Numerical Study of Rheological Natural Convection in a Square Cavity of non-Newtonian Power-Law Fluids Saturated in a Porous Medium

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Received: 12-01-2023 **Accepted: 25-08-2023** **Published: 20-09-2023**

Abstract
In this work, we numerically study thermosolutal convection in an inclined square cavity, the aim being to numerically study natural convection in a square cavity with variable heating and cooling. The cavity is filled with a non-Newtonian power-law fluid and saturated with a porous medium. The active walls are subjected to horizontal pressures, while the other walls are assumed to be adiabitic and impermeable. The equations governing the power-law fluid flow are solved using the finite-difference numerical method in time and space. The study examines the effects of relevant parameters such as Rayleigh number, the buoyancy ratio N, and inclination angel, Φ, and the Lewis number. the effects of the guiding parameters mentioned, flow intensity, apparent viscosity and heat and mass transfer rates, were illustrated and discussed in terms of vapor lines, isotherm, isoconcentration, apparent viscosity contours, flow function, apparent viscosity profiles, mean Nusselt number and mean Sherwood number. The results show that increasing the Rayleigh number improves flow intensity, and that heat and mass transfer are enhanced by increasing the Rayleigh number. also show that the average Nusselt number and sherwood number increase for shear thinning fluids. The Rayleigh number and power law index have significant effects on cavity thermal performance. Heat and mass transfer are enhanced at an angle of inclination of 45°.

Keywords: Thermosolutal convection, non-Newtonian fluid, Power-law, Finite-difference numerical method, Heat and mass transfer

Tob Regul Sci. ™ 2023 ;9(1): 5733-5750
DOI: doi.org/10.18001/TRS.9.1.399

1. Introduction

The study of natural thermo-solutal convection through an enclosure filled with non-Newtonian fluids in porous media exhibits a non-linear behavior that differs from that of Newtonian fluids in porous media. has been the subject of intensive research due to its importance in many natural and industrial problems, and has been the subject of a large number of experimental, analytical and numerical studies in recent years, using a variety of models. Many technical or environmental applications such as chemical engineering, petrochemistry, geophysical systems, certain separation processes, gas storage, oil extraction, thermal engineering and various industrial processes, etc. are based on this technology. Several researchers have studied heat and mass transfer problems for fluids with non-Newtonian rheology. Various models have been proposed to explain the behavior of non-Newtonian fluids. Among these, the power-law model has gained in importance [1-4]

Ostrach et al. [5] have studied heat transfer by natural convection as an important phenomenon in engineered systems, due to its many applications in heat exchangers, electronic cooling, solar collectors and double-glazed windows. Tauton and Lightfoot [6] studied the problem of the stability of a horizontal porous layer saturated with a binary fluid. The horizontal layers were maintained at constant temperatures and concentrations. Heat and mass transfer in a porous layer with imposed temperatures and concentrations on horizontal walls has been studied analytically by Trevisan and Bejan [7]. [8] Have also studied double-diffusive convection in a porous cavity with vertical walls. They studied diffuse convection in a porous cavity whose vertical walls are maintained at constant temperatures and concentrations. Order-of-magnitude analysis for a thrust ratio between -5 and 3, a Lewis number between 0.1 and 10 and a Rayleigh Ra*=200 porous cavity. The order of magnitude of the heat and mass transfer rates was determined as a function of the thermal Rayleigh number, $Ra,$ volume ratio N and Lewis number, Le. These results are in good agreement with those obtained numerically. In addition, they observed a complete disappearance of convective flow when $N = -1$ and $Le = 1$. Chen and Chen [9-10] presented similarity solutions for natural convection of a non-Newtonian fluid over vertical surfaces in porous media. The effects of power-law index n on heat transfer characteristics were also examined.

The problem of natural convection of a non-Newtonian power-law fluid in a porous medium adjacent to a heated impermeable plate with a non-uniform heat flux distribution by these authors. In this study, the power-law model originally proposed by Christopher and Middleman is used. [11] And subsequently modified by Dharmadhikari and Kale [12]. The natural convection of a cylinder and a sphere in a porous medium was presented by Chen and Chen [13-14]. Pericleous [15] examined how shear-thinning and shear-thickening behaviors affected convection in a chamber with isothermally heated vertical walls. Yang et al. [16] analyzed the free convection heat transfer of non-Newtonian power-law fluids over axisymmetric and two-dimensional bodies of arbitrary shape embedded in a fluid-saturated porous. Turki [17] has numerically studied the problem of natural convection in a closed, differentially heated rectangular cavity filled with a non-Newtonian power-law fluid. Nakayama [18] studied free convection from a horizontal heat source in a fluid-saturated porous medium with a power-law. Bian et al. [19] examined the impact of shear behavior on convection in an inclined rectangular porous cavity. Amari et al. [20] propose the study of natural convection of this fluid inside a porous cavity. The configuration of this layer is horizontal and heated from below or from the sides, by a constant heat flux. The problem has been solved analytically in the limit of a thin layer using the modified darcy law. The authors show the sensitivity of the heat transfer to the index of the power-law.

The study of double diffusion from a plate in a porous medium saturated with a non-Newtonian power-law fluid studied by Rastogi and Poulikakos [21]. In a recent study, Younsi et al. [22] investigated the flow behavior flow behavior and transfer in the range of high thermal Rayleigh of large thermal Rayleigh numbers, when the forces of volume forces of thermal and mass origin are opposed. Kim and Hyun [23] studied the effects of heat generation on natural convection in a porous medium saturated with a power-law fluid in an enclosure. Jumah and Mujumdar [24] investigated heat and mass transfer by natural convection of non-Newtonian power-law fluids with flow constraint on a vertical plate in a saturated porous medium at variable temperature and concentration. Non-Newtonian liquids are characterized by materials that thicken and thin under shear, with different influences on heat transport operations. Consequently, highlighting the heat transport characteristics of non-Newtonian liquids at different working stresses helps to increase their heat transport efficiency. Hadim [25] studied the natural convection of non-Newtonian power-law fluids in a porous cavity and described it numerically using the modified Brinkman-Forchheimer model. He found that as the power-law index decreases, the circulation inside the enclosure increases, leading to a higher Nusselt number. These effects are reinforced as the Darcy number increases. The result is a higher vertical velocity near the walls and a higher Nusselt number in the cavity. Cheng [26] studied doubly diffusive natural convection near an inclined corrugated surface in a porous medium saturated with fluid at Constant temperature and concentration. The onset of double-diffusive convection in a shallow enclosure subjected to vertical gradients of heat and solute has been studied analytically numerically by Mamou and Vasseur [27]. Getachew et al. [28] considered the case where the porous medium is saturated by a non-Newtonian fluid characterized by a power-law model. Kalla et al. [29] adopted the Boussinesq approximation in the Darcy model to study natural double-diffusion convective flow in a shallow porous enclosure, In the first case, the porous layer is subjected to vertical gradients of heat and mass, which either help or oppose each other. In the second, heat and mass gradients and imposed horizontally, again either helping or opposing each other. The authors demonstrated the existence of multiple convective solutions for a wide range of governing parameters. Hyun and Lee [30] carried out a numerical study of double-diffusive convection in a rectangular cavity with combined horizontal gradients of temperature and concentration. They imposed boundary conditions on the vertical so that thermal and solutal buoyancy effects cancel each other out. solute buoyancy effects cancel each other out, resulting in an opposite-gradient flow configuration.

The study by Lamsaadi et al. [31] investigated the effect of power index in a rectangular cavity filled with non-Newtonian fluids with adiabatic vertical walls and differentially heated horizontal walls, the bottom wall being at a higher temperature for the high Prandtl number limit. Compared with Newtonian fluids ($n = 1$), a decrease in the power index n (shear-thinning fluids, ($0 < n < 1$) induces an early onset of the single-cell flow regime and increases the rate of convective heat transfer, while an increase in n (shear-thickening fluids, $n > 1$) produces the opposite effect. Cheng was studied the flow of [32] Heat and mass transfer flow by natural convection of non-Newtonian fluids with yield stress in a porous medium from a vertical plate with variable heat and mass fluxes at the wall. He observed that the existence of a threshold pressure gradient in power-law fluids tends to decrease the fluid velocity and local Nusselt and Sherwood numbers. Similarly, an increase in the power-law exponent increases the local Nusselt and Sherwood numbers. Lamsaadi et al. [33] studied heat transfer by natural convection in vertical enclosures filled with non-Newtonian power-law fluids. In their case, the cavity was heated with a constant heat flux, using different boundary conditions. The results obtained show that for non-Newtonian fluids characterized by high Pr values, natural convection in high cavities is essentially controlled by the flow behavior index n and the Rayleigh number Ra. Cheng [34] studied doubly diffusive natural convection near an inclined corrugated surface in a porous medium saturated with fluid at constant temperature and concentration. Makayssi et al. [35] studied the analytical and numerical investigation of natural double-diffusive convection in a rectangular enclosure filled with a non-Newtonian fluid. In the case of a shallow cavity, the authors proposed an analytical solution based on the parallel flow approximation. This analytical solution is in good agreement with the numerical solution. The boundary finite element method for studying the natural convection of a non-Newtonian fluid in a square porous cavity was applied by Jecl and Škerget [36]. The Power-law and Carreau models are used to model non-Newtonian rheological behavior. The results of hydrodynamic and heat transfer evaluations have been reported for the configuration in which the enclosure is heated by a side wall, while the horizontal walls are insulated. Flow in the porous medium was modeled using Brinkman's extended Darcy model.

Kefayati [37] studied fluid flow and heat transfer of non-Newtonian power-law fluids in an inclined porous cavity in the presence of Soret and Dufour parameters , Results indicate that the Improved heat and mass transfer causes the augmentation of the Rayleigh numbers in various power-law indices, Darcy numbers, Soret and Dufour parameter. Makayssi et al. [38] determined the characteristics of thermosolutal natural convection in a rectangular cavity with uniformly heated vertical walls and adiabatic horizontal walls. For a saturated anisotropic porous medium. Rebhi et al. [39-40] presented investigations of the convection generated by various boundary conditions in horizontal and vertical porous enclosures using the Dupuit–Darcy model and the theory of linear stability, the authors predicted explicitly and implicitly the onsets of the triggering point of bifurcation from the steady- to oscillatory-state. They showed a considerable impact of the form drag on the critical conditions and the rates of thermal and mass exchanges. In addition, the authors numerically demonstrated that multiple solutions were possible for a given set of parameters. governing parameters. Hu et al. [41] examined the effect of cavity inclination and magnetic field on double-diffusive convection. For inclined enclosures, they found that in both vertical and inclined enclosures, the increase in thermal Rayleigh number and buoyancy ratio leads to improved heat and moisture transfer rates. As for the magnetic field, they showed that its presence leads to the suppression of convection currents, thus reducing heat and moisture transfer. Nouri et al. [42] reviewed Natural convection for non-Newtonian power-law fluids confined in a square cavity, heated and cooled from its vertical sides with uniform heat fluxes results obtained show that an increase in the index, n, reduces thermal convection, while a decrease in this while a decrease of this parameter produces the opposite effect. The study by Ahmed et al. [43] compares thermosolutal convection in a horizontal annular space filled with a binary fluid (ADI method) and around a hot horizontal cylinder immersed in a saturated porous medium (Keller box method), with both numerical approaches showing that increasing the Lewis number makes the thermal boundary layer increasingly thick. Khali et al. [44] studied doubly diffusive convection on a power-law fluid. The study shows that the fluid structure is more important for the thermal base flow. Sankar et al. [45] have numerically investigated double-diffusion convection in a porous vertical ring subjected to heat and mass fluxes from part of the inner wall, while the outer wall is maintained at uniform temperature and concentration. The influence of key parameters such as thermal Rayleigh number, Darcy number, Lewis number and buoyancy and radius ratios on heat and mass transfer has been investigated. Numerical results show that flow structure and heat and mass transfer rates are strongly dependent on the location of the heat source. In addition, the buoyancy ratio is significantly influenced by the thermal Rayleigh number, Darcy number, Lewis number and source location. Habibi et al. [46] studied heat transfer by natural convection of a non-Newtonian power-law fluid in a square cavity with two concentric hot and cold ducts. Their results revealed that the rate of heat transfer decreases by increasing the power-law index from 0.6 to 1.4. Nayak et al. [47] examined thermosolutal mixed convection of a shear-thinning due to partially active mixed zones inside a square cavity actuated by a lid. The results showed that the location and length of the heating and cooling zones have a significant influence on flow, heat and mass transfer. They found that the rate of heat transfer is maximum for minimum heat source length and maximum power-law index. Manchanda et al. [48] Numerical analysis of natural and forced convection flows in a square cavity closed by a lid on two sides with a heated triangular obstacle for non-Newtonian power-law fluids. They reported that the effect of the mixed convection parameter, i.e., has a negligible impact on the fluidic and thermal structure inside the cavity for fluids with n=0.2. The natural convection of a non-Newtonian fluid in a square cavity was studied by Kefayati et al. [49] They investigated the effect of a porous medium on the rate of heat transfer by adding it to the cavity. The natural convection of a non-Newtonian power-law fluid in a square porous cavity has been studied by Wang et al [50]. The results obtained by these authors are the problem analytically and numerically using the fourth-order Runge-Kutta scheme and the shooting method. The reported numerical results showed that natural convection is triggered if a minimum value of yield stress is provided. Tizakast et al. [51] carried out an analytical and numerical analysis of mixed double-diffusion convection in a closed rectangular cavity filled with a non-Newtonian power-law fluid. The results indicate that increasing the thermal Rayleigh number or Peclet number enhances heat and mass transfer, while the power-law behavior index strongly affects heat and mass transfer, as shear-thinning behavior enhances convection compared to the Newtonian case, while shear-thickening behavior reduces it. The stability of doubly diffusive convection in a vertical rectangular porous enclosure was analyzed by Mamou et al. [52] On the basis of linear stability theory, thresholds for the occurrence of supercritical, oscillating and superstable convection were determined. A threshold for the occurrence of subcritical convection of finite amplitude was calculated analytically as a function of the Lewis number. Earlier studies [Nield 53 and Platten and Legros 54]. revealed the possibility of the existence of oscillating convection regimes below the supercritical Rayleigh number. To determine stable, oscillating, unstable and directly unstable modes a transient linear stability analysis of the problem is required. The finite-amplitude convection threshold in an elongated porous cavity saturated with a binary fluid has been studied by Rudriah et al. [55]. The authors used a nonlinear stability analysis based on a sliced Fourrier series representation. The effects of Lewis and Prandlt numbers on convection were taken into account. Brand and Steinberg [56] studied convection with finite amplitude near the stationary convection threshold. The oscillatory time evolution of heat and mass transfer rates was also demonstrated by these authors. Recently, Rebhi et al. [57-58], have studied asymptotic and numerical double-diffusion convection in the presence of a magnetic field. The study was generalized using Neumann and Dirichlet boundary conditions for temperature and solute concentrations. and solute concentrations, and different convection modes were identified. Nonlinear thermosolutal convection has been extensively studied in a rectangular porous cavity by Rebhi et al. [59]. The authors examined numerically and analytically the Soret and Dufour effect in the presence of a magnetic field, as well as the Hopf bifurcation as a function of the governing parameters, namely the Rayleigh, Hartmann, Soret and Lewis numbers, the buoyancy ratio, the enclosure aspect and the normalized porosity of the porous medium on flow structure, streamlines, isotherm and isoconcertation contours, heat and mass transfer rates and the principles of linear convection, their results showed that the heat transfer rate increases with the Dufour number and decreases with the Soret number, and vice versa for the mass transfer rate. Lounis et al. [60] examined the impact of Dufour and Soret effects on double-diffusive convection inside an angular square enclosure by simulating the rheological behavior of the non-Newtonian fluid using the Carreau-Yasuda model. Their main conclusion is that the $Lewis$ number, the Rayleigh Ra_T number, stimulates heat and mass exchange for different values of the power-law index. The results obtained are as follows for different low power indices, n, the reduced Ra_T increases heat, mass and apparent viscosity exchange. Recently, using the Dupuit-Darcy model, Alilat et al. [61] numerically studied the free convection of an inclined square porous medium filled with a single-walled carbon nanotube and a non-Newtonian nanofluid, water. The Carreau-Yasuda model was used to represent the rheological behavior of non-Newtonian fluids. Their main findings are that increasing the percentage of single-walled carbon nanotubes (SWCNTs) significantly improves heat transfer, even as fluid viscosity increases.

The aim of this work is therefore to study thermosolutal convection in an inclined square space filled with a porous medium saturated with a non-Newtonian fluid such as rheofluidizing fluids modeled by the power-law model, assumed to be incompressible, and subject to Dirichlet-type thermal boundary conditions of constant temperature and concentration, imposed on the active walls of the cavity, as well as on the other walls, is assumed to be adiabatic and impermeable. The aim of this paper is to examine the importance of fluid rheology in porous media on the problem of heat and mass transfer in a laminar boundary layer by natural convection. The basic equations are reduced to a set of ordinary equations that are solved numerically using the finite-difference method. Consequently, the effects of parameters such as Rayleigh number, power-law index, Lewis number, boundary ratio and anchor angle are examined. Numerical results for streamlines, isotherms and isoconcentration, as well as local Nusselt and Sherwood profiles, and apparent viscosity were analyzed.

2. Mathematical Formulation

In this study, we consider the phenomenon of double diffusion convection in an enclosure. The spatial configuration of this problem consists of a square cavity inclined at an angle to the horizontal plane, containing a porous medium and filled with a non-Newtonian fluid of the power-law type, In the Rayleigh-Bénard configuration, the schematic diagram of this configuration is shown in figure 1. The active walls are subject to Dirichlet-type boundary conditions of imposed constant temperature and concentration, while the other walls are assumed to be adiabatic and impermeable.

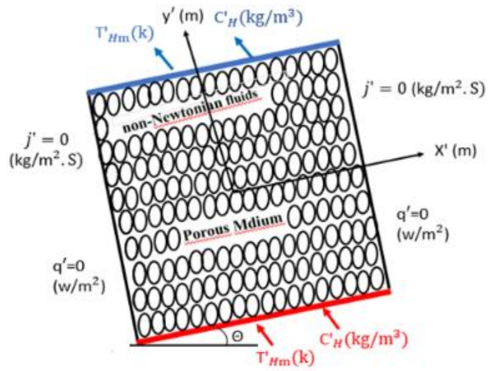

Figure 1. Geometry of the physical problem.

The flow behavior is modeled using the power-law model, The Boussinesq approximation was adopted, where the fluid density variation with temperature is described by the state equation.

The flow is assumed to be two-dimensional, laminar, incompressible and non-Newtonian. The fluid and the porous medium are in local thermodynamic equilibrium. The physical properties of the fluid are assumed constant, and the Boussinesq approximation is adopted; density is constant except in the force density term, where it is assumed to vary linearly with temperature and concentration.

$$\rho = \rho_0 \left[1 - \beta_T (T' - T'_0) + \beta_C (C' - C'_0)\right] \quad (1)$$

Where ρ_0 is the fluid mixture density at reference temperature T and solute fraction, C, β_T and β_C are the thermal and concentration expansion coefficients, respectively.

The non-Newtonian fluids considered here are those whose rheological behavior can be described by the power-law model (also known as the Ostwald-de Waele model or the two-parameter model), which leads to a relationship between shear stress and shear rate. In terms of laminar apparent viscosity, this relationship can be written as follows:

$$\mu = k\left[2\left(\frac{\partial \psi}{\partial x}\right)^2 + \left(\frac{\partial \psi}{\partial y}\right)^2\right]^{\frac{n-1}{2}} \quad (2)$$

where n, is a dimensionless constant called the power-law index, and k is an empirical coefficient known as the consistency factor, which is an indicator of the degree of the fluid viscosity. Note that for $n = 1$, the power-law model reduces to Newton's law by setting $k = m$. Thus, the deviation of n from unity characterizes the degree of non- Newtonian behavior of the fluid. On one hand, when n is in the range $0 < n < 1$, the fluid is said to be pseudo-plastic (or shear-thinning) and the viscosity is found to decrease with increasing shear rate. On the other hand, when $n > 1$, the fluid is said to be dilatant (or shear-thickening) and the viscosity increases with increasing shear rate. Dilatant fluids are in general much less common than pseudo-plastic ones. Though the Ostwald-de Waele model does not converge to Newtonian behavior in the limit of zero and maximum shear rates, it does present the advantage of being simple and mathematically tractable. In addition, the rheological behavior of many substances can be adequately represented by this model for a relatively large range of shear rates (or shear stresses), which makes it useful, at least for engineering purposes, and justifies its use in most theoretical investigations of fluids having pseudo-plastic or dilatant behavior.

Under the convection assumptions commonly used, the dimensionless transport equations for energy, T , concentration, C, and current function the current function, Ψ, are written respectively:

$$\nabla^2 \Psi = -\mu_a^{-1}\left[\frac{\partial \Psi}{\partial y}\frac{\partial \mu_a}{\partial y} + \frac{\partial \Psi}{\partial x}\frac{\partial \mu_a}{\partial x} + R\left(\frac{\partial T}{\partial x} + N\frac{\partial S}{\partial x}\right)\right] \quad (3)$$

$$\varepsilon\frac{\partial C}{\partial t} + \frac{\partial (uC)}{\partial x} + \frac{\partial (uC)}{\partial y} = Le^{-1}\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) \quad (5)$$

where;

$$u = \frac{\partial \Psi}{\partial y} \quad \text{and} \quad v = \frac{\partial \Psi}{\partial x} \quad (6)$$

The apparent viscosity, μ, namely:

$$\mu_a = \left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right\}^{(n-1)/2} \quad (7)$$

The dimensionless variables are obtained using the characteristic scales;

$$(x, y) = \frac{(x', y')}{H'}, (u, v) = \frac{(u', v')H'}{\alpha}, t = \frac{t'\alpha}{H'^2}, \Psi = \frac{\Psi'}{\alpha}, T = \frac{(T' - T'_0)}{\Delta T^*}, C = \frac{(C' - C'^2)}{\Delta C^*} \quad (8)$$

The non-dimensional boundary conditions over the walls of the enclosure are;

$$u = v = 0 \ \text{and} \ \psi = 0 \ \text{at} \ x = \pm\frac{1}{2} \ \text{and} \ y = \pm\frac{1}{2} \quad (9)$$

and the thermal and solutal boundary conditions:

$$\frac{\partial T}{\partial x}=\frac{\partial C}{\partial x}=0 \quad \text{at} \quad x=\pm\frac{1}{2} \tag{10}$$

$$T=C=\pm\frac{1}{2} \quad \text{at} \quad y=\pm\frac{1}{2} \tag{11}$$

In equations (3), (4) and (5), four dimensionless parameters appear: thermal Rayleigh number Ra, the Lewis number Le, the buoyancy ratio N, and the porosity, ε. These parameters are defined as follows;

$$Ra_T=\frac{g\beta_T\Delta T^* H'^3}{\alpha\nu} \quad Le=\frac{\alpha}{D},\ N=\frac{\beta_C\Delta C^*}{\beta_T\Delta T^*} \tag{12}$$

Heat and mass transfer rates on the hot wall can be expressed in terms of average and local Nusselt and Sherwood numbers.

$$Nu=-\left.\frac{\partial T}{\partial y}\right|_{y=\pm 1/2}-\left.\frac{\partial C}{\partial y}\right|_{y=\pm 1/2},\ Nu=\int_{-1/2}^{1/2} Nu\,dx \tag{13}$$

$$Sh=-\left.\frac{\partial C}{\partial y}\right|_{y=\pm 1/2}-\left.\frac{\partial T}{\partial y}\right|_{y=\pm 1/2},\ Sh=\int_{-1/2}^{1/2} Sh\,dx \tag{14}$$

3. Numerical Solution

The complete governing equations associated with their respective boundary conditions are solved numerically using the finite difference method. The equations are discretized using a central second-order finite-difference scheme in time and space on a uniform grid.

The energy and concentration equations, equations (4) and (5), are solved iteratively in a specific time mode using the implicit alternating direction method (A.D.I). The resulting sets of discretized equations for each variable were solved by a line-by-line procedure, using the two-system tri-diagonal matrix (TDMA) algorithm. This method divides a time step into two: in the first half-step, the system is solved implicitly in the x-direction and explicitly in the y-direction. In the second half-step, the procedure is reversed. In the second half-step, the procedure is reversed. the current function equation, which is a second-order partial differential equation containing no transient term, will be solved by the S.O.R. successive block relaxation method proposed by Frankel (1950), which directly gives the value of ip at time $(n + l)\Delta t$, at the block of nodes considered, with a sufficient number of iterations for the desired convergence criterion to be satisfied. The convergence criterion is adopted:

$$\frac{\Sigma_i\Sigma_j\left|\psi_{i,j}^{k+1}-\psi_{i,j}^{k}\right|}{\Sigma_i\Sigma_j\left|\psi_{i,j}^{k}\right|}\le 10^{-8} \tag{15}$$

where $\psi_{i,j}^{k}$ is the value of the stream function at the k^{th} iteration level. The time step size, δt, is varied in the range $10^{-5}\le\delta t\le 10^{-3}$, depending on the values of the governing parameters

The numerical algorithm is tested for the classical problem of natural convection in a square cavity. Clearly, the accuracy of numerical results depends on mesh size. The choice of mesh size depends on the values of the control parameters. To optimize calculation time and preserve the accuracy of the numerical solution. Table 1 shows the sensitivity of the numerical solution to the mesh in terms of Nusselt number, Nu, and flow intensity, Ψ_0, at the center of the cavity for the case of a non-Newtonian fluid saturated by a porous medium. According to this table, it is justified and sufficient to take a uniform mesh of 101×101 to adequately model the flow and heat transfer inside the cavity under consideration.

Table 1. Grid sensitivity study for; $n = 0.6, Ra = 50, Le = 10, N = -0.1, and\ \Phi = 0°$

$Nx \times Ny$	25×25	45×45	75×75	101×101	125×125
Ψ_0	3.1051	2.5711	3.3119	3.7935	3.6958
Nu	4.0943	3.3689	3.4873	3.8798	3.6909
Sh	7.5856	7.7528	11.5782	12.1084	11.5895
⊠	1.8431	2.8967	3.8460	4.4927	5.7479

To assess the accuracy of the solution, the current results for heat transfer coefficients are compared with the results [62-63-64] for natural convection in a cavity for the power-law of non-Newtonian fluids (see Table 1). This indicates that the current numerical results in terms of average Nusselt number are in good agreement with the other studies.

Table 2. Value of the average Nusselt number as a function of the power-law index.

n	Nu_{avg} in [62]	Error (%)	Nu_{avg} in [63]	Error (%)	Nu_{avg} in [64]	Error (%)	Nuavg in this Work
0.6	7.3823	1.07	6.9345	0. 16	7.020	0. 29	6.6578
0.8	5.6201	-	5.5127	-	-	-	-
1	4.7662	0.16	4.6993	0. 07	4.741	0. 12	4.5802
1.2	4.2227	1.9	4.1709	0. 10	-	-	4.0410
1.4	3.8464	3.03	3.7869	2.67	3.770	2.53	3.2161

4. Results and Discussion

In this section, we present numerical results for double diffusion convection. The model studied is an inclined square cavity saturated by a porous medium and filled with a non-Newtonian power-law fluid. The control parameters are the thermal Rayleigh number, Ra, the range the Lewis number Le, the ratio of the solutal and thermal forces of the volume *N*, and angle of inclination. In this study, the values of n is change from from 0.6 to 1.4, including shear-thinning ($0 < n < 1$), Newtonian ($n = 1$) and shear-thickening ($n > 1$) fluids. Streamlines, isotherms and isoconcentration, stream function distributions, apparent viscosity and average Nusselt and Sherwood numbers are shown in the figures.

Figure 2 Shows the effect of the power-law index on the distribution of streamlines, isotherms, and isconcentration contours, respectively. All results presented are associated with single-cell flow, as indicated. Decreasing n intensifies flow patterns and increases thermal amplitudes and iso-concentrations on the hot wall, signifying an improvement in the convection process. This confirms that decreasing the power-law index, n, improves heat and mass transfer. Apparent viscosity also varies as a function of power index, n, with the transition between a shear-thickening fluid *($n = 1.4$)* and a shear-thinning fluid *($n = 1$)* being more complex and distorted.

Ψ_0 T C

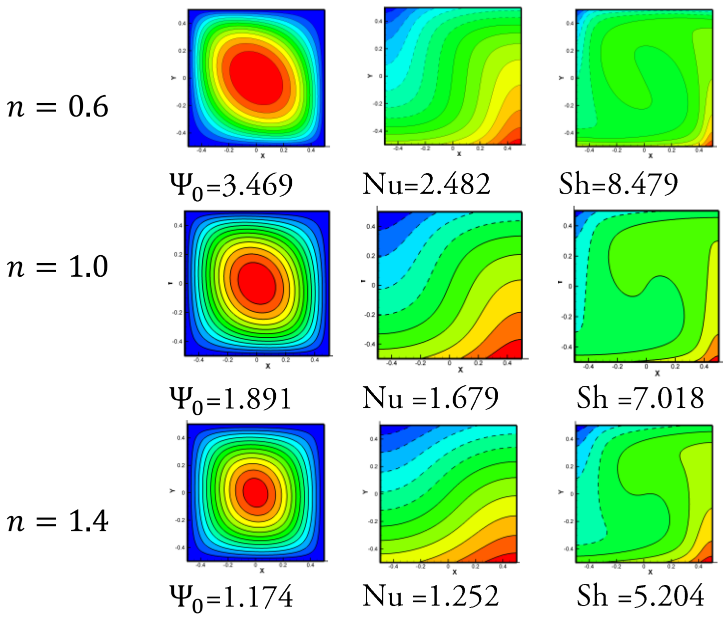

Figure 2. Effect of the power-law index, n, on : (a) streamlines, (b) isotherms, (c) is-concentrations contours for $Ra = 50$, $Le = 10$, $N = -0.5$ and $\Phi = 0°$

The variations of, Ψ_0, μ, Nu and Sh with Rayleigh number Ra are presented in Figure 3 for $Le = 10$, $N = -0.1$ and different values of power-law index, n. This figure also shows the presence of subcritical convection for the different values of the behavior index $n = 1$ (Newtonian fluis) considered in this problem. The, Nu and Sh increase linearly with Ra, and the effect of n is easily observed as shear-thinning fluids have higher heat and mass transfer rates with respect to Newtonian and shear-thickening fluids. In addition, an important observation is that the raise in the heat transfer coefficient due to increase in the viscosity parameter is more significant when considered along with thermal Rayleigh number. Whose main role is to reduce viscous resistance and improve convective flow.

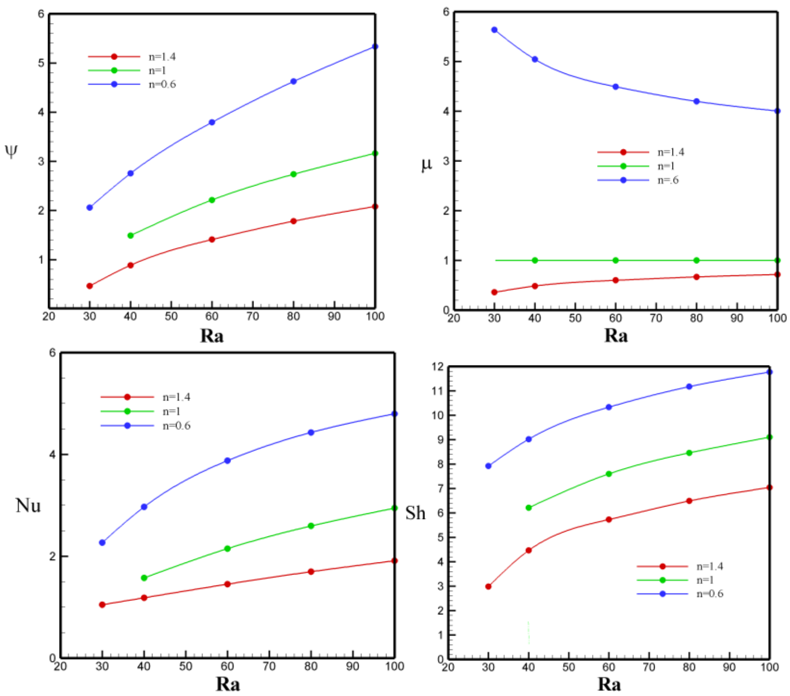

Figure 3. Effect of, Ra, and power-law index, n, on: stream function at the center of the cavity, ψ, apparent viscosity, μ, Sherwood number, Sh, and Nusselt number, Nu, for $Le = 10, N = -0.5$ and $\Phi = 0°$.

The angle of inclination of the cavity has a significant effect on heat and mass transfer and flow. In the following, we will study the effect of four different inclination values on the different variations that develop inside the cavity. Figures 4 show the effects of inclination angle γ on flow functions, apparent viscosity, heat and mass exchanges as a function of power-law n. According to this figure, the current function decreases the shear thickening n (from 1.4 to 0.6) and increases the tilt angle Φ, (from 0° to 90°). Figure 4 shows that viscosity remains constant for ($n = 1 \rightarrow \mu = 1$ and $n = 1.4 \rightarrow \mu < 1$), and increases if $n = 0.6$ with increasing Φ. The results show that the highest values of Nu and Sh are obtained when $n = 0.6$, giving the best heat and mass exchange at $\Phi = 45°$. Resulting in accelerated fluid flows and increased heat and mass exchange rates up to an optimum value Φ.

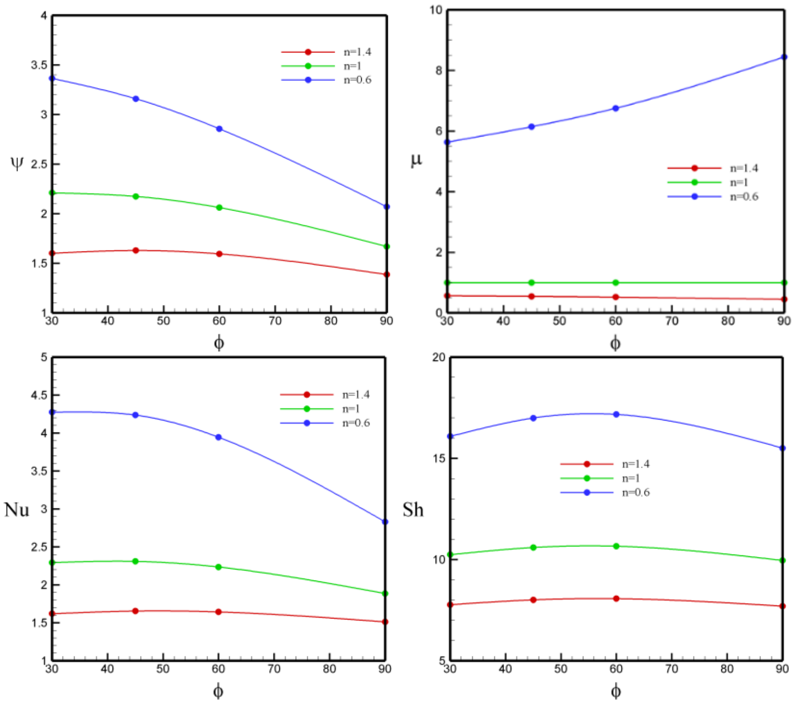

Figure 4. Impact of inclined angel, Φ, and, n, on: stream function at the center of the cavity, Ψ, apparent viscosity, μ, Sherwood number, Sh, and Nusselt number, Nu, for $Le = 10, Ra = 50$ and $N = -0.5$.

The influence of buoyancy ratio N on Ψ_0, Nu, Sh and apparent viscosity is illustrated in Figure 5 for $Ra = 50$, $Le = 10$ and different values of n, the current function, Nu and Sh increase rapidly with decreasing n from $n = 1.4$ to n=1 for positive values of N ($N > 0$), the region where flow is purely dominated by thermal effects (thermal flow $N = 5$). In the case $N = 0$, i.e., when concentration effects are absent, convective motion is induced by temperature gradients alone, leading to counter-clockwise flow ($\Psi > 0$). On the other hand, the flow function increases with decreasing power-law index, n, as shown in Figure 5. This is due to the increase in volumetric forces in the equation of motion. In addition, increased buoyancy improves heat and mass exchange. For negative values of N ($N < 0$), the region where forces are opposed but flux by solute effects (inverse flux $N = -5$). When the thermal effect is dominant, the flux function is positive. In the same interval, the transition from conductive to convective regime when the number of, n, dead for $Ra = 50$ is less than the number of supercritical, Then, for large negative values, the solute effect is clearly dominant and the flux becomes negative. Below the critical value, the solution is purely conductive ($\Psi = 0, Nu = Sh = 1$) whatever the amplitude of the perturbation. Flow behavior is highly dependent on buoyancy rate. Note that mass transfer reaches an asymptotic maximum value faster than heat transfer. This is a direct consequence of the Lewis number ($Le = 10$) on mass transfer. As n decreases from ($n = 1.4$ *to* $n = 0.6$) and, N, increases, a slight difference in viscosity is observed at $\mu = 1$ (Newtonian fluids), then decreases sharply to reach shear-thinning values μ<1. The viscosity of Newtonian fluids is a constant, while the viscosity of non-Newtonian fluids is modified by the partial derivatives of velocity over displacement in the form of a power. On the other hand, shear-thinning fluids have much better heat and mass transfer performance than shear-thickening fluids, i.e. In other words, as the shear-thickening index increases, so does the apparent viscosity.

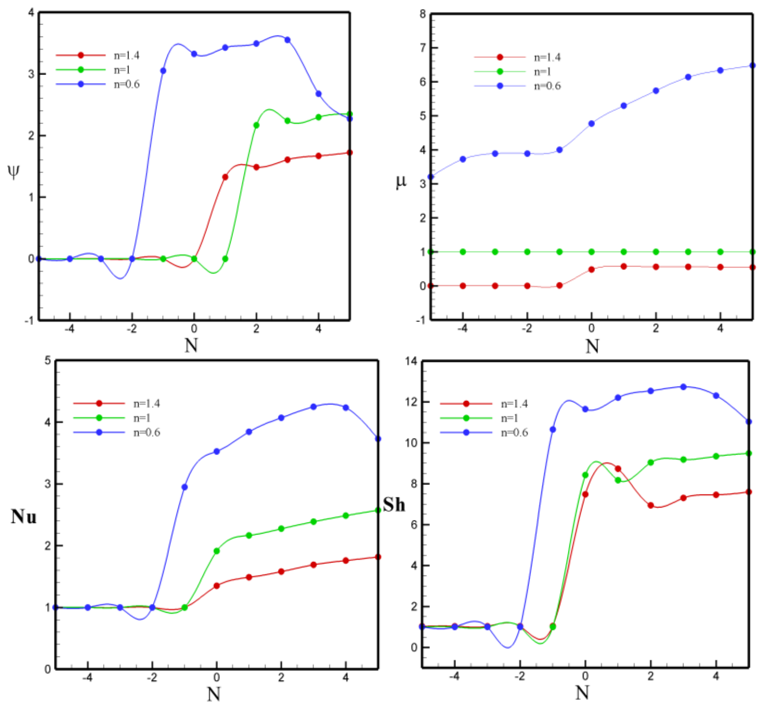

Figure 5. Influence of buoyancy ratio, N, and, n, on: stream function at the center of the cavity, Ψ, apparent viscosity, μ, Sherwood number, Sh, and Nusselt number, Nu, for $Le = 10, Ra = 50$ and $\Phi = 0°$

The influence of Lewis number, Le, and power-law index, n, on the stream function, apparent viscosity, Sherwood number and Nusselt number is illustrated in Figure 6. an increase in the Lewis number, the flux function increases significantly for $n = 0.6$, and for $n = 1$ the current function is zero Ψ=0, and for $n = 1.4$ increases up to $Le = 50$ then decreases from $Le = 100$ for a zero-stream function value. apparent viscosity remains constant for $n = 1$ ($\mu = 1$ Newtonian fluids), for $n = 1.4$ it is less than unity (shear thickening fluids), and greater than unity for $n = 0.6$ (shear thinning fluids). Therefore, Nusselt and Sherwood numbers are low in the range $1 \leq n \leq 1.6$ as Lewis increases, we observe that increasing the Lewis number decreases the number of Nu for the power-law index value $n = 0.6$, It is clear that decreasing the Nusselt number decreases the heat transfer coefficient. In addition, the variation of mass transfer coefficient (Sherwood number, Sh) with Lewis number. It is observed from these figures that increasing Lewis number increases the Sherwood number for power-law index $n = 0.6$. Also, this effect enhances the mass fluxes and lowers the heat fluxes.

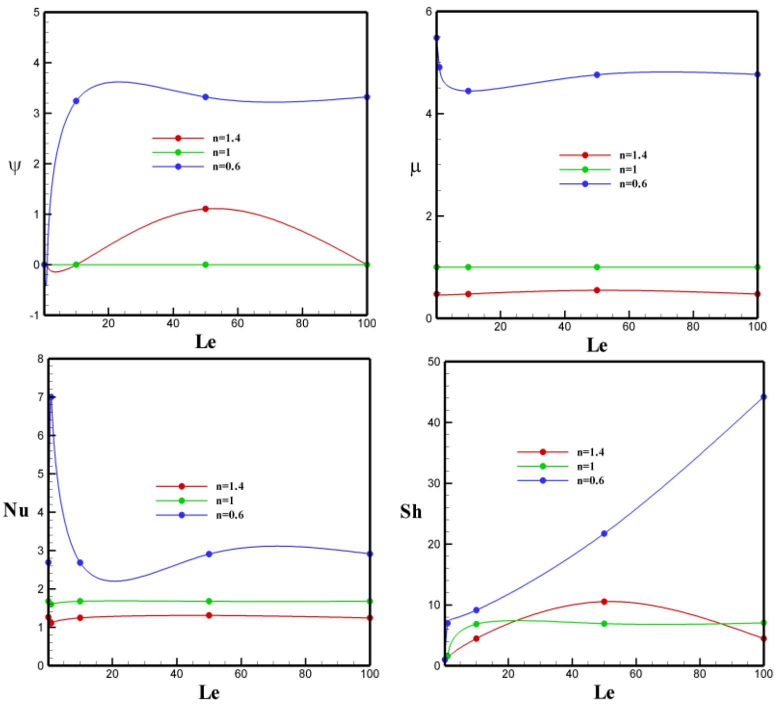

Figure 6. Influence of Lewis number, Le and n, on: stream function at the center of the cavity, Ψ, apparent viscosity, μ, Sherwood number, Sh, and Nusselt number, Nu, for $Le = 10, Ra = 50$ and $\Phi = 0°$

The type of oscillating flow that occurs in the square cavity is illustrated by numerical results in terms of the time evolution of Ψ, Nu and Sh, obtained for $Ra = 50$. The solution shows the formation of cells with inclined and counter-rotating rollers stacked vertically. The resulting oscillatory flow is simply periodic. Snapshots of flow function perturbations (obtained by subtracting the time-averaged solution from the exact solution in time) at the different time steps shown in Figure 7.

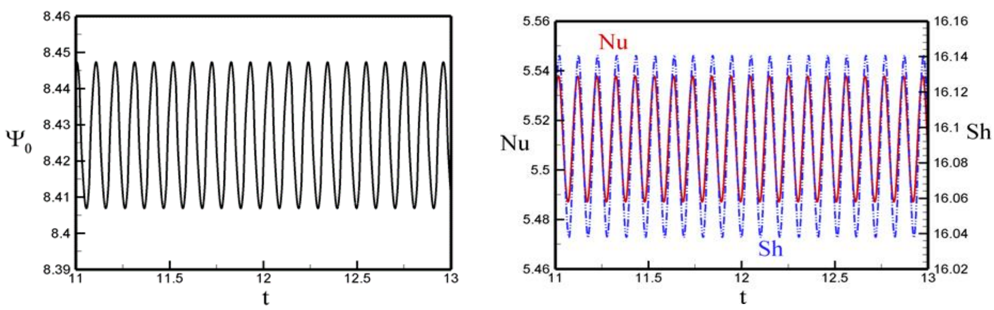

Figure 7. Time history of the Ψ, Nu and Sh, numbers

5. Conclusion

In this paper, heat and mass transfer by natural convection along a square cavity in a porous medium saturated with a non-Newtonian power-law fluid has been studied numerically. We also performed a scaling analysis to predict heat and solute transfer. The main results are summarized below:

1. Increasing the Rayleigh number enhances natural convection inside the cavity, resulting both the flow intensity and heat and mass transfers are promoted upon increasing. At $n > 1$, natural convection is further enhanced when the Rayleigh number is increased.
2. The average Nusselt number increases for shear-thinning fluids and decreases for shear-thickening fluids compared to Newtonian fluids. These changes are more pronounced for shear-thinning fluids ($n < 1$).
3. Reducing the power-law index (n) increase the apparent viscosity of the fluid and enhances natural convection inside the cavity, which then leads to an increased rate of heat and mass transfers. The effect of inclination angle on convective heat and mass transfer convective heat and mass transfer becomes more important as n decreases. Namely, the effect of the load is strongest at 45°.
4. Reducing the power-law index (n) increas the apparent viscosity of the fluid and enhances natural convection inside the cavity, which then leads to an increased rate of heat and mass transfers. Enhancing natural convection as a result of reducing n is more pronounced at higher angle inclination numbers.
5. In the case of opposing thermal and solutal buoyancy forces $N > 0$ are found to be more sensitive, this behavior is not observed in the case of thermal support and solute buoyancy forces $N < 0$. This is due to the fact that for a positive thrust ratio, thermal and solutal forces add up, and flow intensity increases. Increased flow intensity improves heat and mass transfer.
6. The improvement in natural convection resulting from the reduction in n is more pronounced at higher Lewis numbers. The average Nusselt number and Sherwood number increase as the power index decreases for an increasing Lewis number.

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