1 dx_d = \frac{4}{7}l^7 - 2l^7 + \frac{9}{5}l^7 + l^7 - 2l^7 + l^7 = \frac{l^7}{l^6}\left(\frac{13}{35}\right) = \frac{l*156}{420}$$

For the elementN_1N_1, we can do all elements of the equation (18) in the same way to get the global mass matrix (19) for the beam in our system as follows:

$$M = \frac{\rho Al}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & -13l & -3l^2 \\ 54 & -13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} (19)$$

4.Creation of State-space model

The state space representation is a mathematical model of a physical system as a set of input, output, and state variables related by first-order differential equations. The state space is converting the second-order differential equations for dynamic systems in the first-order state space dynamic systems, which has a certain advantage over the second-order form descriptions. The generalized state-space representation is as follows [15]:

$\dot{x}_{ss} = (A_{ss})X_{ss} + (B_{ss})F_{ss}$(20)

$y_{ss} = (C_{ss})X_{ss} + (D_{ss})$u(21)

WhereA_{ss} the system matrix,B_{ss} the input matrix, C_{ss} theoutput matrix, D_{ss} the direct transmission matrix.X_{ss}, u and y_{ss}represent the state vector, input vector, and output vectorrespectively. X_{ss}Used as the state variable to obtain the stateequations. Figure 4 show the system after divided into element. We divide chassis into two elements

Where: -

$$A_{ss} = \begin{bmatrix} 0 & I \\ -M_{sys}^{-1} * K_{sys} & -M_{sys}^{-1} * C_{sys} \end{bmatrix}$$

$$B_{ss} = \begin{bmatrix} 0 \\ -M_{sys} * F_{sys} \end{bmatrix}$$

$$D_{ss} = [0]$$

$$F_{ss} = [f_{sys}]$$

Where: -

$[A_{ss}]$ Matrix (n*n)

$[B_{ss}]$ Matrix (n*m)

$[C_{ss}]$ Matrix (r*n)

$[D_{ss}]$ Matrix (r*m)

$[F_{ss}]$ Matrix(1*n)

(n) The number of Dof,(m) The number of inputs, (r) The number of outputs.

5.MATLAB solution tolinear quadratic regulator problem

Lqr is the type of optimal control that deals with linear systems and the minimization of cost. MATLAB simulation associated with the Algebraic Riccati equation (ARE) to find solutions of Lqr [16]. The value of the weighting parameters Q and R are needed in the determination of optimal control gain of the systems, as they vary the minimization of the quadratic performance index. In this report, Lqr control design is presented for the control of a half-car active suspension system model. The Lqr design problem is to design a state feedback controller (K)_gain such that the objective function J is minimized. In this technique, a feedback gain matrix is calculated to minimize the objective function to achieve some compromise between the use of control, the magnitude, and the speed of response, thus guaranteeing a stable system [7].

5.1 The algebraic Riccati equation.

The algebraic Riccati equation (ARE) has been widely used in control system syntheses, especially in optimal control and robust control. Consider the algebraic Riccati equation as follows

$(A_{ss})^T P + P(A_{ss}) - P(B_{ss})(A_{ss})^T + Q = 0, Q \geq 0$(23)

Where*(A)*and *(B)*are real matrices has dimensions $(n \times n)$and$(n \times m)$respectively.Q is a positive semidefinite real symmetric matrix. It is well known that(23) is associated with the following linear system:

$$\dot{x}(t) = A_{ss}x(\dot{t}) + B_{ss}U_{gain}(t)$$

$x(0) = x_0$(24)

With the state feedback control

$U_{gain}(t) = -K_{gain}\, x(t)$(25)

$K_{gain} = R^{-1}(B_{ss})^T P$ (26)

The performance index

$J = \int_0^\infty \left(x^T Q x + (U_{gain})^T (U_{gain})\right) dt$(27)

5.2Design of control algorithms

5.2.1 Linear quadratic regulator (Lqr) controller.

The suspension system model has been represented in the form of state space as equation (20). TheLqr control algorithm is designed to obtain the value of the control vector ‘u’ such as to reduce the cost function ‘J’that is given as equation (27). The value of the feedback control vector ‘U_{gain}’ is defined as equation (25). The state feedback gain matrix ‘K_{gain}’ is calculated from equation (28). The matrix P is computed by solving the algebraic Riccati Equation (23). Substituting the gain matrix ‘K_{gain}’ and control vector ‘U_{gain}’ in the state equation (25). We obtain

$$\dot{x}_{(t)} = [A_{ss} - B_{ss}K_{gain}]x_{(t)} (28)$$

The four degrees of freedom half-car model is considered to analyze the behavior of the vehicle. Figure (5) shows the schematic diagram of a half-car model. A vehicle model consists of three bodies. One sprung mass and two unsprung masses. For the sprung mass, vertical and pitch movement is allowed, and for the unsprung masses, only vertical movement of the front and rear wheels.

Where:

x_{tf} Front tip displacement

x_{tr} Rear tip displacement

f_{af} Linearrear variable displacement transducer

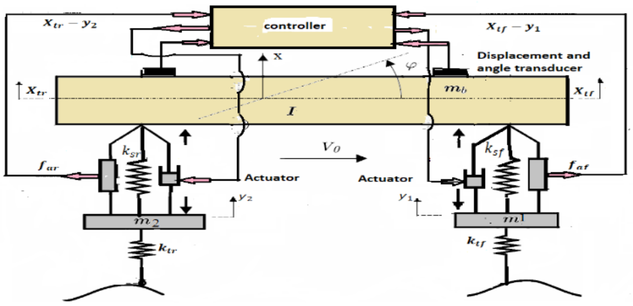

Figure (5) half car model active system

In this paper, the optimal value of (K_{gain}) is obtained from the inbuilt MATLAB command "K_{gain} = lqr (A_{ss}, B_{ss}, Q, R)". The values of the Q and R matrices are chosen iteratively until the desired results are obtained for the specified suspension parameters. The continuous-time, Lqr problem and the associated Riccati equation are solved by the MATLAB command. The previous command calculates the optimal feedback gain matrix "K_{gain}" such that the feedback control law and constraint equation (24) are satisfied as follows.

[P, S, K_{gain}] = lqr (A_{ss}, B_{ss}, Q, R)

Where: -

P = Positive definite symmetric matrix

S = Poles of the system

K_{gain} = Optimal control gain

Q= diagonal one matrix.

6.TheSuitable Rayleigh Damping Coefficients for a Specific System

The Rayleigh damping model is an approximation to viscous damping. It allows modeling the energy dissipation in the material due to internal friction, assuming it is proportional to the strain or deformation rate. There are several general purposes available that have the provision of providing the value of α and β for calculation of the Rayleigh damping matrix for dynamic analysis of systems with multi-degrees of freedom. These systems are not undamped but possess some kind of energy dissipation mechanism or damping. The proportional damping model C_{sys} expresses the damping matrix as a linear combination of the mass and stiffness matrices, that is, acts as the following equation:

$$C_{sys} = \alpha M_{sys} + \beta K_{sys} \quad (29)$$

Where(α&β) is the proportional damping constants that have suitable value for our case is 0.0687 and 2.89e-4 respectively.Where α is stiffness-proportional damping coefficient [sec.] and β is mass-proportional damping [1/sec.].Based on the present technology it is very simple to develop a spreadsheet and arrive at a rational value of α and β. When testing the value of two values of α and β as appeared in figures (6-7).Noticed the value variation of alpha acted strongly in the system while the beta value variation acted weakly in the system.

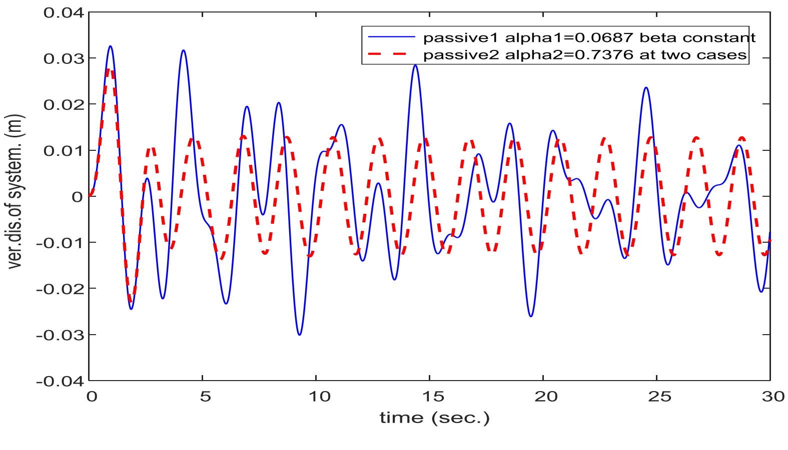

Figure (6) comparative between alpha (1) and alpha (2) for vertical displacement/time relation (Constant beta value)

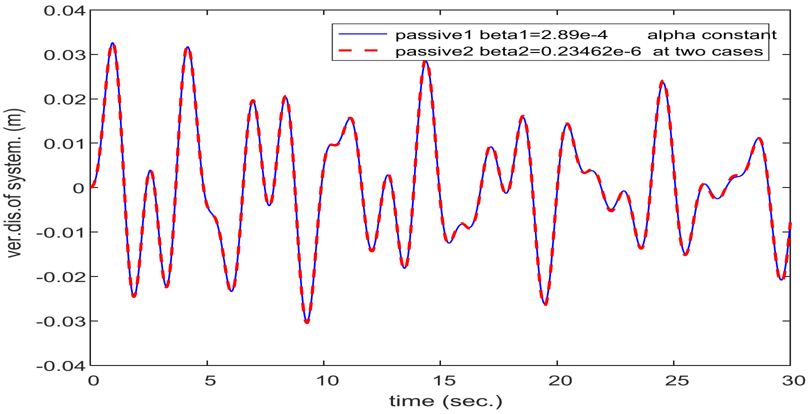

Figure (7) comparative between beta (1) and beta (2) for vertical displacement/time relation (Constantalpha value)

7. Half car model properties

The half-car model properties used for the simulation are shown in Table (1) [12]. The model properties may differ if the vehicle is changed.

Table 1: List of 4 DOF half-car model properties [12]

Front-wheel stiffness	134000 N/m
Rear wheel stiffness	134000 N/m
Front unsprung mass (wheel, axle)	62.2 kg
Rear unsprung mass (wheel, axle)	60 kg
Sprung mass (chassis)	1200 kg
Front suspension stiffness	28000 N/m
Rear suspension stiffness	21000 N/m
Moment of inertia of sprung mass	2100 Kg-m2

8.Results and Discussion

The state-space equation and the linear quadratic regulator are the basic to transform the system from passive to active. Figures (8–13) show the results obtained for homogeneous boundary conditions for the system with three elements of chassis. which appears as the elastic passive system (blue line), rigid passive system (dashed green line), and elastic active system (red line). It appears that the chassis rigidity affects the behaviors of the system. From the figures, it appears that the passive system is greater than the active system (red line) three times. Figures (8) and (9) show the system vertical displacement with time and the system vertical acceleration with time. Figures (10) and (11) show chassis vertical displacement and acceleration respectively with the time. Figures (12) and (13) show chassis angular displacement and angular acceleration with time. That, the amplitude of displacement and acceleration reduces to a third of the passive value depending on the active controller of the system.

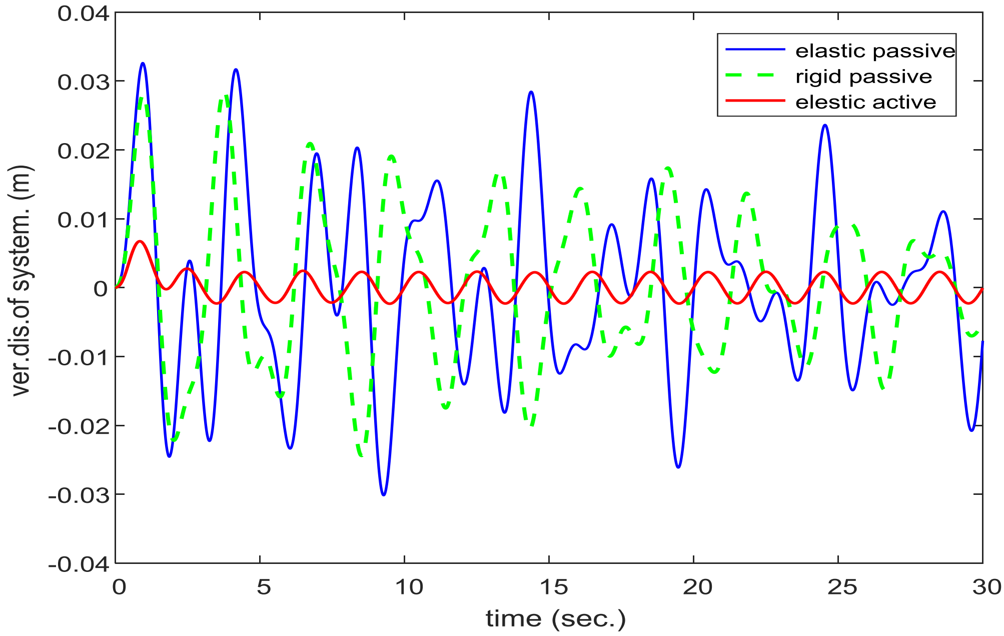

Figure (8) System vertical displacement-time relation

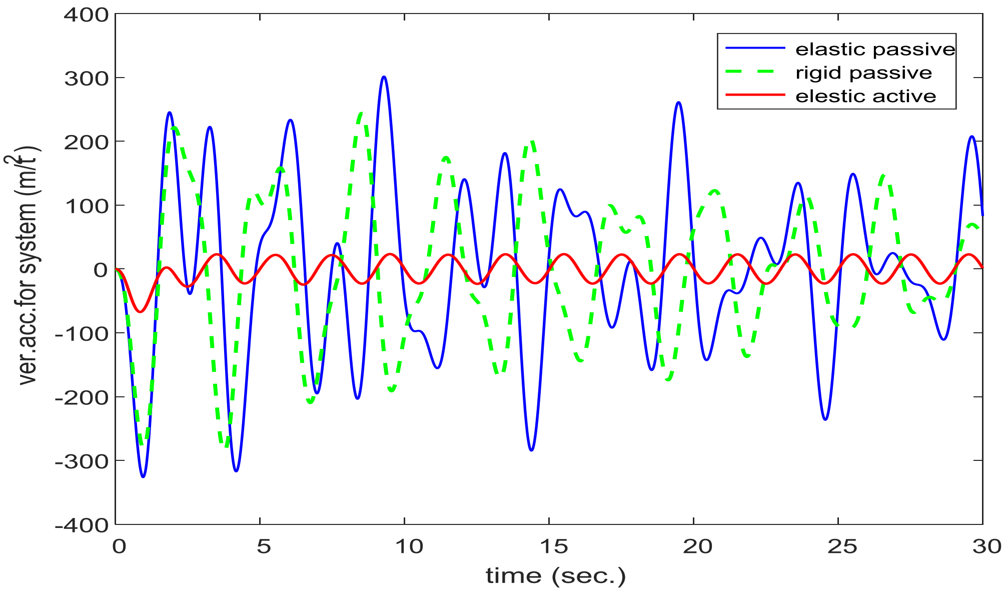

Figure (9) system vertical acceleration-time relation

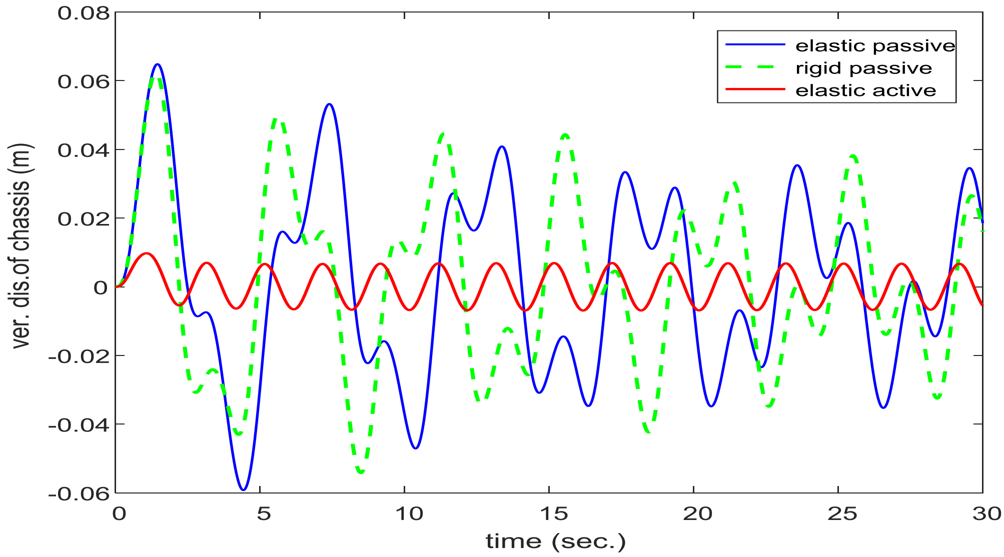

Figure (10) chassis vertical displacement-time relation

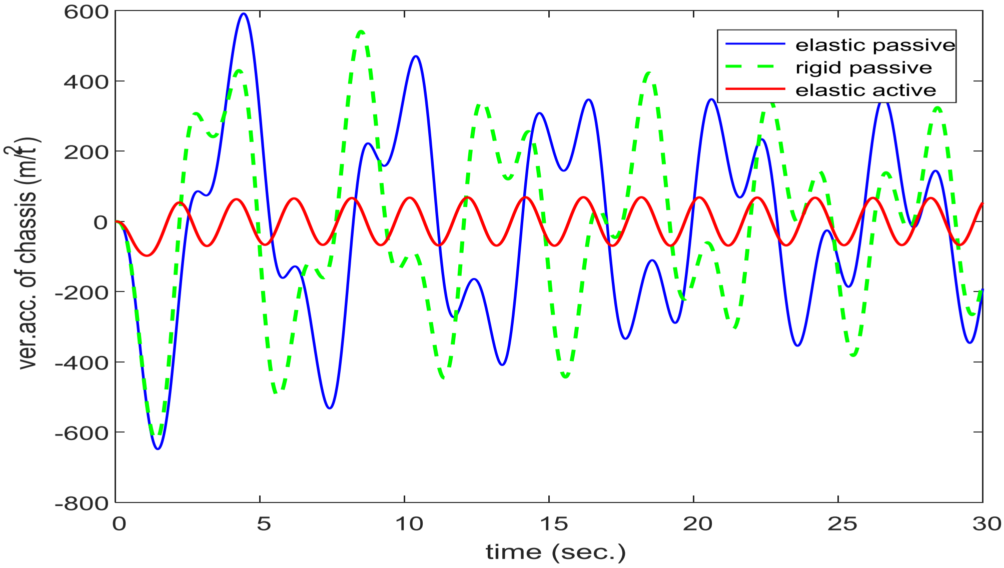

Figure (11) Chassis vertical acceleration-time relation

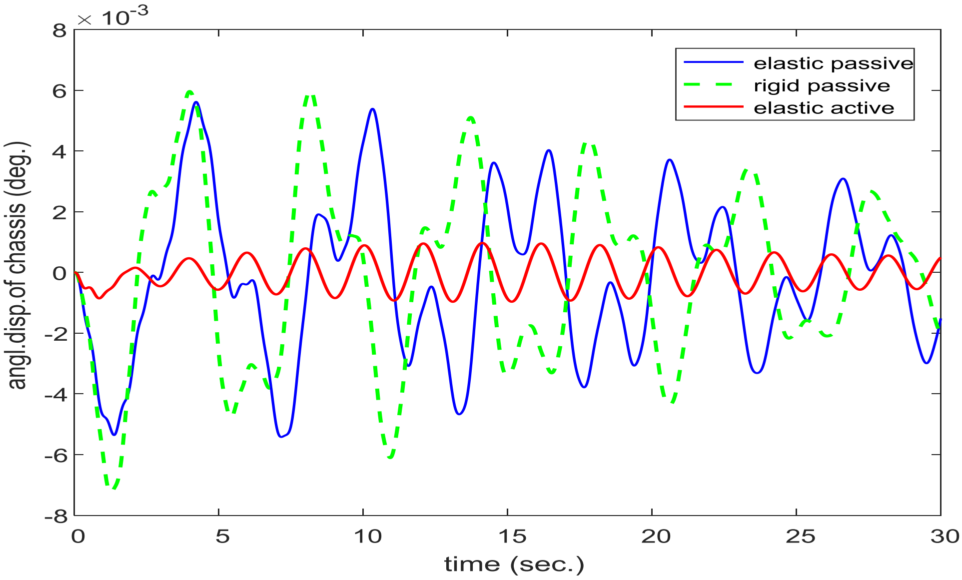

Figure (12) Chassis angular displacement-time- relation

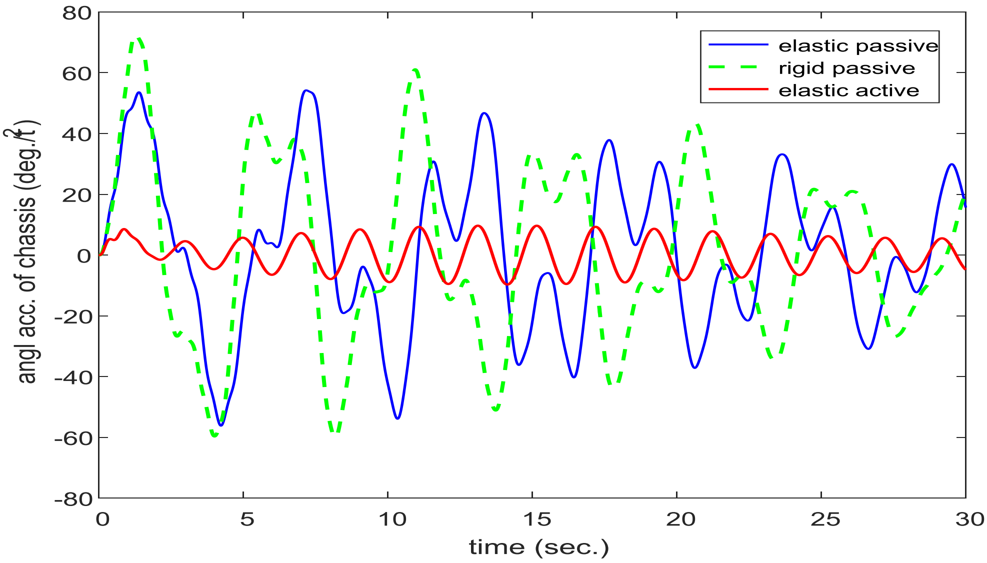

Figure (13) Chassis angular acceleration-time relation

9.Conclusions

The present paper's objectives are an analysis of the suspension system behavior for a half-vehicle model which is influenced by road irregularities. From the presented Graphs discussed before. We can say that the chassis elasticity is acting on the suspension system. So, we must consider chassis elasticity in vehicle design to reach the real case of vehicles. That gives us more control of the vehicle, more comfort on the ride, and insurance contact between tires and the road all the time.

10. References

[1] Hac., A.," Stochastic Optimal Control of Vehicles with Elastic Body and Active Suspension ", ASME J. Dynamic system measurement and control,108, 1986 pp106-110.

[2] Anupam M., Saurav T.&Karuna K. "Comparative Analysis of an Elastic 2 Degree of Freedom Half Car Model with a Rigid two Degree of Freedom Model", SAE International 2012-01-0057.

[3] Ibrahim, I.M., Crolla, D.A. and Barton, D.C., "Effect of Frame Flexibility on the Ride Vibration of Trucks", Computers and Structures 58(4): 709-713,

[4] A. Mansour Abdelhaleem and D. A. Crolla "Analysis and Design of Limited Bandwidth Active Hydropneumatic Vehicle Suspension Systems" Downloaded from SAE International by Univ of California Berkeley, Saturday, August 04, 2018.

[5] S. Talukdar, A. Mazumdar, K Kalita "COMPARATIVE STUDY OF A FLEXIBLE AND RIGID TWO DEGREE OF FREEDOM HALF CAR MODEL" International Journal of Research in Engineering and Technology e-ISSN: 2319-1163 p-ISSN: 2321-7308

[6] Y. Wang, Z. Dimitrovová, and Jong-D. Yau "Dynamic response of a vehicle with flexible car body moving on a ballasted track" MATEC Web of Conferences 211, 11003 (2018) VETOMAC XIV https://doi.org/10.1051/matecconf/201821111003.

[7] Aref M. A. Soliman "Adaptive LQR Control Strategy for Active Suspension System "Published April 12, 2011 by SAE International View list of content related to the publisher(s) in the United States.

[8] A. B. Sa, V. A, and N. J La "Simulation and Analysis of Integral LQR Controller for Inner Control Loop Design of a Fixed Wing Micro Aerial Vehicle (MAV)" Procedia Technology 25 (2016) 76 – 83

[9] Euler-Bernoulli Beam Equation, http://en.wikipedia/Euler-Bernoulli-beam equation E.P Popov., Mechanics of Materials, Prentice-Hall, 1978. K.S.

[10] Andreas Öchsner- Esslingen "Classical Beam Theories of Structural Mechanics" ISBN 978-3-030-76035-9 Germany- April 2021.

[11] CG du Toit "THE GLOBAL GALERKIN FINITE ELEMENT METHOD AS APPLIED TO THE 1D BEAM EQUATION: A SYSTEMS APPROACH" Fifth South African Conference on Computational and Applied Mechanics SACAM06 Cape Town, 16-18 January 2006 ©SACAM

[12] C. B. Patel1, P. P. Gohil1 and B. Borhade2 "MODELLING AND VIBRATION ANALYSIS OF A ROAD PROFILE MEASURING SYSTEM" International Journal of

Automotive and Mechanical Engineering (IJAME)ISSN: 2229-8649 (Print); ISSN: 2180-1606 (Online); Volume 1, pp. 13-28, January-June 2010

[13] S Rao. Gunakala, D.M.G. Comissiong, K. Jordan and Alana Sankar "A Finite Element Solution of the Beam Equation via MATLAB" International Journal of Applied Science and Technology Vol. 2 No. 8; October 2012

[14] Syed F. R., Ali B., Saulat S. C & Herbert W. "A Tool for Converting FEM Models into Representations Suitable for Control Synthesis "The International Federation of Automatic Control Seoul, Korea, July 6-11, 2008

[15] Ammar M. H. Al-Ghanim, &Ameen A. N. "Modeling, Simulation, and Control of Half Car Suspension System Using Matlab/Simulink "International Journal of Science and Research (IJSR) Index Copernicus Value (2016): 79.57 | Impact Factor (2015): 6.391

[16] Aleksandar K., Milan Z., Cvijetin M., Aleksandar Ž.&Saša P. "STATE SPACE MODELING FROM FEM MODEL USING BALANCED REDUCTION "V International Conference Industrial Engineering and Environmental Protection 2015 (IIZS 2015) October 15-16th, 2015, Zrenjanin, Serbia

[17] A. Afis, and Sebiotimo, Abdul-AzeezArisekola "THE USE OF MATLAB IN THE SOLUTION OF LINEAR QUADRATIC REGULATOR (LQR) PROBLEMS" Computing, Information Systems, Development Informatics & Allied Research Journal Vol. 5 No. 4. December 2014 – www.cisdijournal.net