The Field of Automatic Control Systems Using Fractional Calculus

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Abstract:

In this paper I have proposed improvements in adaptive algorithms by the introduction of fractional order operators.

That controllers based on a fractional order calculation have gained more and more interest from the community of control. In this area of research, Developed fractional order controllers include operators and or systems in their structure or implementation. These controllers have been introduced into control applications in an ongoing effort to improve control system performance and robustness. The problem was solved using the proposed algorithm (MOWOA) to make a comparison between the solution with some other methods. Also, a numerical example is shown with data in their real form to compare the effectiveness of the proposed classification technique. The proposed obfuscation and ordering technique can be used in other areas of decision-making that deal with data in the same fashion. The results were obtained by various numerical examples and applied to the fractional order filter. The contribution presents an optimal tuning of the classical PID controller and the fractional order $PI^{l}D^{n}$ for brushless DC motor speed control, using single-and multi-objective optimization. Control parameters were synthesized using the MOWOA (Multi-Objective Whale Optimization Algorithm) algorithm.

Keywords: Fractional order calculus, fractional order controllers, control systems, MATLAB toolboxdiffer integration

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1. Introduction

The idea of partial calculus was proposed more than 3 centuries ago. However, the first successful application of fractional calculus dates back to the 1960s [13],[17],[22].In the last decades, fractional calculus has been really developed and widely used in different fields of science and engineering [28]. Recently, the great interest in calculus gave a stimulus through its applications in various fields of systems and control. Some work was done by bode who proposed the function of transferring the order of open-loop fractions while maintaining the stable operation of the loop amplifiers.

Fractional calculus has been used in the field of automatic control systems dating back to the early 1960s [13][12].But only in consoles based on fractional order arithmetic and in recent decades has gained increasing attention [18],[24]. In this area of research, all fractional order controllers developed involve fractional order operators and/or systems in their structure or implementation. These controllers have been introduced into control applications in an ongoing effort to improve control system performance and robustness.

Austalope was the first to truly introduce the partial order controller, who developed the CRONE controller (robust non-whole order command) and he applied in various fields of control systems [17].A fractional order $PI^{\lambda}D^{\mu}$ controller; It was suggested by Podlubny, comprising an integration action of λ order and an action of μ order differentiation [24].In [4],a very good summary of fractional calculus can be found in the field of control; the basic definitions of fractional calculus, fractional order dynamic systems and fractional order control was also present, in addition, several known typical fractional order controllers were presented and discussed.

Controllers based a fractional order calculation are gaining more and more interest among researchers in the control field. These controllers may have fractional order operators and/orsystems in their structure or implementation. They have been introduced into control applications in an ongoing effort to improve system control performance.

My research at present focuses on the design and analysis of new robust fractional order controllers based on the invalid commands approach [16].

The objective is to extend the derivation or integration of the fractional system by using not only the whole order but also non-integer orders [16].

The history of fractional calculus began with a key question from Leibniz, to whom we owe the idea of fractional derivation [26]. He introduced the derivation symbol of order n.

$$\frac{d^n y}{dx^n} = D^n y$$

Where n is a positive integer. It was perhaps a naive play of symbols that prompted Hospital to question the possibility of having n in Q. He asked the question: what if $n=\frac{1}{2}$

In 1695, in a letter to Hospital, Leibniz prophetically wrote: “So it follows that $d^{1/2}x$ will be equal to

$\sqrt[\frac{1}{2}]{dx;x}$The difference is clear, from which we can draw useful conclusions."On these questions, we find the contributions of great mathematicians such as Euler or Lagrange in the 18th century, Laplace, Fourier, Liouville (1832; 1837) or Riemann (1847) in the 19th century, as well as Grünwald (1867) and Letnikov (1868), at the end of the same century. It seems that a contradiction in the definitions prevented a greater success of the theory, which is admittedly not yet further unified; the early absence of a clear geometric or physical interpretation of the derivative fractionalization of a function has been instrumental in keeping exciting fields of research in the dark. The paradox of distinct definitions was resolved by understanding the nonlocal character of the noninteger derivation operator. For more historical details, we refer to [8], [12], [19and 27].During the past three decades, more interest has been given to fractional calculus, and the fields of application have diversified.

In recent years considerable interest has been shown in fractional calculus through the application of these concepts in different fields of physics and engineering, where a significant advance in theoretical work has been found which can serve as a foundation for a number of applications in these fields. So, a great effort has been made to try to put into practice the results already established, and intensive research work is still underway in several fields of engineering for the application of these fractional order concepts.

In this paper, I proposed a new approach for applying fractional order systems in control engineering that contrasts symmetrically with previous classical methodologies.

My objective in this work is to use partial order systems and operators in adapting control schemes. Our interest in operators and fractional order systems is driven by the very good performance of the latter relative to those of the whole order. Indeed, new control diagrams have been proposed while showing their advantages by applying them to different types of dynamic systems. In this context, I introduce a new adaptive control system with high partial gain and with the output notes.

With the entry of the fractional order integral with the regular integral of System output in adaptive gain of control strategy. Simulate and the results, I got were satisfactory.

We also provide large gain adaptive controller with modulation of modulation concept by calculating the fractional order and this modification, we intend to maintain the robustness in the presence of disturbances and the elimination of inappropriate behavior in the absence of perturbations by introducing the derivative of the fractal order into the adaptive gain of Control strategy instead of regular derivatives. Simulation was performed to show the seriousness and efficacy.

2. Fractional order modeling

The mathematical representation of fractional systems in the domain frequency gives irrational functions which, in the time domain, correspond to differential equations which are difficult to use. Given the absence of mathematical methods, fractional-order dynamic systems had until then been studied only marginally, either in theory or in application. For reasons of analysis, synthesis, and in simulating such systems, the use of rational functions for the approximation is of great importance. So to analyze and design, fractional order control systems we must approximate them by rational functions. Fractional order modeling consists of describing the physical phenomena associated with devices whose behavior can be governed by derivative equations partial. Fractional infinitesimal calculus (differential and integral) marks its beginning in the 17th century, after some work by Gottfried Wilhelm Leibniz (1697) and Leonhard Euler (1730). One hundred years later it begins to be studied again by a large number of famous mathematicians such as P. S. Laplace (1812), J. B. J. Fourier (1822), N. H. Abel (1823-1826), J. Liouville (1832-1873), B. Riemann (1847).

Today, the fractional approach is thus applied for the modeling of electrical devices [1, 2, 3, 14, 25], for the modeling of the consequences of natural disasters [24] or for the synthesis of the command [11, 15, 21].Order modeling fractional is also present in the field of biological sciences (models of human body parts) [5] or even human and social sciences (modeling of market behavior) [9].

3. Mathematical definitions

Fractional calculus (integration and differentiation of fractional order) is the field mathematical analysis and the application of integrals and derivatives of arbitrary order. Fractional calculus can be considered an old topic and still new. In recent years the considerable interest in fractional calculus has been stimulated by its applications in the various fields of physics and engineering. The mathematical representation of fractional systems in the frequency domain gives irrational functions which, in the time domain, correspond to differential equations that are difficult to exploit. Due to the absence of mathematical methods, dynamic fractional order systems were previously studied in a marginal way. Only, whether in theory or in application. For reasons of analysis, synthesis and simulation of such systems, the use of rational functions for approximation is of great importance. So to analyze and design fractional order control systems we have to approximate them by rational functions [10].

The integral-differential operator c$D_t^m$where c and t are the operator's terminals are defined as:

$$\text{c}D_t^m = \begin{cases} \frac{d^m}{dt^m} \dots\dots\dots\dots\dots\dots\dots R(\alpha) > 0 \\ 1 \dots\dots\dots\dots\dots\dots R(\alpha) = 0 \\ \int_c^t (dt)^{-m} \dots\dots\dots\dots\dots.. R(\alpha) < 0 \end{cases} \quad \dots\dots\dots\dots\dots\dots..(3.1)$$

In general m is the order of operation$\boldsymbol{\alpha} \in \boldsymbol{R}$

There are several mathematical definitions of integration and fractional order derivation. These definitions do not always lead to identical results, but are equivalent to a wide range of functions [4, 22].

3.1. Definition of Riemann-Liouville

Let C and $\Re$ be the rings of complex and real numbers respectively, $\Re$ (.) Symbolize the real part of a complex number.

Let$\alpha\epsilon\mathrm{C}$ and$R(\alpha) > 0$, $\boldsymbol{t_0} \in \boldsymbol{R}$ f a locally integrals function defined on [$\boldsymbol{t_0}$,+∞[

The integral of order α of with lower bound t_0 is defined by:

$$\frac{\mathbf{1}}{\boldsymbol{\gamma(\alpha)}}\int_{\boldsymbol{t_0}}^{\boldsymbol{t}}(\boldsymbol{t}-\boldsymbol{\tau})^{\boldsymbol{\alpha}-\mathbf{1}}\boldsymbol{f}(\boldsymbol{\tau})\boldsymbol{d\tau}\text{.............}(3.2)$$

$$\boldsymbol{\gamma}(\boldsymbol{x}) = \int_{\mathbf{0}}^{\infty}\boldsymbol{e}^{-\boldsymbol{t}}\boldsymbol{t}^{\boldsymbol{x}-\mathbf{1}}\text{........................}(3.3)$$

Let$\alpha\epsilon\mathrm{C}$ and$R(\alpha) > 0$, $\boldsymbol{t_0} \in \boldsymbol{R}$ f a locally integrals function defined on [$\boldsymbol{t_0}$,+∞[

The integral of order α of with lower bound t_0 is defined by:

$$\mathrm{RL}\boldsymbol{D}_{\boldsymbol{t_0}}^{\boldsymbol{\alpha}}\mathrm{f(t)}=\frac{\mathbf{1}}{\boldsymbol{\gamma(n-\alpha)}}\frac{d^n}{dt^n}\int_{\boldsymbol{t_0}}^{\boldsymbol{t}}(\boldsymbol{t}-\boldsymbol{\tau})^{\boldsymbol{n}-\boldsymbol{\alpha}-\mathbf{1}}\boldsymbol{f}(\boldsymbol{\tau})\boldsymbol{d\tau}\text{..................}(3.4)$$

$(n-1) < \alpha < n$.

Note: to simplify the writing, one will note in the followingI^αForI_0^α and D^α for D_0^α

3.2. Definition of Caputo

Caputo introduced another formulation of the defined fractional order derivative:

$$\mathrm{C}\boldsymbol{D}_{\boldsymbol{t_0}}^{\boldsymbol{\alpha}}\mathrm{f(t)}=\frac{\mathbf{1}}{\boldsymbol{\gamma(n-\alpha)}}\int_{\boldsymbol{t_0}}^{\boldsymbol{t}}(\boldsymbol{t}-\boldsymbol{\tau})^{\boldsymbol{n}-\boldsymbol{\alpha}-\mathbf{1}}\boldsymbol{f}(\boldsymbol{\tau})^{\boldsymbol{n}}\boldsymbol{d\tau}\text{......................}(3.5)$$

$(n-1) < \alpha < n$.

$\boldsymbol{f}(\boldsymbol{\tau})^{\boldsymbol{n}}$is the derivative of integer order n of the function$\boldsymbol{f}(\boldsymbol{\tau})$

3.3. Definition of Gründwald-Leitnikov

The fractional order derivative of order $\boldsymbol{\alpha}$> 0 of Gründwald-Leitnikov (G-L) is given by:

$$D^{\alpha}\mathrm{f(t)}=\frac{d^\alpha}{dt^\alpha}\mathrm{f(t)}=\lim_{h^{-\alpha}--0}\sum_{j=0}^{k}(-1)^j\binom{\alpha}{j}\mathrm{f(kh\text{-}jh)}\text{.....................}(3.6)$$

Where he is the sampling period and the coefficients .

$$w_j^{(\alpha)}=\binom{\mu}{j}=\frac{\alpha+1}{(j+1)(\alpha-j+1)}$$

With$w_0^{(\alpha)}=\binom{\alpha}{0}=1$

Are the coefficients of the following binomial.

$$(1\text{-}z)^{u}\sum_{j=0}^{\infty}(-1)^j\binom{\alpha}{j}z^j=\sum_{j=0}^{\infty}(w_j)^{\alpha}z^j\text{..................}(3.7)$$

The Grünewald-Leitnikov definition of fractional order integral is formulated as follows:

I^{γ}f(t)=$D^{-\gamma}$f(t) =$\frac{d^{\gamma}}{dt^{\gamma}}$ f(t)=$\lim_{h^{-\mu}--0}\sum_{j=0}^{k}(-1)^{j}\binom{-\gamma}{j}$f(kh-jh)

Where h is the sampling period and the coefficients

$w_j^{(-\gamma)}\binom{-\gamma}{j}$=1

Are the coefficients of the following binomial:

(1-z)$-\gamma\sum_{j=0}^{\infty}(-1)^{j}\binom{-\gamma}{j}z^{j}$=$\sum_{j=0}^{\infty}(w_j)^{-\gamma}z^{j}$…………………(3.8)

3.4 Fractional order high gain adaptive control

I proposed a schematic diagram of a high-gain adaptive fractional order high gain controller with feedback output for a class of linear, time constant, minimum phase of the maneuverable system (single input single output, SISO) and one relative degree. A high-gain adaptive controller has also been proposed by modifying the concept of calculus for fractions.

4. Problem Position

Hypothesis 1: The above system we assume is controllable and can be observed at a minimum phase with gain at high positive frequencies and at first relative degree.

The system of an equation (3.4) fulfills Hypothesis 1, this system can be established by the relative adaptive control law of equations (3.5) and (3.6); but an annoying feature of this law-control is that the gain k (t) diverges in the presence of any output error y (t), for example, by measurement noise. In the case of the problem of controlling the output feedback with a gain constant, this can be easily overcome by introducing the integral term into the control law. The aim of this work is to suggest a modification in the equation's gain conditioning.

$\frac{dk(t)}{dt}$=y^{2}(t)………………..(4.1)

The initial character is arbitrary, in this case, conditionsx_0=x(t_0)ϵR^{n},k_0=k(t_0)ϵR, the problem at the boundaries of the closed-loop system has a unique solution (x (t), k (t)) with the following properties:

$$\lim_{t\to\infty} x(t) = 0, \lim_{t\to\infty} k(t) = k_{\infty}$$

From Equation (5) and the Control Law to maintain the stability of a closed-loop control system and force its output to converge without constant error and maintain its finite gain; without prior knowledge of the system parameters and has a request. So the primary objective of this new modification in the design of the control law is the beginning of the fractal system integration as well as the regular integration of the system outputs into the adaptive gain of the control strategy in equation (9), for the uncertain maneuvering of.

$\frac{dx(t)}{dt}$=Ax(t)+Bu(t)……….(4.2)

$Y(t)=C\,x(t)\,,x(t)\epsilon R^{n},t\in R,$

X: state vector for dimension n

U: vector of commands for dimension p

Y: vector of measurements for dimension m, the command scalar $u(t)\in R, y(t)\in R$. Satisfactory hypothesis 1, uniformly stable by high-gain proportional adaptive control in Equation (4.3).*A, B*, and C are unknown matrices of suitable dimensions. ***u(t)=- K(t)y(t)*.....(4.3)**

Whose law of adaptation is given as follows:

e(t) = y(t) - r(t)

u(t) = -k(t) e(t)

5. Application

I give an illustrative example to illustrate the efficiency and improve the quality of control using the proposed control scheme. We will observe in an unstable continuous system with a minimum phase of the first relative degree.

Numerical arithmetic of a fractional integral of order $\frac{155}{100}$ of the equation

$k(t) =\frac{40}{10}\{I^{1}[y^{2}(t)]\}+\frac{120}{10}\{I^{\frac{155}{100}}[y^{2}(t)]\}$……….(5.1)

The classic case given in [28] was used for comparison. And we mentioned that if$\gamma_1=\frac{40}{10}\neq 0$ ET$\gamma_2=\frac{0}{10}$,Equation Control Scheme

$k(t) =\gamma_1 I^{1}[y^{2}(t)] +\gamma_2 I^{(1-\alpha)}[y^{2}(t)]$………(5.2)

It is the classic case. And then, the classic control gain kc(t) is given by:

$k_c(t) =\frac{40}{10}\{I^{1}[y^{2}(t)]\}$

The numerical calculation of the fractional integrator of order$\frac{155}{100}$of equation (5.1) was done using a digital FIR filter obtained by [6].The simulation results are obtained by taking the reference signal r(t) asunit step. Figure 1 shows the plots of the outputs y(t) and yc(t) of the closed-loop control system using, respectively, the proposed fractional order control scheme and the classical control scheme. From Figure (1), it can be seen that the output y(t) of the closed loop control system using.

The proposed fractional order control scheme is faster; it has smaller overshoot and rise time compared to the output yc(t) of the closed-loop control system using the classical control scheme.

The plot of the control signals u(t) and uc(t) of the closed-loop control system using the proposed fractional and classical order control schemes, respectively, are given in Figure (2). The plots of the fractional and classical control gains k(t) and kc(t) are shown in Figure .3.

Suggested partial command control scheme and closed-loop system outputs y (t) and yc (t)

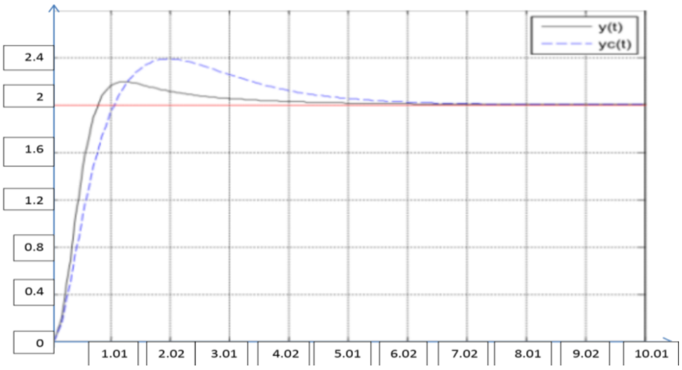

Figure1

classic control system

The proposed partial order and closed-loop order scheme u(t) and uc(t).

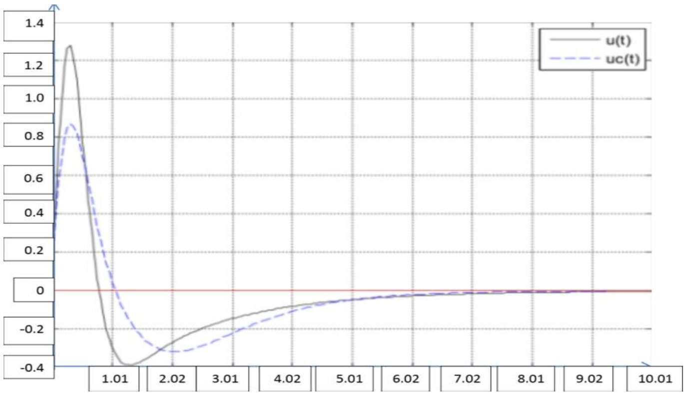

Figure.2

classic diagram command

Closed-loop system and control gains k (t) and kc (t).

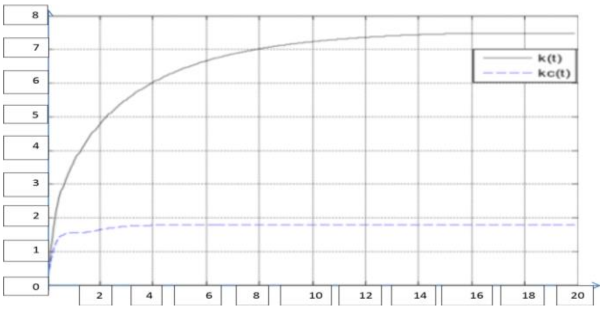

Figure 3.

Proposed partial order control scheme and classical control system

6. Conclusion

Studying the field of partial order adaptive control allowed me, to propose improvements in adaptive algorithms by introducing fractional order operators, thus extending the theory of this matter by developing new algorithms while demonstrating their goals by applying them to a variety of different operations. We made a significant contribution related to partial order adaptive control to improve the quality of control in dynamic systems. My contribution includes a part based on numerical simulation examples, a theoretical part based on contextual analysis and the development of a fractal system stability theory for adaptive large gain control.

My contribution is to propose a new high-gain adaptive micro-system control system with output feedback for a class of systems, linear, time-invariant, with a minimum phase, single-variable, and relative degree unit. The main objective of this new scheme is to use partial system integration along with the regular integration of system outputs in the adaptive gain of the control strategy. The presented partial integration improved the behavior of the control system. We have established that the console can also install the class of studied systems. The proposed adaptive controller can also solve the unit step reference tracking problem.

I do a simulation, which has a satisfactory behavior of the controllers. The presented simulation results show the improvement in the quality of control by comparing the proposed control scheme with the traditional control scheme.

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