Damage Development Analysis in Composite Materials by the BEM

Rai Naceur Bouharkat[1,2], Medjahed Driss Meddah[3], Ait Amar Meziane Mohamed[1], Medjadji Nassira[4], Kaddour Abdelmadjid[5*], Tlili Salah[6]

[1] University Ibn Khaldoun, Tiaret, Algeria, Tiaret 14000, Algeria

[2] Laboratory of Industrial Technologies Research, University Ibn Khaldoun, Tiaret, Algeria

[3] Mechanical Engineering Department, University Center Salhi Ahmed Naama, Naama 45000, Algeria

[4] Electrical Engineering Department, University Center Salhi Ahmed Naama, Naama 45000, Algeria

[5*]Unité de Recherche Appliquée en Energies Renouvelables, URAER, Centre de Développement des Energies Renouvelables, CDER, 47133, Ghardaïa, Algeria

[6]Univ. Ouargla, Fac. Mathematics and Material Sciences, Lab. Development Of New and Renewable Energies In Arid And Saharan Zones Laboratory (LARENZA), Algeria

***Email corresponding author:** kaddour.majid@gmail.com

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Abstract

Continuum mechanics hinges on the concept of a Representative Volume Element (RVE) playing the role of a mathematical point of a continuum field approximating the true material microstructure. The RVE is very clearly defined in two situations only: (i) unit cell in a periodic microstructure, and (ii) volume containing a very large (mathematically infinite) set of microscale elements (e.g. grains), possessing statistically homogeneous and ergodic properties. The RVE or unit cell approach currently gains more and more importance in the numerical determination of generalized material behavior of multiphase materials, which are of sufficient length to capture all the average details of the microstructure.

One important goal of the mechanics and physics of heterogeneous materials is to derive their effective properties from the knowledge of the constitutive laws and spatial distribution of their components. Homogenization methods have been designed for this purpose. The basic idea behind RVEs is that the elastic energy stored inside a unit cell is identical to the one stored inside the represented homogenized continuum.

This paper deals with a multiscale approach to model heterogeneous materials, and using failure criteria, Tsai-Hill and maximum deformation, to detect any failure of the matrix and fiber, respectively. In the macroscale (global scale), the Boundary Element Method (BEM) for anisotropic plane elasticity was used to evaluate strain and stress fields in the domain of the lamina. These fields represent the macroscopic tensor of the structure, which is used to evaluate the boundary conditions in the microscale (local scale). Computer implementation of BEM and multi-scale approaches, to perform defect analyzes on composite laminates; Discussion of the results obtained from the multi-scale analysis and the failure are compared with the results obtained by other analyzes.

Keywords: Composite materials, Multiscale modeling, Boundary Element Method, Representative Volume Elements, Failure Criteria.

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Introduction

With the identification of damage mechanisms at different length scales, experimental approaches have been developed to detect and monitor their occurrence, evolution and interaction. The revealed relations have been covered by appropriate mathematical descriptions and the associated parameters have been determined for various material systems like wood, short and long fiber reinforced polymer composites or steel reinforced concrete. The resulting theories span from statistical descriptions of the successive fiber fracture (fiber bundle models), over the description of stress concentration fields around successively failing fibers up to macroscopic post-failure degradation models. The sub-project Numerical simulation of damage evolution included the implementation of the generated knowledge into application-oriented engineering tools.

Numerous studies have been carried out on the homogenization method and the mechanical behavior of corrugated structures for general applications.

Going from structures to materials requires a careful investigation of the effective material properties resulting from the arrangement of stiff inclusions in a compliant matrix with the overall composite undergoing finite strains, which is the main focus of this contribution. The extraction of effective properties encourages the use of homogenization techniques to determine the full elasticity tensor, which is generally anisotropic.

We will consider composite materials in which microstructural characteristics (such as inclusion size) are orders of magnitude smaller than the body's macroscopic extensions, so that we may assume a separation of scales as in the classical theory of composites [1]. Consequently, the composite can be treated as a homogeneous solid with an effective response to be determined from a microstructural representative unit cell. [2] One-way out is what is commonly known as multi-scale modeling, where macroscopic and microscopic models are coupled to take advantage of the efficiency of macroscopic models and the accuracy of microscopic models. The goal of this multiscale modeling is to design combined macroscopic-microscopic computation methods that are more efficient than solving the complete microscopic model and at the same time to provide the information we need at the desired precision [3 – 8].

In homogeneous materials such as a fiber reinforced composite material, granular media or biomaterials generally possess a complex micro structure. In accuracies or stochastic variation (uncertainty) of a material property, geometry or topology sometimes arises in the microstructure. This in accuracy will have influence on a homogenized material property, macroscopic or microscopic mechanical response of a structure. In general, stochastic characteristics such as expectation or variance of a homogenized property of composites will be unknown even if those stochastic characteristics of a mechanical property of each component material are known.

In order to improve reliability of a composite structure, therefore, the influence of the inaccuracy in a microstructure on a homogenized mechanical property should be investigated.

The prediction of the mechanical properties of the composites has been an active research area for several decades. Except for the experimental studies, either micro- or macro mechanical methods are used to obtain the overall properties of composites.

Micromechanical method provides overall behavior of the composites from known properties of their constituents (fiber and matrix) through an analysis of a periodic representative volume element (RVE) or a unit-cell model [9, 10].

In the macro mechanical approach, on the other hand, the heterogeneous structure of the composite is replaced by a homogeneous medium with anisotropic properties. The advantage of the micromechanical approach is not only the global properties of the composites but also various mechanisms such as damage initiation and propagation, can be studied through the analysis [11, 12].

In the micromechanics of the continuum, each material point is considered as a finite volume of a homogeneous material that has macroscopically zero structural dimensions but represents a finite microscopic size with a certain microstructure. For a non-periodic microstructure, the RVE isdefined as a volume containing a very large number of elements at the microscopic scale. This definition is valid only in the case of an ergodic material, that is, the ergodic hypothesis implies that the heterogeneous material is supposed to be statistically homogeneous. This fact also implies that sufficiently large volume elements selected at random positions in the sample of the material considered have statistically equivalent component arrangements and contain the same averaged material properties. Such material properties are referred to as the effective material properties of the inhomogeneous material. Therefore, the volume in the homogenization / localization procedure must be chosen to be an appropriate RVE, with the size sufficient to contain all the information needed to describe the behavior of the composite. Thus, such a choice largely determines the accuracy of the model of a heterogeneous material [4].

The aim is therefore to define the range of the possible effective behavior in terms of limits, which depend on certain parameters characterizing the microstructure, such as, for example, the volume ratio of the inclusions in a matrix. To this end, many homogenization methods have been developed. We mention the pioneering studies of Voigt [13] and Reuss [14], which formulated rigorous limits for the effective moduli of the prescribed volume fraction composites. A few decades later, Hashin and Shtrikman [15 – 16] presented an extension of the method based on variational formulations. If the microstructure is composed of a matrix and spherical or spheroidal inclusions, the effective behavior of the composite can be obtained using the self-coherence method [17 – 21].

This work aims to develop a numerical failure analysis in unidirectional laminates, based on multiscale modeling of heterogeneous materials, and using failure criteria, Tsai-Hill and maximum deformation, to detect any failure of the matrix and fiber, respectively.

1. Failure criteria

The concept of failure in laminated materials can be classified into three levels: lamina failure criterion, the laminate failure criteria, and the criteria for structural failure. The study of a failure criterion of a lamina defines the possible states of stresses in which the failure occurs.

When evaluating the failure of the laminate, it is questioned if this occurs when only one of the laminae fails or when all fail. In the latter case, the numerical analysis must consider the progressive failure of the laminas, being necessary to use theories of damaging the lamina.

The last level of failure criterion is structural. The focus of this criterion is the fulfillment of the objective of the structural part in a project; it defines a permissible damage criterion in designed part [22 – 24].

2. Basic relations for anisotropic elasticity

Using the notation reduced tensor proposed by Lekhnitskii [25], the equation for anisotropic elasticity may be written as

$$a_{11}\mu^4 - 2a_{16}\mu^3 + (2a_{12} + a_{66})\mu^2 - 2a_{26}\mu + a_{22} = 0 \quad (1)$$

Where $a_{ij}$is the material compliance matrix given by [25]:

$$a_{11} = \frac{1}{E_1}, \qquad a_{12} = -\frac{\nu_{12}}{E_1} = -\frac{\nu_{21}}{E_2}$$

$$a_{16} = \frac{\eta_{12,1}}{E_1} = \frac{\eta_{1,12}}{G_{12}}, \quad a_{22} = \frac{1}{E_2}, \quad a_{26} = \frac{\eta_{12,2}}{E_2} = \frac{\eta_{2,12}}{G_{12}}, \quad a_{66} = \frac{1}{G_{12}} \quad (2)$$

Where $E_k$are Young's moduli referring to axes xk, G_{12} is the shear modulus for the plane,$\nu_{ij}$are Poisson's ratios and $\eta_{jk,l}$and $\eta_{l,jk}$are mutual coefficients of the first and second kind, respectively, and μ the roots of the equation, always complex or pure imaginary, occurring in pairs (μk and $\bar{\mu}$k) as shown by Lekhnitskii [26].

3. Boundary integral equation for anisotropic materials

The integral equation, which relates the fundamental state with any other state in a body with domain Ω and boundary Γ, can be written for an interior domain point as (see for example Reference [27 - 28]):

$$u_i + \int_\Gamma T_{ik} u_k d\Gamma = \int_\Gamma U_{ik} t_k d\Gamma \quad (3)$$

whereu_i is the displacement vector, t_k is the traction vector, U_{ik} and T_{ik} are the displacement and traction anisotropic fundamental solutions for elastostatics, respectively.

The anisotropic displacement fundamental solution for elastostatics can be written as

$$U_{ji} = 2Re\left[q_{i1}A_{j1}ln(z_1 - z_1') + q_{i2}A_{j2}ln(z_2 - z_2')\right] \quad (4)$$

Where $q_{ik}$is equal to

$$q_{ik} = \begin{bmatrix} a_{11}\mu^2 + a_{12} - a_{16}\mu \\ a_{12}\mu + \frac{a_{22}}{\mu} - a_{26} \end{bmatrix} \tag{5}$$

z, z' the complex variables Defining as [36]:

$$z = \begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix} = \begin{Bmatrix} x_1 + \mu_1 x_2 \\ x_1 + \mu_2 x_2 \end{Bmatrix} \tag{6}$$

$$z' = \begin{Bmatrix} z'_1 \\ z'_2 \end{Bmatrix} = \begin{Bmatrix} x'_1 + \mu_1 x'_2 \\ x'_1 + \mu_2 x'_2 \end{Bmatrix} \tag{7}$$

x_1 and x_2 are the field point co-ordinates, x'_1 and x'_2 are the source point co-ordinates.

A_{ik} is the solution vector of the linear system.

4. Results and discussion

In this section, different microstructures (RVEs) were adopted, whose homogenized constitutive responses are compared with those obtained from the FEM model for the microstructure proposed in [16], [18], and [19] in order to validate the formulation developed here. In this comparison, the same mesh was always adopted for both BEM and FEM models, that is, for the finite element model, the triangular elements were adopted coinciding with the triangular cells of the model developed with the BEM. It is worth mentioning that although they are not presented here, convergence tests of the results with the mesh refinement were tested for both the BEM and the FEM model. As the two numerical models are quite different, a mesh was adopted that presented good results for both methods, in order to compare the results. Thus, in order to simulate the incremental loading process related to the problem of the macro-continuum in a multi-scale analysis, any deformation is imposed on the RVE in increments. Then, after solving the RVE for each strain increase, the homogenized values of the stress vector and the constitutive tensor are obtained. It is important to note that an analysis of the convergence of the results with the mesh refinement was done in all examples. Thus, when this study is not presented, we always used a mesh whose results had already achieved this convergence.

4.1 Influence of the volume fraction of inclusions

In this first set of RVEs, an elastic inclusion was defined at the center of the RVE, the behavior of the matrix material being governed by the constitutive model of von Mises. Different volume fractions were considered for this inclusion, in order to verify how it influenced the constitutive response of the RVE. In all calculations, the model of periodic fluctuations in the contour of the RVE was adopted. The following volume fractions were adopted: fv = 10%, fv = 30% and fv = 37% (see Figure 1). The following properties were adopted for the inclusions, which are adopted as elastic: Elastic modulus E = 200GPa and Poisson's coefficient ν = 0.2. For the matrix it was adopted: E = 70GPa, ν = 0.3, yield stress σy = 243MPa and hardening module K = 2.24GPa. The following strain vector was imposed on the RVE in 25 increments: {ε} = {ε1 ε2 ε12} = {- 0.0015 0.0048 0}.

The meshes considered for the RVE (with fv = 10%, fv = 30% and fv = 37%) contain, respectively:

a) 220 cells and 131 nodes (40 contour elements and 12 interface elements),

b) 204 cells and 123 nodes (40 contour elements and 20 interface elements) and

c) 236 cells and 139 nodes (40 contour elements and 24 interface elements).

In Figure (1), the tension is homogenized in the x2 direction along the incremental process and considering the three different RVEs. As expected, the most rigid response refers to RVE with fv = 37% and the most flexible response refers to RVE with fv = 10%. In addition, in Figure (1)it is noted that the results of the BEM and the FEM are very similar.

4.2 Calculation of the tensors of macroscopic deformations

Using the geometric data shown in Fig. 2, referring to the AS4 epoxy laminate 3501-6 with respect to the orientation of the fibers and with a thickness of 1 mm, the analysis was carried out in the plane of the laminate, using the BEM for the anisotropic elasticity, and one deduces the tensors of deformations associated with internal material points. For this, we consider the laminate with the outer contour and the central hole discretized by 7 discontinuous quadratic elements and 52 internal material points.

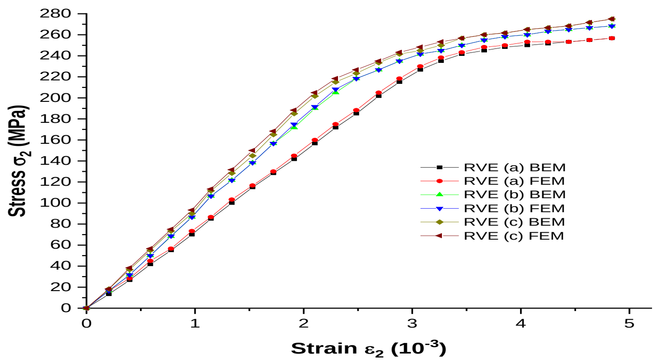


Fig 1. Homogenized stress in the x2 direction, considering different volume fractions for the inclusions

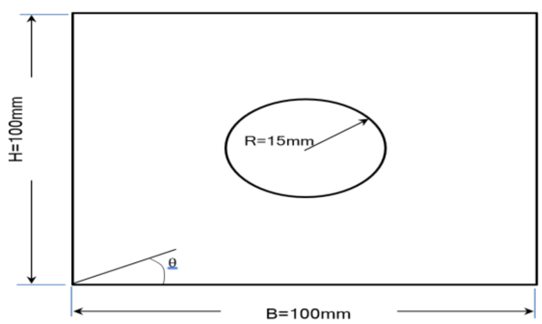


Fig 2 . Unidirectional laminate with central hole subjected to a positive displacement, θ being the orientation angle of the fibers.

For a laminate with θ = 0 ° fibers subjected to positive displacement in the right edge along the 0.20 mm axis and restrictions on movement in and from the left and / or right directions, as shown in Figure 3. The strain tensors were obtained from each internal material point, which refers to the tensors of the macroscopic deformations, which are identical to the tensors of the homogenized EVRS deformations. The graphs shown in Fig. 4 and Fig. 5 show the values obtained from the deformation tensors for the internal points of the laminate, with respect to the same values obtained using the ABAQUS finite element software, where the laminate was discretized by a linear quadrangular finite element mesh at 4 knots, a length of each element of 2. The figures also show the average percentage of relative errors obtained for each point, which notes that the results are consistent and the average percentage of errors is the lowest.

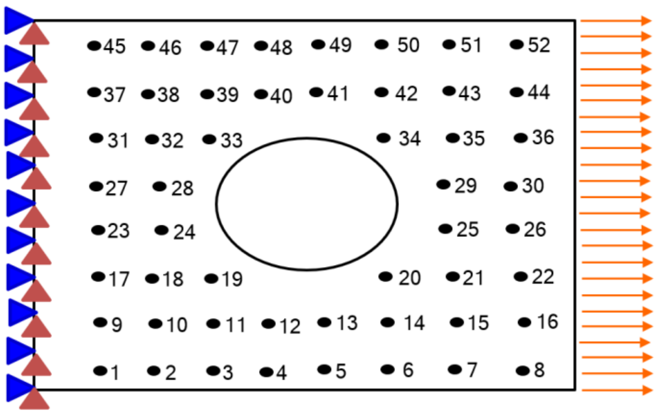
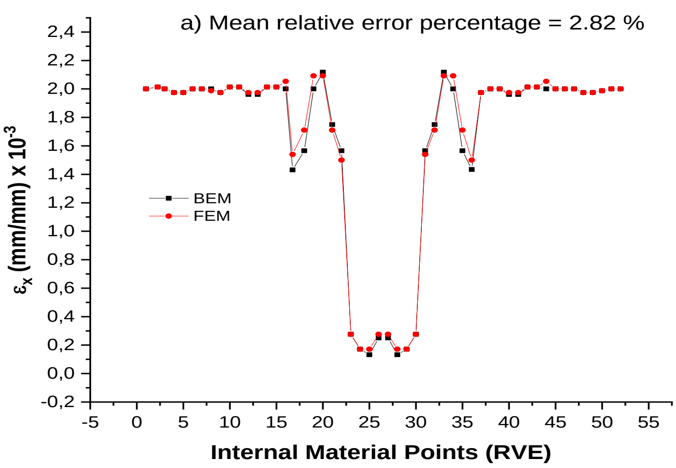
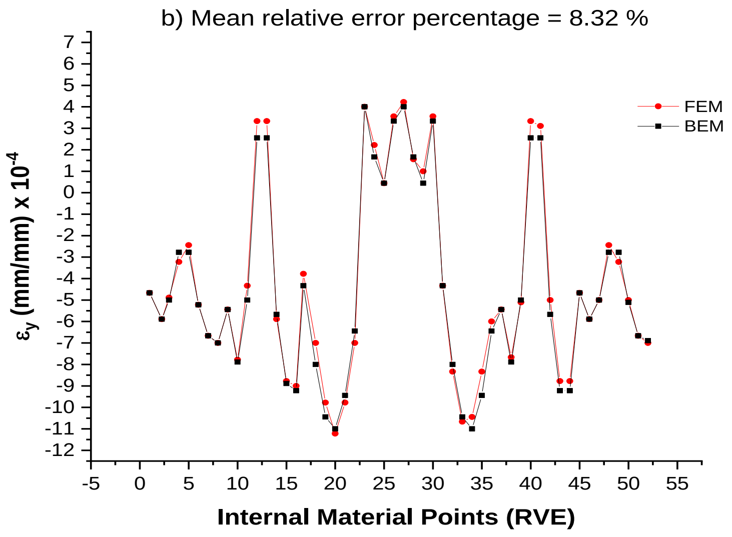


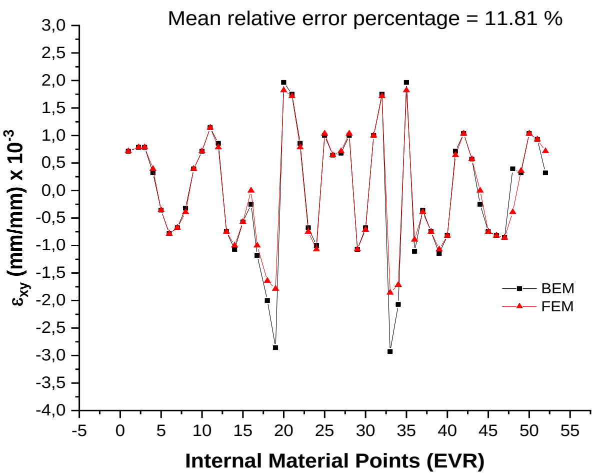
Fig 3. Discretization of the laminate and boundary conditions.





(a) Average percentage of relative error = 2.82% (b) Average percentage of relative error = 8.32%

Fig 4. Deformation of internal points in (a) The x direction. (b) The y direction.



Average percentage of relative error = 11.81%

Fig 5. Deformation of internal points in xy direction.

4.3 Square plate clamped on all four sides under uniformly distributed load

Consider a recessed plate (Figure 6) loaded at time τo = 0 s by a step-type load q = 2, 07 × 106 N / m2. The plate is orthotropic and has the following properties and dimensions:

E2 = 6895 MPa, E1 = 2E2, G12 = 2651.9 MPa, ν12 = 0, 3, ρ = 7166 kg / m3,

a = 254 mm and thickness h = 12, 7 mm.

The static bending moment of the plate's central node is given by mstatx = 4,06 × 103N.m / m and the time normalization factor by

$$t_0 = \frac{a^2 \sqrt{\frac{\rho h}{a_{11}}}}{4}$$

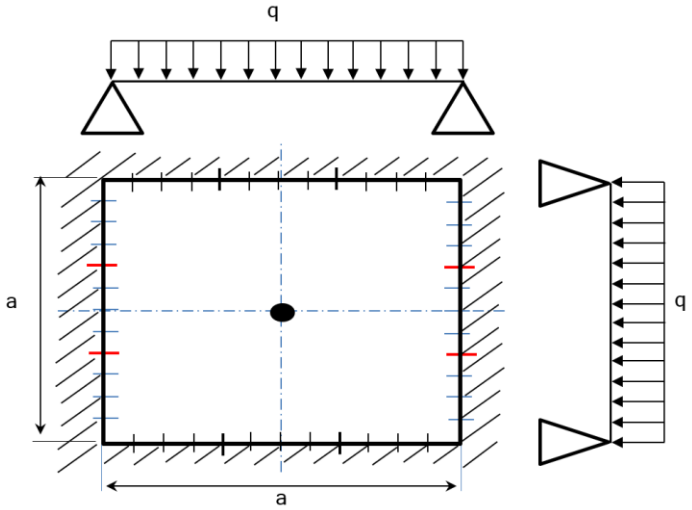


Fig 6. Square orthotropic clamped plate.

The plate was discretized using 12 discrete square contour elements of the same length and time step Δτ = 2, 1915 × 10−5 s. The problem was analyzed using 1, 9 and 25 internal points evenly distributed. Figure 7 shows the bending moment of the central node of the plate as a function of time. In addition, results using the MLPG and finite elements presented by Sladek et al. [29].

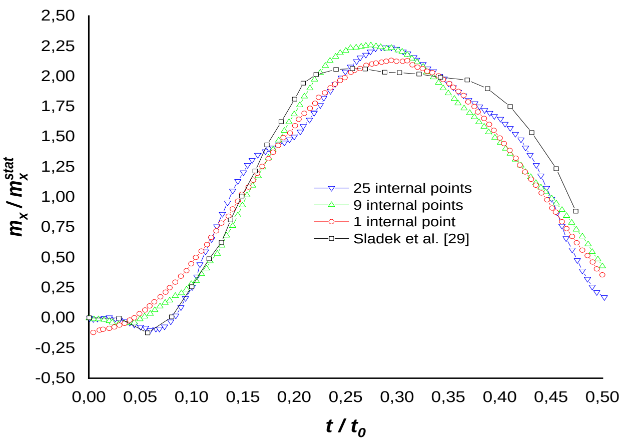


Fig 7. Bending moment of the central node of the plate as a function of time, varying the number of internal points.

It can be seen that internal points are necessary to obtain a greater precision of the results, as shown in Figure 6. With only one internal point there is an expressive difference in relation to the other results. The result with 25 internal points proved to be closer to the solutions obtained by

Sladek et al. [29]. The result with 9 internal points had a good agreement with the results of the literature.

5. Conclusion

Failure analysis of multi-layered composite structure has been studied in the present work. The laminate considered for the present failure analysis is a ten-layered ply. The Tsai-Hill and maximum deformation failure criteria are in good agreement since the AS4 epoxy laminate 3501-6 structure will not fail under the given set conditions. The results obtained indicate that the failure criteria used are good and can be used to predict interfiber failure in multi-layered composite structure.

The results presented in this paper show that computational micromechanics has become a powerful tool for linking the microstructural characteristics of composites reinforced with unidirectional fibers with the properties of macroscopic layers. These capacities were gathered in a kit of calculation tools for Abaqus, BEM, Matlab.Which allows us to carry out the design of laminated composites, by calculating the macroscopic properties (rigidity, force) from the mechanical properties of the fibers, of the matrix and of the interface and of the volume fraction, of the shape and of the spatial distribution of the fibers. This tool is very useful from an industrial point of view to select new material configurations with properties optimized for specific applications and to provide the input data for the structural analysis of laminates in the framework of computational mechanical meso.

The microstructure-based numerical modelling concept presented in this pater constitutes a promising method for the design of new composites. The potential of the approach is its suitability to be applicable to any complex microstructure. A microstructure-based model is much more versatile and universal than an analytical approach, since arbitrary microstructures with any number of phases, morphology and properties can be modelled.

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