Nonlinear Voltage Controller-based Model Predictive Control with a Prescribed Degree of Stability Using Takagi-Sugeno Fuzzy Control System and Laguerre Functions for Multimachine Power Systems

Amel Abbadi[1, *], Fethia Hamidia[1], Abdelkader Morsli [1], Mohamed Redha Skender[2], Faouzi Didi[3], Farouk. Bettache[4]

[1]Electrical Engineering Department, Research Laboratory in Electrical Engineering and Automatic (LREA), University of Médéa, Médéa, Algeria

[2]Electrical Engineering Department,Renewable Energy and Materials Laboratory (REML). University of Médéa, Médéa, Algeria

[3] Common-Core Department, Renewable Energy and Materials Laboratory (REML). University of Médéa, Médéa, Algeria

[4]Electrical Engineering Department, Laboratory of Mechanics, Physics and Mathematical Modeling,University of Médéa, Médéa, Algeria

(Corresponding Author): *E-mail: amel.abbadi@yahoo.fr

Received: 07-01-2023 **Accepted:** 07-07-2023 **Published:** 09-07-2023

Abstract

This article presents the design of a control approach based on the continuous-time predictive control model (MPC) with prescribed degree of stability, the Takagi–Sugeno fuzzy model, and the Laguerre functions. This controller is offered to improve the transient stability and voltage regulation of multi-machine power systems. We first used the Takagi–Sugeno fuzzy model. It is a strategy for converting a nonlinear system into a linear system so that linear control techniques can be applied. Model Predictive Control (MPC) is an advanced control scheme based on optimal control. Laguerre functions are used to approximate the control signal to reduce the high computational cost of this technique. The proposed controller is also designed to acquire a wider stability margin through the implementation of the concept of the prescribed degree of stability. This approach will allow the poles of the closed-loop system to be assigned to the desired locations in the left half of the S-plane. The proposed approach is applied to the two-generator infinite bus power system. This multi-machine power system is chosen to highlight the effectiveness of the proposed approach in improving transient stability and voltage regulation with respect to the various imposed faults.

Keywords: Continuous-time model predictive control, Prescribed degree of stability, T-S fuzzy model, Laguerre function, Multi-machine power system.

Tob Regul Sci. ™ 2023;9(1): 3441-3458
DOI: doi.org/10.18001/TRS.9.1.241

1. Introduction

Modern electrical networks are huge and complex systems, whose management must be safe, reliable and economical [1]. Maintaining transient stability and voltage regulation are fundamental requirements when operating an interconnected electrical system, since it concerns the ability of this system to withstand severe disturbances while ensuring the perpetuation of service.

In recent years, much attention has been paid to the application of advanced control techniques to improve the transient stability and voltage regulation of multi-machine power systems [2-8]. This is mainly due to the impact of changes in operating conditions: variations in production/load with the possibility of the appearance of various disturbances which can affect the stability and quality of the energy.

A new control technique among the advanced control methods, has appeared and aroused great interest, whether in the field of academic research or in practical applications. This technique is the Model Predictive Control (MPC) algorithm [9-14]. This technique uses the process model to predict the future behavior of the controlled system by solving a potentially constrained optimization problem. Optimization is an inherent capability of an MPC controller. The combination of prediction and optimization is the main difference with classical control approaches, which use precomputed control laws.

The MPC technique has proven its effectiveness also in the control of electrical power systems. In [15], a deep Koopman model predictive control (MPC) strategy is used to improve the transient stability of power grids. The deep neural network method is employed to map the original nonlinear dynamics into an infinite dimensional linear system. The MPC strategy should handle high dimensional linear system. To improve the transient stability of the power system, a coordinated control structure is proposed in [16]. This technique is based on two MBC controllers (tube-based MPCs technique) and aims to determine the control signals of the excitation system of the synchronous generators and the steam turbine. Bonfiglio et *al*. in [17] proposed a decentralized and optimal emergency control strategy operating on the mechanical power in order to ensure the transient stability of the electrical network. This controller is an explicit MPC. The control laws employed are Piecewise affine (PWA) functions based on general polytopic partitions. This technique is often prohibitively complex for fast systems such the power systems.

A distributed model predictive control-based load frequency control (MPC-LFC) scheme was proposed in [18], to improve the performance of the frequency regulation of power system. The orthonormal Laguerre functions are employed to estimate the predicted control trajectory.

Inspired by these approaches, this article aims to use MPC to address the complex problem of transient stability and voltage regulation of a multi-machine power system that exhibits a naturally nonlinear dynamic. With this control technique, we will calculate the optimal future behavior of the electrical system in order to ensure the stability of the system even in the presence of faults.

To smoothly deal with the nonlinearity problem of the multi-machine power system, so-called Takagi–Sugeno (T-S) fuzzy models are generally used [19-20]. In the literature, T-S fuzzy logic and model predictive control via PDC approach applied to discrete-time nonlinear systems have received great attention from researchers [21-23]. However, it is well known that the continuous-time model is more suitable for the exploitation of physical systems. Therefore, the use of the continuous-time MPC model is more appropriate when applied to industrial processes [24].

This article focuses on the use of the continuous-time MPC strategy based on the Laguerre function to improve the transient stability and voltage regulation of a multi-machine power system. This controller is designed to be an optimal controller with a prescribed degree of stability. The prescribed degree of stability criterion is used to guarantee a certain rate of convergence. The continuous-time MPC algorithm is introduced using the Parallel Distribution Compensation (PDC) method. The global controller is constructed by combining local MPC controllers through fuzzy inference.

The performance of the proposed control scheme is evaluated through simulations on a two-machine infinite bus power system. The continuous-time MPC is applied as a decentralized controller since only local measurements are used.

The article is structured as follows: Section 2 discusses the T-S Fuzzy model and control power system. Section 3 describes the design of the continuous-time model predictive control with a specified degree of stability based on Takagi-Sugeno fuzzy logic. Section 4 will present the simulation results. In the final section, some conclusions are discussed.

2. Fuzzy model and Control of Power System

2.1 T-S Fuzzy model of power system

The primary objective of this article is to perform the voltage regulation and achieve transient stability of "c" generator of a multi-machine power system. This nonlinear system can be converted into a closed-loop linear dynamic system with the Direct Feedback Linearization (DFL) approach. This aim can be achieved by employing the state vector.

$$\boldsymbol{x}_{mk} = \left[\Delta V_{tk}\left(t\right) \quad \Delta\omega_{k}\left(t\right) \quad \Delta P_{ek}\left(t\right)\right]^{T} \qquad (1)$$

where

$$\Delta V_{tk}\left(t\right) = V_{tk}\left(t\right) - V_{tk0}; \Delta\omega_{k}\left(t\right) = \omega_{k}\left(t\right) - \omega_{0}; \Delta P_{ek}\left(t\right) = P_{ek}\left(t\right) - P_{mk} \quad (2)$$

The DFL-compensated model under review is as follows:

$$\sum : \begin{cases} \Delta\dot{V}_{tk}(t) = f_{k_1}\Delta\omega_k(t) - \frac{f_{k_2}}{T'_{dok}}\Delta P_{ek}(t) + \frac{f_{k_2}}{T'_{dok}} v_{fk}(t) \\ \Delta\dot{\omega}_k(t) = -\frac{D_k}{2H_k}\Delta\omega_k(t) - \frac{\omega_0}{2H_k}\Delta P_{ek}(t) \\ \Delta\dot{P}_{ek}(t) = -\frac{1}{T'_{dok}}\Delta P_{ek}(t) + \frac{1}{T'_{dok}} v_{fk}(t) \end{cases} \quad (3)$$

with

$$f_{k_1}(t) = -\frac{\left(1 + x'_{dk}B_{kk}\right)\left[-E'^2_{qk}B_{kk} - Q_{ek}(t)V_{tqk}(t)\right]}{V_{tk}(t)I_{qk}(t)} - \frac{x'_{dk}\left(1 + x'_{dk}B_{kk}\right)}{V_{tk}(t)} \quad (4)$$

$$f_{k_2}(t) = -\frac{\left(1 + x'_{dk}B_{kk}\right)V_{tqk}(t)}{V_{tk}(t)I_{qk}(t)} \quad (5)$$

From (4) and (5), it is clear that $f_{k_1}(t)$ and $f_{k_2}(t)$ are extremely nonlinear functions. But they operate in a defined domain, they depend on the operating conditions. The following restrictions are used:

$$-3.526 \le f_{1_1} \le -0.259\ ;\ 0.266 \le f_{1_2} \le 3.794$$
$$-2.832 \le f_{2_1} \le -0.233\ ;\ 0.241 \le f_{2_2} \le 3.670$$

The DFL compensating law considered is

$$u_{fk}(t) = \frac{1}{k_{ck}I_{qk}(t)}\left(v_{fk}(t) - T'_{dok}E'_{qk}\dot{I}_{qk} + P_{mk}\right) + \frac{1}{k_{ck}}\left(\left(x_{dk} - x'_{dk}\right)I_{dk}(t)\right) \quad (6)$$

where $v_{fk}(t)$ is the feedback control law to be evaluated

$$v_{fk}(t) = -k_{V_{tk}}\Delta V_{tk}(t) - k_{\omega_k}\Delta\omega_k(t) - k_{P_{ek}}\Delta P_{ek}(t) \quad (7)$$

and ω_k(t) the relative speed of the *k*-th generator, in rad/sec; $P_{mk}$the mechanical input power, in p.u.; P_{ek} (t) the electrical power, in p.u.; ω_0 the synchronous machine speed**,** in rad/sec ,$\omega_0 = 2\pi f_0$; D_k the per unit damper constant; $H_k$the inertia constant, in sec**.**E'_{qk}(t) the transient EMF in quadrature axis, in p.u.; T'_{dok} the direct axis transient open circuit time constant, in second; V_{tk} the generator terminal voltage , in p.u.; x_{dk} the direct axis reactance, in p.u.; x'_{dk} the direct axis transient reactance, in p.u.; I_{dk} the direct axis current, in p.u. ; I_{qk} the quadrature axis current, in p.u. ; u_{fk} the input of the SCR amplifier; $Y_{ij}=G_{ij}+jB_{ij}$ the *i*-th row and *j*-th column element of nodal admittance matrix, in p.u.; $Q_{ek}$the reactive power, in p.u.

The mathematical parameters of this model as well as the physical hypotheses are reported in [8,18-19].

To develop the fuzzy T-S model, we use the sum of the products of linearly independent functions [18-19]. Indeed, the DFL-compensated model (3) can be represented by

$$\dot{\boldsymbol{x}}_{mk}(t)=\left[\mathbf{F}_{k_0}+\sum\nolimits_{l=1}^{2} f_{k_l}\mathbf{F}_{k_l}\right]\boldsymbol{\eta}_k(t) \qquad (8)$$

Where $\eta_k(t)=\left[\Delta V_{tk}(t) \quad \Delta\omega_k(t) \quad \Delta P_{ek}(t) \quad v_{fk}(t)\right]^T$ (9)

$$F_{k_0}=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -D_k/2H_k & -\omega_0/2H_k & 0 \\ 0 & 0 & -1/T'_{d0k} & 1/T'_{d0k} \end{pmatrix}; F_{k_1}=\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix};$$
$$F_{k_2}=\begin{pmatrix} 0 & 0 & -1/T'_{d0k} & 1/T'_{d0k} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad (10)$$

The state and the input matrices A_{mk_i} , B_{mk_i} are developed as

$$\left[A_{mk_1} \quad B_{mk_1}\right]=F_{k_0}+f_{k_{10}}F_{k_1}+f_{k_{20}}F_{k_2}, \left[A_{mk_2} \quad B_{mk_2}\right]=F_{k_0}+f_{k_{10}}F_{k_1}+f_{k_{21}}F_{k_2}$$
$$\left[A_{mk_3} \quad B_{mk_3}\right]=F_{k_0}+f_{k_{11}}F_{k_1}+f_{k_{20}}F_{k_2}, \left[A_{mk_4} \quad B_{mk_4}\right]=F_{k_0}+f_{k_{11}}F_{k_1}+f_{k_{21}}F_{k_2}$$

It is assumed that $\left(A_{mk_i} \quad B_{mk_i}\right)$ is a controllable pair.

Four fuzzy rules are employed in this instance.

Plant Rule *i*:

$$IF \;\; z_1(t) \;\; is \;\; M^i_{k_{1p}} \;\; and \;\; z_2(t) \; is \; M^i_{k_{2p}}$$
$$THEN \;\; \begin{cases} \dot{\boldsymbol{x}}_{mk}(t)=\boldsymbol{A}_{mk_i}\boldsymbol{x}_{mk}(t)+\boldsymbol{B}_{mk_i}\boldsymbol{u}_k(t), & i=1,...,4; \\ \boldsymbol{y}_{mk}(t)=\boldsymbol{C}_{mk_i}\boldsymbol{x}_{mk}(t), & k=1,...,c;\; p=0,1 \end{cases} \qquad (11)$$

Where $\boldsymbol{u}_k(t)$ is the control vector. $\boldsymbol{y}_{mk}$ and C_{mk_i} are the output vector and the output matrix respectively. '*c*' is the number of generators in the multi-machine power system. $\boldsymbol{z}(t)=\{z_1(t),z_2(t)\}$ are recognized as premise variables (in our case $z_1(t)=f_{k_1}(t)$ and $z_2(t)=f_{k_2}(t)$) and $M^i_{k_{lp}}$ is the fuzzy set '*lp*' in rule '*i*' for the generator '*k*'.

The membership functions $M_{k_{l0}}$ and $M_{k_{l1}}$ are identified as

$$\begin{cases} M_{k_{l0}}(z(t))=\left(f_{k_{l1}}-f_{k_l}(z(t))\right)/\left(f_{k_{l1}}-f_{k_{l0}}\right) & l=1,2. \\ M_{k_{l1}}(z(t))=\left(f_{k_l}(z(t))-f_{k_{l0}}\right)/\left(f_{k_{l1}}-f_{k_{l0}}\right) & k=1,....,c \end{cases} \qquad (12)$$

Several linear models are interpolated to establish the global fuzzy model.

$$\dot{\boldsymbol{x}}_{mk}(t)=\sum_{i=1}^{4}h_{k_i}\left(\boldsymbol{z}(t)\right)\left(\boldsymbol{A}_{mk_i}\boldsymbol{x}_{mk}(t)+\boldsymbol{B}_{mk_i}\boldsymbol{u}_k(t)\right) \qquad (13)$$

The output is inferred as follows:

$$\boldsymbol{y}_{mk}(t)=\sum_{i=1}^{4}h_{k_i}\left(z(t)\right)\boldsymbol{C}_{mk_i}\boldsymbol{x}_{mk}(t) \qquad (14)$$

where:

$$h_{k_1}=M_{k_{10}}M_{k_{20}},h_{k_2}=M_{k_{10}}M_{k_{21}},h_{k_3}=M_{k_{11}}M_{k_{20}},h_{k_4}=M_{k_{11}}M_{k_{21}} \qquad (15)$$

2.2 Fuzzy T-S PDC Controller of power system

The PDC's fundamental idea is straightforward. Each fuzzy control rule is derived from the relevant T-S fuzzy model rule (11), as indicated in (16)

Control rule *i*:

$$\begin{array}{ll} IF \quad f_{k_1}(t) \ is \ M_{k_{1p}}^{i} \ \ and \ \ f_{k_2}(t) \ is \ M_{k_{2p}}^{i} & i=1,\ldots,\ 4; \\ THEN \quad \boldsymbol{u}_k(t)=-\boldsymbol{K}_{mk_i}\boldsymbol{x}_{mk}(t) & k=1,\ldots,c;\ p=0,1 \end{array} \qquad (16)$$

where $\boldsymbol{K}_{mk_i}$ is a linear state feedback gain for the *i*-th fuzzy subsystem.

For the T-S model (13), the inferred PDC controller is designed as

$$\boldsymbol{u}_k(t)=-\sum_{i=1}^{r}h_{k_i}\left(\boldsymbol{z}(t)\right)\boldsymbol{K}_{mk_i}\ \boldsymbol{x}_{mk}(t) \qquad (17)$$

From (13) and (17), the resulting closed-loop control system is determined as follows

$$\dot{\boldsymbol{x}}_{mk}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_{k_i}\left(z(t)\right)h_{k_j}\left(z(t)\right)\left(\boldsymbol{A}_{mk_i}-\boldsymbol{B}_{mk_i}\boldsymbol{K}_{mk_j}\right)\boldsymbol{x}_{mk}(t) \qquad (18)$$

3. Model Predictive Control using Laguerre Functions with a Prescribed degree of stability based on T-S Fuzzy logic

3.1 Model Structure of the Continuous-Time Model Predictive Control system

This section will discuss the problem formulation of the continuous-time predictive control model based on the Takagi-Sugeno (T-S) fuzzy system with a specified degree of stability.As know, the T-S fuzzy model can be used to describe a local linear system using IF-THEN fuzzy rules to represent a nonlinear system. For a continuous fuzzy system, the *i*-th rule of the T-S fuzzy models is given as follows [25]

$$\begin{array}{l} IF \quad z_1(t) \ is \ \ M_1^i \ \ and \ldots z_p(t) \ is \ M_p^i \\ THEN \quad \begin{cases} \dot{\boldsymbol{x}}_m(t)=\boldsymbol{A}_{mi}\boldsymbol{x}_m(t)+\boldsymbol{B}_{mi}\boldsymbol{u}(t), \\ \boldsymbol{y}_m(t)=\boldsymbol{C}_{mi}\boldsymbol{x}_m(t), \end{cases} \quad i=1,2,\ldots r \end{array} \qquad (19)$$

where $x_m(t)$ is the state vector of dimension 'n_1'. In (19), $\boldsymbol{A}_{mi}, \boldsymbol{B}_{mi}$ and C_{mi} dimension are $n_1 \times n_1$, $n_1 \times m$ and $q \times n_1$, respectively.

To establish the predictive model based on (19), we define the following auxiliary variables [26]

$$w(t) = \dot{x}_m(t) \text{ and } \quad y(t) = C_{mi} x_m(t) \qquad (20)$$

The new state variable vector is chosen as $\boldsymbol{x}(t) = \begin{bmatrix} w(t) & y(t) \end{bmatrix}^T$. With these auxiliary variables, the augmented state-space model obtained for the i-th rule is as follows

Plant Rule i:

$$\begin{aligned} &IF \;\; z_1(t) \;\; is \;\; M_1^i \;\; and ... z_p(t) \;\; is \;\; M_p^i \\ &THEN \;\; \begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}_i \boldsymbol{x}(t) + \boldsymbol{B}_i \dot{\boldsymbol{u}}(t), \\ \boldsymbol{y}(t) = \boldsymbol{C}_i \boldsymbol{x}(t), \end{cases} \quad i = 1, 2, ...r. \end{aligned} \qquad (21)$$

Where $\boldsymbol{A}_i = \begin{bmatrix} \boldsymbol{A}_{mi} & \boldsymbol{0}_m^T \\ \boldsymbol{C}_{mi} & \boldsymbol{0}_{q\times q} \end{bmatrix}$, $\boldsymbol{B}_i = \begin{bmatrix} \boldsymbol{B}_{mi} \\ \boldsymbol{0}_{q\times m} \end{bmatrix}$, $\boldsymbol{C}_i = \begin{bmatrix} \boldsymbol{0}_m & \boldsymbol{I}_{q\times q} \end{bmatrix}$

and $\boldsymbol{0}_m$ is a $q \times n_1$ zero matrix, $\boldsymbol{0}_{q\times q}$ is a $q \times q$ zero matrix, $\boldsymbol{0}_{q\times m}$ is a $q \times m$ zero matrix, and $\boldsymbol{I}_{q\times q}$ is the identity matrix with dimensions $q \times q$.

The continuous-time Takagi-Sugeno fuzzy model of the model predictive control system is represented as

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r} h_i(z(t))\left(\boldsymbol{A}_i \boldsymbol{x}(t) + \boldsymbol{B}_i \dot{\boldsymbol{u}}(t)\right) \\ \boldsymbol{y}(t) = \sum_{i=1}^{r} h_i(z(t)) \boldsymbol{C}_i \boldsymbol{x}(t) \end{cases} \qquad (22)$$

3.2 PDC Controller Design for the Continuous-Time Model Predictive Control System using Laguerre Functions with Prescribed degree of stability

The design principle of the continuous-time model predictive control (CMPC) is very similar to the one used in the T-S Model-Based fuzzy control system. The overall control is denoted as follows:

$$\dot{\boldsymbol{u}}(t) = -\sum_{i=1}^{r} h_i(z(t)) \boldsymbol{K}_i \, \boldsymbol{x}(t) \qquad (23)$$

Here, it suffices to integrate to extract the control law [22]

$$\boldsymbol{u}(t) = \int_0^t \dot{\boldsymbol{u}}(\tau) d\tau \qquad (24)$$

The feedback gain $\boldsymbol{K}_i$ is calculated using the lowest value of the objective function J, as shown in (25):

$$J=\int_0^{Tp}\left(\boldsymbol{x}^T(t)\boldsymbol{W}\,\boldsymbol{x}(t)+\dot{\boldsymbol{u}}^T(t)\boldsymbol{R}\dot{\boldsymbol{u}}(t)\right)dt \qquad (25)$$

The penalty on states and input energy are expressed by the matrices $\boldsymbol{W}$ and $\boldsymbol{R}$, respectively. The weight matrix $\boldsymbol{R}$ is positive definite symmetric, it is determined as a diagonal matrix, where each element weighs the corresponding control signal. Whereas $\boldsymbol{Q}$ is symmetric, constant, and non-negative matrix.

As is well known, when the prediction horizon 'T_p' is large, the model predictive control algorithm becomes numerically ill-conditioned. To solve this issue, the exponential data weighting strategy [27] is used.

The optimization will be ensured with a prescribed degree of stability. To be able to minimize the quadratic performance index (25) and, at the same time, to ensure that the poles of the closed loop are to the left of a line Re (s) = - β, for an $\beta > 0$, it suffices to consider the following new variables.

$$\hat{\boldsymbol{x}}(t)=e^{\beta t}\boldsymbol{x}(t)\ ,\ \dot{\hat{\boldsymbol{u}}}(t)=e^{\beta t}\dot{\boldsymbol{u}}(t)\ ,\ \hat{\boldsymbol{y}}(t)=e^{\beta t}\boldsymbol{y}(t) \qquad (26)$$

The problem of minimizing (25) is identical to the issue of minimizing the objective function:

$$\hat{J}=\int_0^{T_p}e^{2\beta t}\left(\boldsymbol{x}^T(t)\boldsymbol{W}\,\boldsymbol{x}(t)+\dot{\boldsymbol{u}}^T(t)\boldsymbol{R}\dot{\boldsymbol{u}}(t)\right)dt \qquad (27)$$

The objective function is then represented by

$$\hat{J}=\int_0^{T_p}\left(\hat{\boldsymbol{x}}^T(t)\boldsymbol{W}\,\hat{\boldsymbol{x}}(t)+\dot{\hat{\boldsymbol{u}}}^T(t)\boldsymbol{R}\dot{\hat{\boldsymbol{u}}}(t)\right)dt \qquad (28)$$

Subject to

$$\begin{cases}\dot{\hat{\boldsymbol{x}}}(t)=\sum_{i=1}^{r}h_i\left(z(t)\right)\left(\boldsymbol{A}_{\beta i}\hat{\boldsymbol{x}}(t)+\boldsymbol{B}_i\dot{\hat{\boldsymbol{u}}}(t)\right)\\ \hat{\boldsymbol{y}}(t)=\sum_{i=1}^{r}h_i\left(z(t)\right)\boldsymbol{C}_i\hat{\boldsymbol{x}}(t)\end{cases} \qquad (29)$$

Where $\boldsymbol{A}_{\beta i}=\boldsymbol{A}_i+\beta\boldsymbol{I}_n$ and β is the desired prescribed degree of stability. The overall control law is expressed as:

$$\dot{\hat{\boldsymbol{u}}}(t)=-\sum_{i=1}^{r}h_i\left(z(t)\right)\boldsymbol{K}_i\,\hat{\boldsymbol{x}}(t) \qquad (30)$$

From (29) and (30), the closed loop control system is defined as follows

$$\dot{\hat{\boldsymbol{x}}}(t)=\sum_{i=1}^{r}h_i\left(z(t)\right)\sum_{j=1}^{r}h_j\left(z(t)\right)\left(\boldsymbol{A}_{\beta i}-\boldsymbol{B}_i\boldsymbol{K}_j\right)\hat{\boldsymbol{x}}(t) \quad (31)$$

Remark 1:

We deduce from (26) that $\boldsymbol{x}(t)=e^{-\beta t}\hat{\boldsymbol{x}}(t)$, this means that **x**(t) decays at least as fast as the rate of $e^{-\beta t}$ [26].

The feedback gain matrix $\boldsymbol{K}_i$ for the lowest value of $\hat{J}$ is defined as

$$\boldsymbol{K}_i=\begin{bmatrix} L_1(0)^T & o_2 & \cdots & o_m \\ o_1 & L_2(0)^T & \cdots & o_m \\ \vdots & \vdots & \ddots & \vdots \\ o_1 & o_2 & \cdots & L_m(0)^T \end{bmatrix}\Omega_i^{-1}\Psi_i \quad (32)$$

where $L_1(0)$, $L_2(0)$,….. $L_m(0)$ are the initial conditions of first, second, third, …, and m *th* inputs, respectively. They are expressed as:

$$L_g(0)=\sqrt{2p_g}\,[1 \quad 1 \quad . \quad . \quad . \quad 1]^T \quad (33)$$

It is a column vector of 'N_g' elements, in which 'N_g' defines the number of Laguerre networks (g=1, …, m). p_g denotes the scaling factor for Laguerre functions. The row vector o_g contains zero elements and has a dimension identical to $L_g(0)^T$.

The matrices Ω_i and ψ_i are constant. They are formulated as:

$$\begin{cases} \Omega_i=\sum_{k=0}^{M}\varphi_i(kh)W\,\varphi_i(kh)^T\,h+R_L \\ \Psi_i=\sum_{k=0}^{M}\varphi_i(kh)W\,e^{\boldsymbol{A}_{\beta i}kh}h \end{cases} \quad (34)$$

M is the number of samples; it is denoted as: $M=T_p/h$ where h is the sampling interval within the optimization window, T_p is the prediction horizon.

$\varphi_i(kh)^T$ is the Linear Algebraic Equation (LAE) solution.

$$\boldsymbol{A}_{\beta i}\,\varphi_i(kh)^T-\varphi_i(kh)^T A_{p_g}^T=-B_i\,L_g(kh)^T+e^{\boldsymbol{A}_{\beta i}kh}B_i\,L_g(0)^T \quad (35)$$

where $L_g(kh)$ is the set of Laguerre functions, denoted as

$$L_g(kh)=e^{\left(\boldsymbol{A}_{p_g}kh\right)}L_g(0) \quad (36)$$

And $\boldsymbol{A}_{p_g}$ is a lower triangular matrix of ($N_g \times N_g$) order and it is formulated as

$$\boldsymbol{A}_{p_g} = \begin{bmatrix} -p_g & 0 & . & . & . & 0 \\ -2p_g & -p_g & . & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ -2p_g & . & . & . & -2p_g & -p_g \end{bmatrix} \quad (37)$$

R_L is a block diagonal matrix. It contains R_g number of block diagonal matrices such that ($\boldsymbol{I}_{N_g \times N_g}$ is the identity matrix with order of ($N_g \times N_g$).

3.3 Asymptotic Stability of the Continuous-Time Model Predictive Control System

This section addresses the equivalence between the proposed controller (model predictive control with a prescribed degree of stability based on the Takagi-Sugeno fuzzy control system and Laguerre functions) and the classical optimal fuzzy control (model predictive control with a prescribed degree of stability based on the Takagi-Sugeno fuzzy control system).

The stability conditions of the classical optimal fuzzy control system are derived from Lyapunov's stability theory and a set of LMI-based stability conditions are derived to establish the conditions under which the closed loop system (31) ensures asymptotic stability with the control law (30).

Theorem 1. Feedback gains that ensure minimization of the upper bound of the performance function

$$\hat{J} = \int_0^{\infty} \left(\hat{\boldsymbol{x}}^T(t) \boldsymbol{W} \, \hat{\boldsymbol{x}}(t) + \dot{\hat{\boldsymbol{u}}}^T(t) \boldsymbol{R} \dot{\hat{\boldsymbol{u}}}(t) \right) dt \quad (38)$$

can be obtained by solving the following LMIs

$$\underset{\boldsymbol{Q}, \boldsymbol{Y}_1, \ldots\ldots, \boldsymbol{Y}_r}{\text{minimize}} \ \alpha$$

subject to

$$\boldsymbol{Q} > 0,$$

$$\begin{bmatrix} 1 & \boldsymbol{x}^T(0) \\ \boldsymbol{x}(0) & \boldsymbol{Q} \end{bmatrix} \geq 0 \quad (39)$$

$$\begin{bmatrix} \boldsymbol{QA}_{\beta i}^T + \boldsymbol{A}_{\beta i}\boldsymbol{Q} - \boldsymbol{Y}_i^T\boldsymbol{B}_i^T - \boldsymbol{B}_i\boldsymbol{Y}_i & \boldsymbol{QC}_i^T\sqrt{\boldsymbol{W}} & \left(\boldsymbol{Y}_i^T\right)\sqrt{\boldsymbol{R}} \\ \sqrt{\boldsymbol{W}}\boldsymbol{C}_i\boldsymbol{Q} & -\alpha\boldsymbol{I} & 0 \\ \sqrt{\boldsymbol{R}}\left(\boldsymbol{Y}_i\right) & 0 & -\alpha\boldsymbol{I} \end{bmatrix} < 0 \quad (40)$$

$$\begin{bmatrix} \boldsymbol{T}_2 & \boldsymbol{QC}_i^T\sqrt{\boldsymbol{W}} & \boldsymbol{K}_i^T\sqrt{\boldsymbol{R}} & \boldsymbol{QC}_j^T\sqrt{\boldsymbol{W}} & \boldsymbol{K}_j^T\sqrt{\boldsymbol{R}} \\ \sqrt{\boldsymbol{W}}\boldsymbol{C}_iQ & -\alpha I & 0 & 0 & 0 \\ \sqrt{\boldsymbol{R}}\boldsymbol{K}_i & 0 & -\alpha I & 0 & 0 \\ \sqrt{\boldsymbol{W}}\boldsymbol{C}_jQ & 0 & 0 & -\alpha I & 0 \\ \sqrt{\boldsymbol{R}}\boldsymbol{K}_j & 0 & 0 & 0 & -\alpha I \end{bmatrix} < 0 \quad (41)$$

Where $T_2 = \boldsymbol{QA}_{\beta i}^T + \boldsymbol{A}_{\beta i}\boldsymbol{Q} + \boldsymbol{QA}_{\beta j}^T + \boldsymbol{A}_{\beta j}\boldsymbol{Q} - \left(\boldsymbol{Y}_j^T\boldsymbol{B}_i^T + \boldsymbol{B}_i\boldsymbol{Y}_j\right) - \left(\boldsymbol{Y}_i^T\boldsymbol{B}_j^T + \boldsymbol{B}_j\boldsymbol{Y}_i\right)$

The feedback gains are obtained as

$$\boldsymbol{K_i} = \boldsymbol{Y}_i\boldsymbol{Q}^{-1} \quad (42)$$

for all *i* since $\boldsymbol{Q} = \alpha\boldsymbol{P}^{-1}$. and the performance index satisfies

$$\hat{J} < \hat{\boldsymbol{x}}^T\left(0\right)\boldsymbol{P}\,\hat{\boldsymbol{x}}\left(0\right) \le \alpha \quad (43)$$

The prediction horizon T_p is generally considered to be large enough so that state variable predictions are numerically correct. However, the model predictive control algorithm becomes numerically ill-conditioned in this case. The exponential data weighting strategy is used first to eliminate the numerical ill-conditioning problem from the design, in second to provide a design that leads to asymptotic closed-loop stability, and finally to achieve a solution with a specified degree of stability. which means that the results given by the model predictive control with a prescribed degree of stability are equivalent to those of the fuzzy optimal control with a prescribed degree of stability.

4. Simulation

The simulation results are exhibited in this section to demonstrate the efficacy of the proposed technique for transient stability enhancement and voltage regulation of multi-machine power system. The multi-machine benchmark network used is the two-machine infinite bus power system. Figure 1 depicts the one-line diagram for this system.

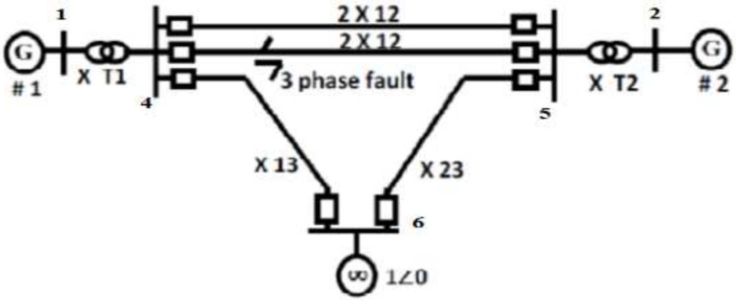

Fig. 1. Two machine infinite bus example power system.

The proposed technique aims to optimize the control action of the generator excitation systems in the multi-machine power system. Since each local controller solves its own optimal control problem using only local measurements, they are referred to as decentralized controllers.

References [8,18-19] provide additional information about the system under consideration.

The initial conditions for the different cases are included in Table 1.

Table 1. Initial conditions.

	Machine	$\delta(\circ)$	$P_m\left(p.u\right)$	$V_t\left(p.u\right)$
Case 1	*1*	*52.72*	*0.95*	*1.00*
	2	*54.48*	*0.95*	*1.02*
Case 2	*1*	*46.00*	*0.87*	*1.02*
	2	*44.69*	*0.86*	*1.1*

To validate the proposed control scheme, two different types of contingencies are proposed.

- **First contingency:** The considered contingency is that two symmetrical three-phase short-circuit faults occur successively at the fault location λ = 0.06. The first fault occurs after 3s from the beginning of the simulation and it is cleared after 0.2 s by opening the line. The transmission lines are re-established at t = 4.5 s. The second one is introduced at 5.5 s and it is cleared after 0.2s. The system is in a post-fault state.
- **Second contingency:** The considered contingency is a three-phase symmetrical fault occurring at the fault location λ = 0.05 after 3 s from the beginning of the simulation and it is cleared after 0.2s. This fault is followed by a 45% drop in mechanical power. The system is in a post-fault state.

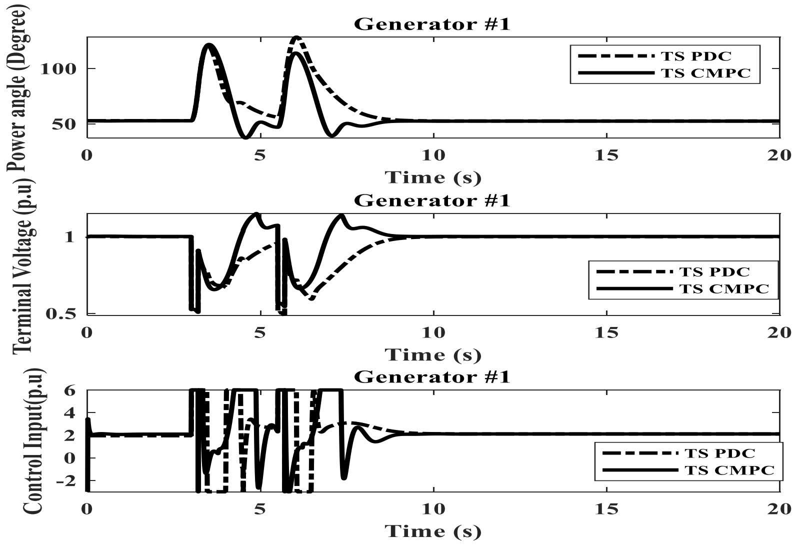

Fig.2. System state responses of the generator 1 for the first contingency

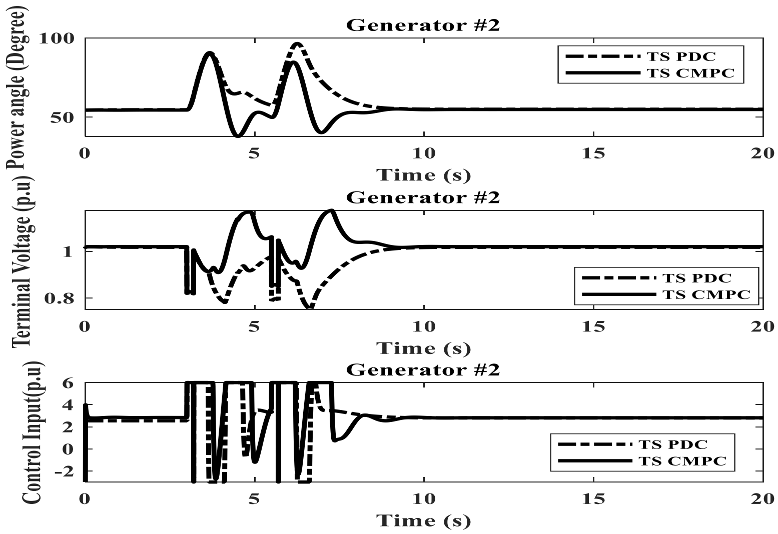

Fig.3. System state responses of the generator 2 for the first contingency

Figures 2-5 shows the power angle, the terminal voltage and the control action of each machine of the considered contingencies.

The faults considered have caused significant disturbances since the operating points have been modified. However, with the proposed MPC controller, it is clear that the system can withstand all of the considered contingencies.

The objective of ensuring transient stability after the occurrence of faults is greatly improved since the power angles always keep constant values. And simultaneously, it is seen that the output voltages are kept fixed and the voltage drop was quickly rejected after the occurrence of symmetrical three-phase short-circuit faults. It can be noticed also that the power angles and terminal voltages of the machines exhibit fewer post-fault oscillations with the proposed control scheme compared to the controller based on the fuzzy model of Takagi-Sugeno with pole placement (TS-PDC) [19]).

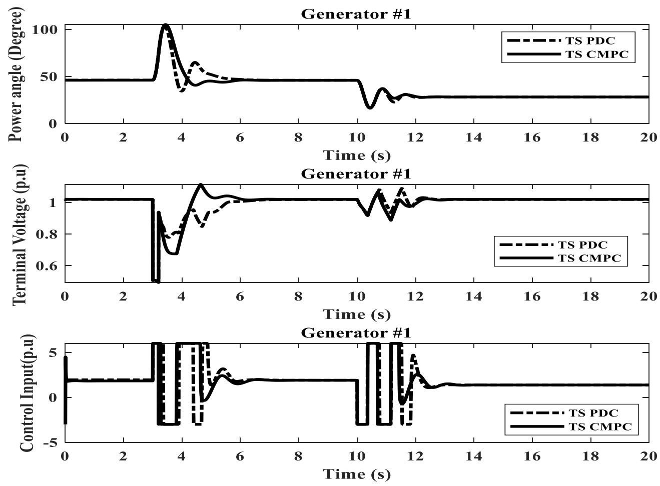

Fig.4. System state responses of the generator 1 for the second contingency

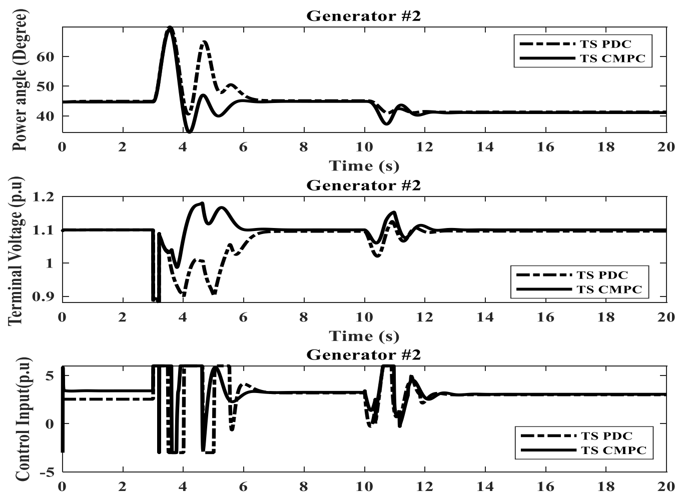

Fig.5. System state responses of the generator 2 for the second contingency

In order to evaluate the ability of CMPC controllers to maintain the power angle and the terminal voltage as close as possible to the pre-contingency values, a comparative study of the performance of two control techniques (the proposed controller (TS-CMPC) and the TS-PDC controller) was carried out.

Figures 6 and 7 summarize the results obtained for the squared integral error (ISE), the integral of the absolute error (IAE) and the integral of the time multiplied by the absolute error (ITAE) during the evaluation of the transient stability and voltage regulation of the multi-machine power system of the TS-CMPC and TS-PDC controllers.

The comparison results in Figures 6 and 7 show that the proposed CMPC controller outperforms the TS-PDC controller significantly. Indeed, for all contingencies considered, the proposed control scheme provides lower error values in terms of voltage deviation than the TS-PDC controller. The proposed CMPC controller can greatly improve transient stability regardless of operating points and fault locations.

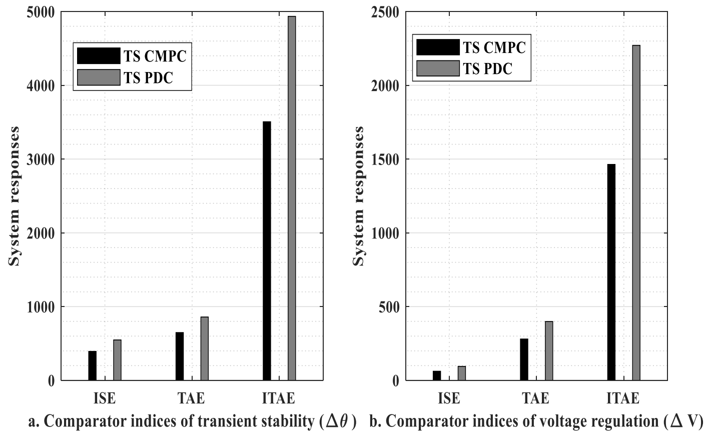

Fig.6. Values of performance indices under different conditions for transient stability evaluation (a) and voltage regulation (b) for the first contingency

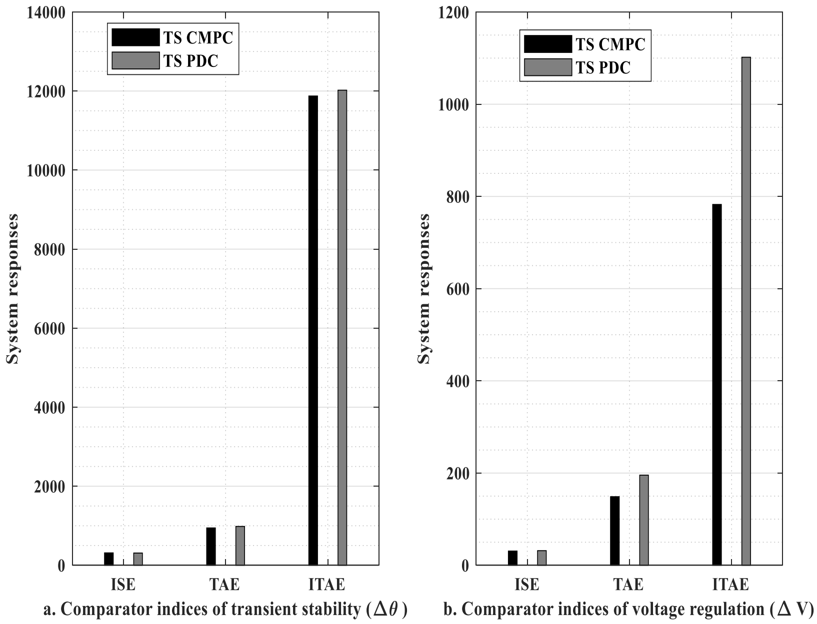

Fig.7. Values of performance indices under different conditions for transient stability evaluation (a) and voltage regulation (b) for the second contingency

5. Conclusion

This article deals with the design of a decentralized nonlinear voltage control scheme based on continuous-time MPC strategy with a prescribed degree of stability. The main objectives of the control scheme are to improve the transient stability and voltage regulation of the multi-machine power system. The proposed controller is designed to be an optimal controller with a prescribed degree of stability. A specific rate of convergence is ensured using the degree of stability criterion. The continuous-time MPC algorithm is integrated as a regulator in the multi-machine power system using the parallel distribution compensation (PDC) method. Indeed, the design principle of the regulator with the fuzzy model-based control system is used as the basic idea for the conception of this regulator. Laguerre functions are utilized in MPC design to lessen the computational load needed for control signal optimization. An extremely nonlinear multi-machine benchmark network (the two-generator infinite bus example system) is used in computer simulation to show the controller's effectiveness. It was proven that the proposed decentralized controller maintains the overall stability of multi-machine power systems even in the presence of severe contingencies. Indeed, despite the change in initial operating points, mechanical input power, and fault sites, transient stability is noticeably enhanced, and the post-fault voltage level is well appreciated. The comparative results between the proposed controller and the TS-PDC controller confirmed that the performance of the proposed MPC controller outperforms that of the second controller.

REFERENCES

1. P. Kundur, Power system stability and control. McGraw-Hill, New York, 1994.
2. D.Bandekas, G. Tsirigotis, P.Antoniadis, and N. Vordos, “A Robust Controller Design for a Multimachine Power System,”. ElektronikaIrElektrotechnika, vol. 65, no.1, pp.20-24,2006.
3. S. Wang and Z. Wang: “Sliding Mode Dynamic Surface Control for Multi-Machine Power Systems with Time Delays and Dead-Zones,” Cybernetics and Systems, vol.52, no.1, pp.1-15,2020.
4. S.Keskes, N.Bouchiba, S. Kammoun , S.Sallem , L.Chrifi-Alaoui , and M.B.A. Kammoun : “ An improved Backstepping technique using sliding mode control for transient stability enhancement and voltage regulation of SMIB power system”, International Journal of Systems Science, vol. 49, no. 9, pp.1964-1973 , 2018.
5. T.K. Roy, M.A. Mahmud, W.X. Shen, and M.T. Amanullah: “An adaptive partial feedback linearizing control scheme: An application to a single machine infinite bus system”, IEEETransactions on Circuits and Systems II: Express Briefs, vol. 67, no. 11, pp. 2557 – 2561, 2019.
6. A.Ganesh, R.Dahiya, and G. K. Singh, “A Novel Robust STATCOM Control Scheme for Stability Enhancement in Multimachine Power System”, ElektronikaIrElektrotechnika, vol.20, no.4, pp. 22-28, 2014.
7. P. Zhao, W. Yao, J. Wen, L. Jiang, S. Wang, and S. Cheng: “Improved Adaptive non-linear voltage controller for multi-machine power system synergetic excitation control for transient stability enhancement and voltage regulation of power systems”, *Int. J Electr. Power Energy Syst.*, vol.68, pp. 44–51,2015.
8. C.Zhu, R. Zhou, Y. Wang, “A new decentralized nonlinear voltage controller for multimachine power systems”, IEEE Trans Power Syst, vol.13, no.1, pp. 211–216, 1998.
9. J. Rawlings, “Tutorial overview of model predictive control”, IEEE Control Systems Magazine, vol.20, no.3, pp. 38–52, 2000.

10. V.Slapak, K. Kyslan, and F. Durovsky, "Position Controller for PMSM Based on Finite Control Set Model Predictive Control", ElektronikaIrElektrotechnika, vol.22. no.6 pp. 17 – 21, 2016.
11. E. Camacho and C. Bordons, Model Predictive Control, 2nd ed., ser. Advanced Textbooks in Control and Signal Processing. London: Springer-Verlag, 2004.
12. J. B. Rawlings, D.Q. Mayne and M. M. Diehl, *Model Predictive Control: Theory, Computation, and Design*（2nd Ed.）, Nob Hill Publishing, LLC, ISBN 978-0975937730, 2017.
13. S. Vazquez, José R. Rodríguez, M. Rivera, L. Franquelo and M. Norambuena, "Model Predictive Control for Power Converters and Drives: Advances and Trends", IEEETransactions on Industrial Electronics, vol.64, no.2, pp. 935-947, 2017.
14. J Xin et al. "Research on Adaptive Optimization Trajectory Tracking Algorithm based on MPC 7151", Tob Regul Sci., vol.7,no 6-1, pp.7151-7165,2021.
15. Z. Ping, Z. Yin, X. Li, Y. Liu, and T. Yang, "Deep koopman model predictive control for enhancing transient stability in power grids," Int Jl Robust Nonlinear Control, vo.31, no.6, pp 1964-1978, 2020.
16. A.B.Rehiara , N.Yorino, Y.Sasaki , and Y.Zoka : " A novel adaptive LFC based on MPC method", IEEJ Transactions on Electrical and Electronic Engineering,vol.14,no.8,pp 1145-1152, 2019.
17. A. Bonfiglio, F. Cantoni, A. Oliveri, R. Procopio, A. Rosini, M. Invernizzi, and M. Storace , "An MPC-based approach for emergency control ensuring transient stability in power grids with steam plants", IEEE Trans. Ind. Electron., vol. 66, no. 7, pp. 5412-5422,2019.
18. Y.Zheng, J. Zhou, Y. Xu,Y. Zhang, Z. Qian, "A distributed model predictive control-based load frequency control scheme for multi-area interconnected power system using discrete-time Laguerre functions". ISA Trans, vol.68, pp.127–140,2017.
19. A. Abbadi, J. Boukhetala, L. Nezli and A. Kouzou, "A nonlinear voltage controller using T-S fuzzy model for multi-machine power systems". In: Proceedings of the 9th annual IEEE international multi-conference on systems, signals and devices, Chemnitz, Germany*,* pp.1-8,2012.
20. A. Abbadi, F.Hamidia, "Transient stability and voltage regulation in the framework of the interval type-2 fuzzy model-based control systems", IEEJ Transactions on Electrical and Electronic Engineering,vol.14, no.7 ,2019.
21. Y.L.Kuo, and I.E. Citra Resmi , " Model Predictive Control Based on a Takagi–Sugeno Fuzzy Model for Nonlinear Systems", Int. J. Fuzzy Syst., vol.21,pp. 556–570 ,2019.
22. N. Liu N, X.Tang,L. Deng, " Constrained model predictive control for T–S fuzzy system with randomly occurring actuator saturation and packet losses via PDC and non-PDC strategies", Transactions of the Institute of Measurement and Control, vol.42, no.8, pp.1437-1447, 2020.
23. B.W.L.Margolis, M.A. Ayoubi, and S.S Joshi, "Nonlinear Model Predictive Control of Reentry Vehicles Based on Takagi-Sugeno Fuzzy Models", J Astronaut Sci, vol.67, pp.113–136,2020.

24. L.Magni, R.Scattolini, "Model predictive control of continuous-time nonlinear systems with picewise constant control ", IEEE transactions on automatic control, vol. 49, no. 6, pp. 900 – 906, 2004.
25. K. Tanaka and O. Wong, Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach. John Wiley & Sons Inc., 2001.
26. L. Wang, "Model Predictive Control System Design and Implementation Using MATLA", Springer Science & Business Media, 2009.
27. B. D. O. Anderson and J. B. Moore, Linear Optimal Control, Prentice-Hall, 1971.