# A theoretical and experimental study of the hydraulic jump evolving in a rough bottom channel

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**Abstract**

**This paper conducts a theoretical and experimental analysis of the hydraulic jump controlled by a threshold in rectangular channel with a rough bottom for various tested roughnesses namely ε = 6, 8, 10 and 12mm. We obtained a theoretical relationship, in dimensionless terms, linking the various parameters of the jump, showing the effect of the roughness of the bottom of the channel. It is essential to adjust at the end of this experiment the relation which is drawn by the theoretical development of the equation of the quantity of movement. The results showed that the presence of roughness positively influences the jumps which tend to shorten and dissipate more energy, a relation was proposed for this purpose to facilitate the determination of the dimensions of the damping basin.**

**Keywords: Hydraulic jump; Rectangularchannel; Roughnesses;rough bottom; Damping basin**

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## 1. Introduction

Knowing the position of the jump is very important, indeed determining the latter will guarantee the length of protection against erosion necessary for the soils and banks just downstream of a dam. The review of the literature shows that the dimensionless characteristics involved in the jump phenomenon are essential, the number of Froude of the incident flow ($Fr_1$), the initial heights ($h_1$) and final ($h_2$) of the jump and the ratio of the combined heights of the jump (Y).

The main objective is to establish an empirical relationship between dimensionless hydraulic and geometric parameters translated respectively by the upstream and downstream conditions of the flow and the nature of the control structure, including the upstream height of the structure and the Froude number and the effect of the roughness. In order to size a more compact and reduced sinking basin, which is very economical in terms of construction.

## 2. Position of the problem

The jumps occur during the transition from the torrential regime to the fluvial regime. In nature, they frequently appear in mountain streams or rivers or a sink or lavatory, and also from the discharge of a flood flow through a weir. We can recreate hydraulic jumps in the laboratory. They are called classic when they form in a rectangular channel with a smooth bottom with no slope. They are called controlled when finding an obstacle to the downstream flow. The type of jump that will be studied in this paper configures in a rectangular channel with a rough bottom and is controlled by a threshold.

The search for the parameters of a jump has been the subject of several studies but in vain, there is unfortunately no empirical formula allowing the determination of the exact parameters of the jump. The purpose of this paper is to find an empirical relation indicating the effect of roughness on the characteristics of the hydraulic jump.

## 3. Theoretical analysis

Figure 1 represents a diagram of definition of a step controlled by threshold in a channel of rectangular section with rough bottom and a zero slope.

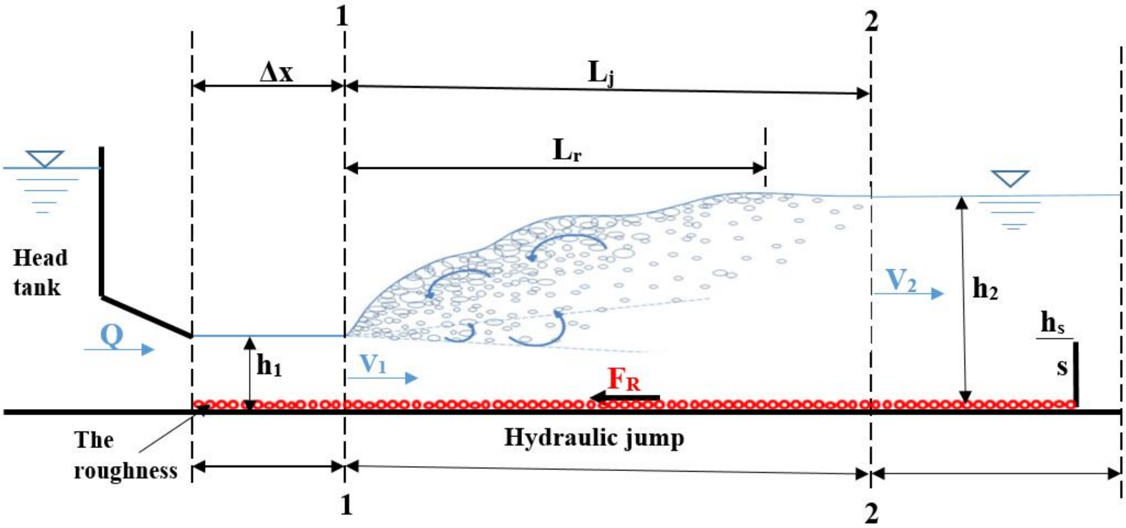


**Fig. 1.**A jump controlled by a threshold in a rectangular channel with a rough bottom.

Given the following simple hypotheses:

- The hydrostatic pressure distribution in the upstream (1-1) and downstream (2-2) sections, and the vertical profile velocity distribution is uniform.
- The friction between the water particles is negligible.
- Air resistance is negligible, and the flow is permanent.

If we applied the momentum equation between two initial (1-1) and final (2-2) sections of the jump, we can write the following equation:

$$F_1 - F_2 - F_R = \rho Q V_2 - \rho Q V_1 \quad (1)$$

Where:

$F_1$ and $F_2$ are the pressure forces exerted on the upstream and downstream sections of the jump, respectively:

$F_1 = P_1A_1$,with$P_1=\rho g \hbar_1$ and $\hbar_1=h_1/2$, $A_1= bh_1$

$F_2 = P_2A_2$, with $P_2=\rho g \hbar_2$ and $\hbar_2=h_2/2$, $A_2= bh_2$

$V_1= Q/bh_1$ and $V_2= Q/bh_2$ are respectively the speeds at the upstream and downstream sections of the jump,

$\rho$the density of the liquid.

Qis the volume flow.

$F_R$ is the resistance force due to the roughness of the walls of the channel, which is expressedas follows: $F_R=CF_N$, with $F_N=AP_N$, and $A=bL_j$, and $P_N=\rho g\hbar$, $\hbar=V_1^2/2g$

To deduce:$F_R = C\rho gbL_j\left(\frac{V_1^2}{2g}\right)$

By replacing all the expressions in equation (1), we obtain:

$$\frac{\rho gb}{2}(h_2^2 - h_1^2) = \frac{\rho Q^2}{b}\left(\frac{1}{h_2} - \frac{1}{h_1}\right) + C\rho L_j\frac{Q^2}{2bh_1^2} \tag{2}$$

We have $Y=h_2/h_1$, therefore $h_2=Yh_1$, if we replace the value of $h_2$ in equation (2), we obtain:

$$\frac{1}{2}(1 - Y^2) = \frac{Q^2}{gb^2h_1^3}\left(\left(\frac{1}{Y} - 1\right) + \frac{CL_j}{2h_1^2}\right) \tag{3}$$

We propose$K = \frac{CL_j}{2h_1^2}$, and for a rectangular channel$Fr_1^2 = \frac{Q^2}{gb^2h_1^3}$, equation (3) becomes:

$$\frac{1}{2}(1 - Y^2) = Fr_1^2\left(\left(\frac{1-Y}{Y}\right) + K\right) \tag{4}$$

$$\frac{1}{2}(1 - Y)(1 + Y) = \frac{(1 - Y)}{Y}Fr_1^2\left(1 - \frac{YK}{Y - 1}\right) \tag{5}$$

We subtract (1-Y) from two sides of the equation (5):

$$Y(1 + Y) = 2Fr_1^2\left(1 - \frac{YK}{Y - 1}\right) \tag{6}$$

We propose$C_R = \left(\frac{YK}{Y-1}\right)$

The final equation becomes:

$$Y = \frac{1}{2}\sqrt{8Fr_1^2(1 - C_R) + 1} - 1 \quad (7)$$

For a coefficient ($C_R$) of zero, the expression (7) returns to the theoretical equation of Bélanger (1928) of a hydraulic jump moving in a rectangular channel with perfectly smooth walls.Indeed, this expression is theoretical because the coefficient of resistance ($C_R$), can only be found from experimental data.

## 4. Experimental protocol

### 4.1 Description of the model

The physical model that served as a test bench (Figure 2) and (photo 1) consists essentially of a rectangular channel 10 meters long, 25 centimeters wide and 50 centimeters in depth. A supply basin is connected to the canal by means of a circular pipe 150 mm in diameter. The latter is connected to the channel by means of a closed metal box, on which is inserted a converging sheet of rectangular section opening directly into the channel.

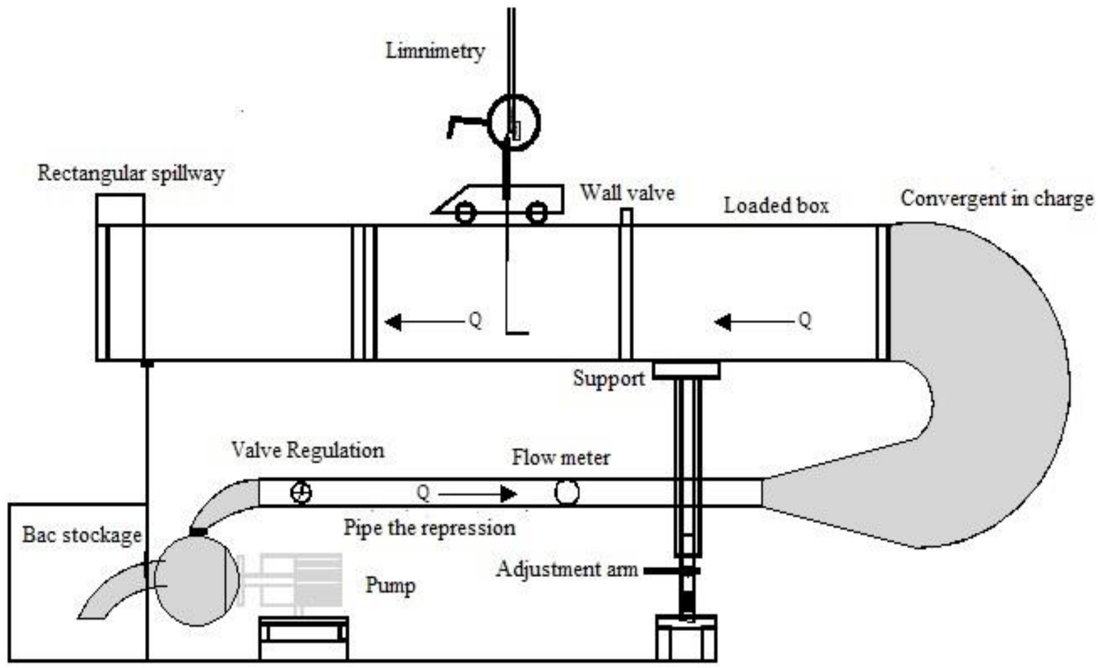


**Fig. 2.**Diagram of the device setup used in the experimental mode.

The convergent and the part of the channel upstream of the wall valve constitute the load box which ensures the high speed of the incident flow. The role of the wall valve will be used to adjust the height of the incident flow h1 of the jump. The volume flow rates are adjusted by manipulating the valve and the supply is effected by means of a pump delivering up to 44 l/s.

**Photograph. 1.**Channel used.

4.2 Experimental procedure

The experiment was carried out at four initial heights ($h_1$ (cm) = 2; 3; 4; 5). A large range of incident Froude numbers was thus obtained ($2 \leq Fr_1 \leq 14$). Thresholds of different heights were tested, the height s of which varied between 3 cm and 20 cm. Indeed, four absolute roughnesses were tested: ε (mm) = 6; 8; 10; 12.


**Photograph. 2.**Different roughnesses tested.

In this experiment, for each value of the absolute roughness we fixed the opening of the wall valve ($h_1$), then we varied the threshold height and adjusted the flow rate until the hydraulic jump is created. Once the experiment is completed for the first opening, the opening will be successively changed from 3 to 5 cm. For each test the following measures will be taken:

1) The final height ($h_2$)
2) The length of the jump ($L_j$)
3) The flow rate (Q)

A sample made up of several experimental measurement points, for each characteristic, thus made it possible to achieve significant results.

**Photograph. 3.**Hydraulic jump moving through a rough-bottomed canal.

## 5. Experimental results

5.1 Variation of the ratio of the combined heights of the jump (Y) according to the resistance coefficient ($C_R$) and the Froude number ($Fr_1$)

The theoretical equation (7) being implicit in (Y), we have found an approximate relation which makes it possible to easily find the ratio (Y) as a function of the coefficient ($C_R$) of resistance at the bottom of the channel and of the Froude number of l incident flow ($Fr_1$).

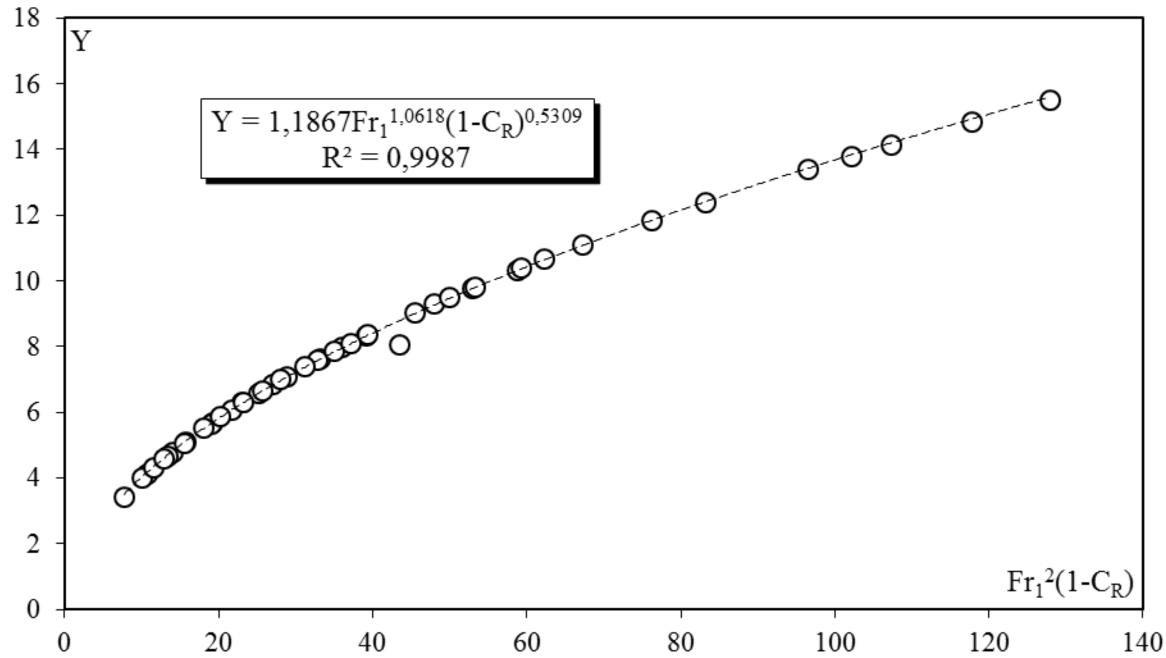


**Fig. 3.**Variation of the ratio of the conjugated heights (Y) as a function of $Fr_1^2$ ($1-C_R$).

(o) experimental measurement points. (...) Adjustment curve.

Figure 3 shows a cloudof points that perfectly follows the shape of a single curve. The adjustment of the measurement points by the nonlinear least squares method gives with a very good correlation the following power relation:

$$Y = 1{,}1867 Fr_1^{1{,}0618}(1 - C_R)^{0{,}5309} \qquad (8)$$

5.2 Variation of the resistance coefficient ($C_R$) according to the absolute roughness (ε)

In order to find the expression of the coefficient of resistance ($C_R$) as a function of the absolute roughness (ε), we will represent in Figure 4, the variation of the term

([(Y + 1)*2]$^2$-1) / (8) as a function of ($Fr_1^2$) for the four absolute roughnesses: ε(mm) = 6, 8, 10 and 12.

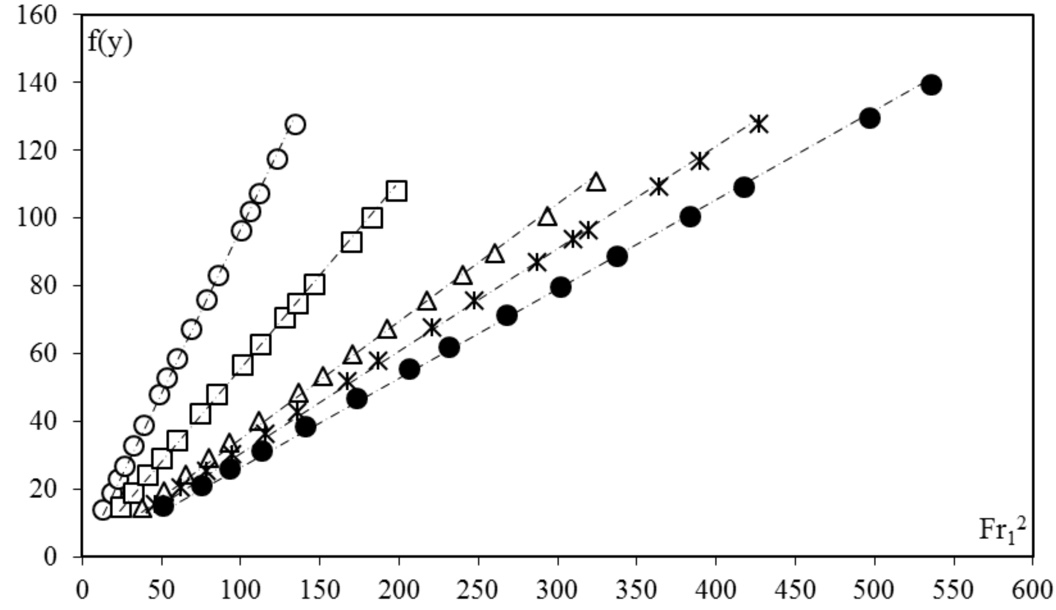


**Fig. 4.**Variation of f (Y) as a function of the squared Froude number ($Fr_1^2$).

(—) Adjustment curve.

Figure 4 shows four distinct clouds of points, each corresponding to a well-determined value of the absolute roughness (ε). For each series of measurements, each time the value of the absolute roughness increases for the same Froude number ($Fr_1$) it causes the decrease in the ratio of the conjugated heights of the jump (Y), which reflects the positive effect of the roughness on the jump parameters.

The analysis of the points of experimental measurements of the jump, shows that each cloud of points can fit with a line of the form $([(Y + 1)*2]^2-1) / (8) = d\ Fr_1^2$. In comparison with equation (7), the term "d" represents the factor ($1-C_R$).

Table 01 presents the values of the coefficients ($C_R$).

| **ε (mm)** | **Coefficient ($C_R$)** | **$R^2$** |
|---|---|---|
| 0 | 0,04 | 0,99 |
| 6 | 0,447 | 0,96 |
| 8 | 0,653 | 0,98 |
| 10 | 0,697 | 0,99 |
| 12 | 0,737 | 0,97 |

**Table 1.**Coefficient (CR) of the adjustment curves.

Table 01 shows that the coefficient ($C_R$) increases with the increase of absolute roughness (ε).The statistical adjustment of the pairs of values (ε, $C_R$) by the method of least squares gives a linear type relation in the following form: $C_R$=0,069ε. This is shown in Figure 5.

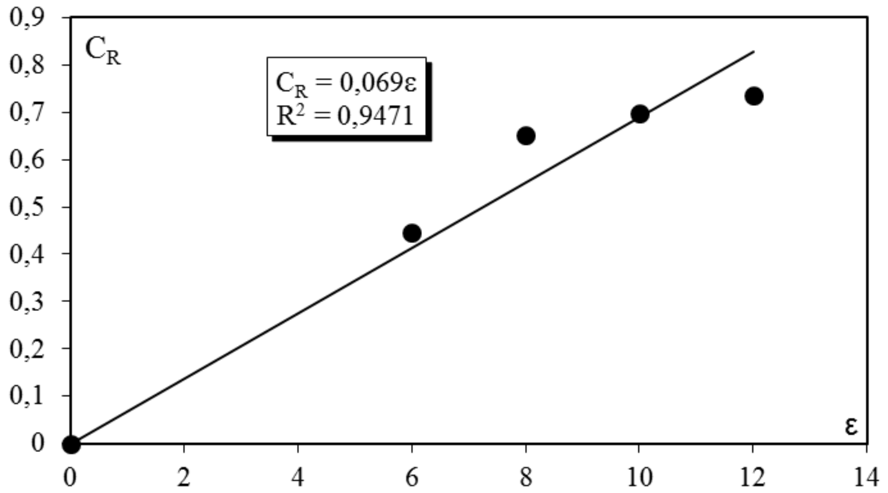


**Fig. 5**.Variation of the coefficient ($C_R$) as a function of the absolute roughness (ε).

By replacing the coefficient ($C_R$) by its expression in the equation:

([(Y + 1)*2]²-1)/(8) = (1-$C_R$) $Fr_1^2$, the theoretical equation becomes:

$$Fr_1^2(1-0{,}069\varepsilon) = \frac{((1+Y)*2)^2-1}{8} \quad (9)$$

Figure 6 shows that the relation f(Y) = ξ(ε, $Fr_1$) adjusts the experimental measurement points with a good correlation and the latter perfectly follow the first bisector, thus showing the reliability of relation (09).

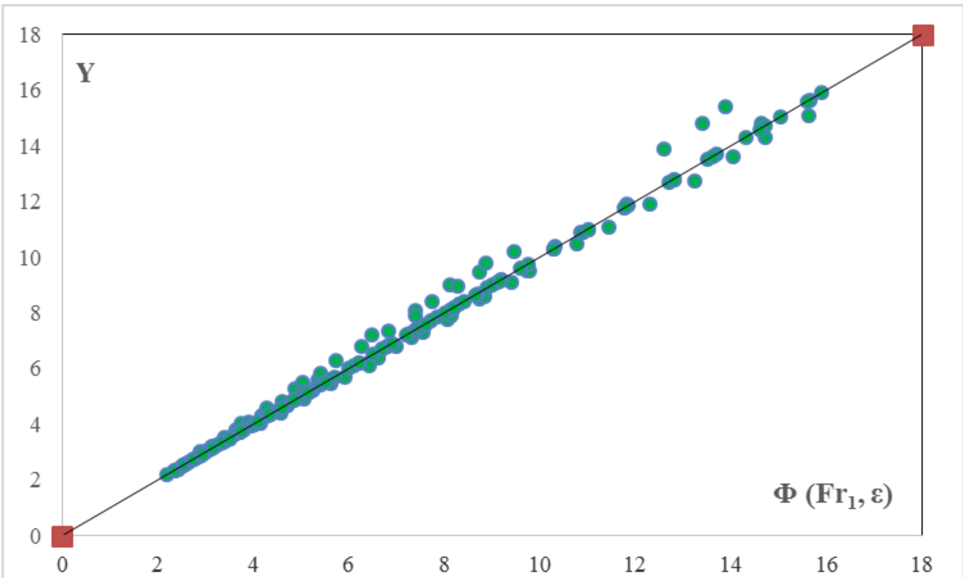

**Fig. 6.**Variation of f (Y) according to ξ(ε, $Fr_1$).

## 6. Hydraulic jump performance

The efficiency of the hydraulic jump or the dissipation of hydraulic energy is defined as the ratio between the loss of the load ΔH and the upstream load $H_1$, and their formula is written as follows:

$$\eta = \frac{\Delta H}{H_1} = \frac{H_1 - H_2}{H_1} = \frac{Y + \frac{Fr_1^2}{2Y^2}}{1 + \frac{Fr_1^2}{2}} \quad (10)$$

Figure 7 shows that the measurement points of the rough-bottomed jump are above their smooth-walled counterparts for a range of incident Froude numbers 2 ≤ $Fr_1$ ≤ 14. However, for the Froude numbers $Fr_1$ ≤ 14, all the measurement points tend to come together to form a single cloud of points.

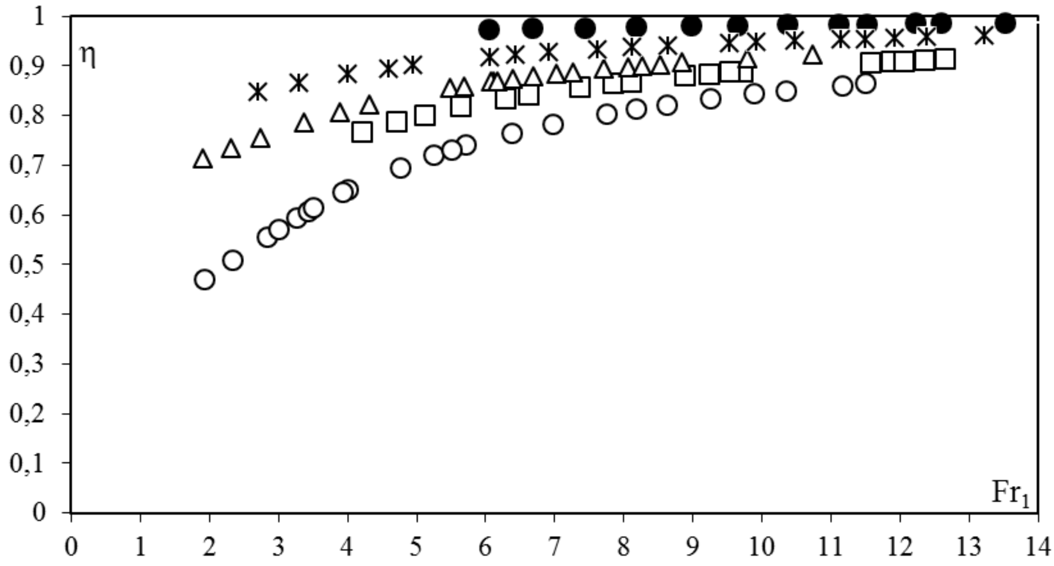


**Fig. 7.**Variation of the yield as a function of the Froude number $Fr_1$.

Indeed, the hydraulic jump in the rough-bottomed channel dissipates the load better than its smooth-walled counterpart. In addition, Figure 7 shows that the dissipation of the hydraulic head increases with increasing roughness. However, for high Froude numbers, the measurement points join together and the effect of roughness is attenuated. This can be explained by the fact that beyond a certain practical Froude number, the jump becomes choppy and cannot constantly adhere to the bottom of the channel (According to the classification of Bradley and Peterka, 1957).

## 7. Conclusions

A theoretical power-type relationship, linking the ratio of the combined heights of the jump (Y), the Froude number ($Fr_1$) and the resistance coefficient ($C_R$) was found for different roughnesses tested.Statistical analysis of the experimental measurement points makes it possible to find the value of the coefficient of resistance ($C_R$), which is equal to the absolute roughness multiplied by 0,069.

It has been observed that for a value of the Froude number ($Fr_1$), the increase in absolute roughness leads to a decrease in the ratio of the conjugated heights of the jump (Y). The graphical representation of the variation in the yield of the jump as a function of the Froude number ($Fr_1$) has shown that the dissipation of the hydraulic head increases with the increase in roughness.

We can conclude that for a series of incident Froude numbers, the hydraulic jump evolving in a rectangular channel with a rough bottom, reduces the effect of the ratio of the conjugated heights of the jump more than its counterpart with a smooth bottom.

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