Proportional Derivative (PD)-Based Interval Type-2 Fuzzy Control Design of a Quadrotor Unmanned Aerial Vehicle

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Abstract

For a decade, the studies and development of dynamic control for quadrotor unmanned aerial vehicles (UAVs) took a large interest and has become one of the most fruitful research areas, where different approaches are proposed dealing with the stability and the nonlinearity issues, which present the most important features of this system. The main goal of this paper is to propose an interval type-2 fuzzy proportional derivative (IT2-FPD) controller for the the position and attitude tracking control of an underactuated quadrotor UAV system in the presence of parametric uncertainties based on fuzzy control theory without the need for model identification. In this context, we will firstly present the analytical formulation of the used IT2-FPD control structure and its output in closed-loop. We will then evaluate the gain adjustments with respect to the Footprint of Uncertainty (FOU) design parameter of the IT2-FLPD controller. The proposed control method has a two-loop structure: inner loop for attitude control and an outer loop for position and altitude control.

The effectiveness of the proposed IT2-FPD control approach is examined using MATLAB/Simulink simulations of a quadrotor system. By comparing the numerical results obtained via different strategies, it can be concluded that our proposed controller offers the following main advantages: (1) steady-state behaviors and outstanding transient, (2) susceptibility to parameter variations, and (3) performance robustness and remarkable stability. From the numerical results of position and altitude tracking senario, we have found a 12.6% and 11.29% improvement in Root Mean Square Error (RMSE) compared to type-1 fuzzy PD controller and classical PID

controller, respectively. Interval Type-2 fuzzy logic proportional derivative controllers are capable of managing different types of uncertainties that occur naturally in most real-world situations. Hence, for operational tasks in which the accurate and fast response are of significant importance, using the IT2-FPD control approach is recommended.

Keywords: -quadrotor; position stabilization; interval type-2 fuzzy controller; PD controller;Uncertainties; attitude tracking.

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1. Introduction

Quadrotors in particular are the most popular used unmanned aerial vehicles (UAVs) due to their small size, high maneuverability, low cost, hovering, vertical take off and landing capability, instantaneous acceleration in three dimensions, and their simple mechanical design [1, 2]. For this reason, they are widely applied for traffic management, security, combat, agriculture, aerial photography, and a variety of other applications [3, 4, 5]. The quadrotor is classified as an underactuated system. This is due to the fact that only four actuators (rotors) are used to control all six degrees of freedom (DOF). The four actuators directly impact z-axis translation (altitude). The other two DOF are translation along the x and y-axis. Additional quadrotor benefits are swift maneuverability and increased payload. These capabilities make quadrotors ideal for developing aerial robotic applications and testing new control strategies [6, 7].

The quadrotor's applications are increasing rapidly, which has motivated researchers to develop robust and reliable control methods to satisfy application requirements and perform successfully the assigned tasks. Stabilizing a UAV's attitude and position at a desired reference value is required for control. Prior to developing the controller, most conventional UAV control strategies require mathematical modeling of the UAV.

Due to the high nonlinearity of its dynamics, coupling dynamics features, unknown and unbounded parameter uncertainties, and unmodeled dynamics, controlling quadrotors is a challenging task [8]. These undesired aspects might result in imprecise quadrotor trajectory tracking and, eventually, an unstable control system. To deal with these challenges, a variety of robust control strategies, including feedback linearization [9], proportional-integral-derivative (PID) control [10, 11], linear-quadratic-regulator control (LQR) [12, 13], sliding mode control (SMC) [14–15], model predictive control [16–17], adaptive SMC control [18, 19], and robust backstepping control [20, 21], have been developed for nonlinear UAVs systems, including the quadrotor system.

This model-based control techniques, only display good control performance to a limited extent because they are primarily predicated on the suppositions that the system model is accurate. However, quadrotors are susceptible to a number of operational uncertainties, including measurement noises. Given its capacity to manage uncertainties and design nature, fuzzy control theory is an effective alternative for such uncertain systems. The ability of FLCs (particularly type-2 FLCs), to emulate human decision-making through the use of membership functions and rule-based inference mechanisms is exceptionally powerful [22]. Furthermore, without having considerable knowledge of the mathematical model, they can effectively capturing and accommodating uncertainties [23, 24]. These strong FLC advantages can ease the development process of UAVs and enhance their performance in the presence of uncertainties whitch arise from environmental conditions, noise, and sensor measurement errors.

The most challenging aspect of piloting a UAV is maintaining its attitude. A simple controller can be used to maneuver the UAV to a desired position once a stable attitude control system has been developed [25, 26]. In order to improve the proposed UAV's behavior in uncertain environments, we propose to build a type-2 fuzzy logic controller (T2-FLC) for the inner-loop (attitude) control of the UAV and apply a PD controller for controlling it's position.

Through there have been past attempts for developing UAVs with FLCs [27], they are mainly concentrated on applying computationally expensive Mamdani fuzzy inference methods.

Hence, in this paper, a new interval type-2 fuzzy proportional derivative controller (IT2-FPDC) is developed and implemented to address the quadrotor's position and tracking altitude control. Furthermore, each input variable is defined by five triangular IT2-MFs to enhance the flight efficiency of a quadrotor aircraft [28], [29]. The performance of the proposed controller is compared to performances of a classical PID controller, a type-1 fuzzy PD controller (T1-FPD) to prove the effectiveness of the proposed approach.

This paper presents an interval type-2 fuzzy-PD (IT2-FPD) controller for the position and attitude tracking problem through UAVs. The proposed methodology is compared with a conventional PID controller and a type-1 fuzzy-PD (T1-FPD) controller. The fuzzy logic algorithm determines the proportional, derivative gains as the UAV performs tracking.

The rest of the paper is organised as follows. The mathematical model of a quadrotor is discussed in Section 2. Following this, the proposed control strategy is shown in Section 3. Next, a comprehensive simulation of the quadrotor UAV attitude tracking and position is constructed in Section 4, based on the plant dynamics in order to test and compare the proposed controller technique. Finally, conclusions on the approach proposed and results achieved is presented Section 5.

2. Dynamic description and problem statement

2.1 Dynamic modelling of Quadrotor

The quadrotor is a highly nonlinear, multivariable, strongly coupled, and underactuated system. The main forces and moments acting on the quadrotor are produced by propellers. The schematic diagram of a quadrotor with its earth-fixed and body-fixed reference frames is given in Fig. 1. Where$E(O_{xyz})$and $B(O_{xyz})$represent the Earth-fixed frame and Body-fixed frame of the quadrotor, respectively.

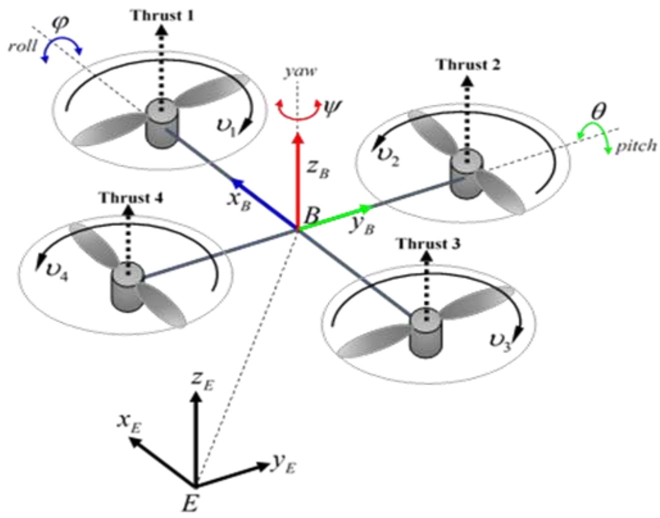


Figure 1. Quadrotor model.

To simplify the complexity of the quadrotor dynamic model, the following assumptions are considered in this study [30]:

1. The framework of quadrotor is intended to be rigid.
2. The fixed frame of the body aligns with the center of gravity.
3. The quadrotor design is rigid and symmetric (inertia product = 0).
4. The pitch angle of the blades is fixed.

Using these assumptions, the flight dynamics of a rigid body subjected to the aerodynamic forces and moments produced by the propeller's rotation can be used to represent the flight dynamics of a quadrotor.

The orientation of the robot is given by the rotation matrix $R: E \rightarrow B$, where Rdepends on the 3 Euler angles, roll(ϕ), pitch $(\theta)$and yaw(ψ). These angles are quadrotor orientation in the body frames, they are bounded and meet satisfaction of:$(-\pi/2 < \phi, \theta < \pi/2)$and $(-\pi < \psi < \pi)$[30]. The rotation matrix between$E$and$B$is given below:

$$\begin{cases} R = R_x(\phi) + R_y(\theta) + R_z(\psi) \\ R = \begin{pmatrix} c(\theta)c(\psi) & s(\phi)s(\theta)c(\psi) - c(\phi)s(\psi) & c(\phi)\, s(\theta)\, c(\psi) + s(\phi)s(\psi) \\ c(\theta)s(\psi) & s(\phi)\, s(\theta)\, c(\psi) + c(\phi)s(\psi) & c(\phi)\, s(\theta)\, s(\psi) - s(\phi)c(\psi) \\ -s(\theta) & s(\phi)c(\theta) & c(\phi)c(\theta) \end{pmatrix} \end{cases} \quad (1)$$

where$s$denotes$\sin(.)$and $c$denotes $\cos(.)$. The angular velocities$[p, q, r]^T$ in the body frame can be obtained by converting the angular velocity$[\dot{\phi}, \dot{\theta}, \dot{\psi}]^T$in the inertial frame as:

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 1 & 0 & -s(\theta) \\ 0 & -c(\phi) & s(\phi)\, c(\theta) \\ 0 & -s(\phi) & c(\phi)c(\theta) \end{pmatrix} \begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} \quad (2)$$

The gyroscopic effect resulting from the rigid body rotation, the gyroscopic effect resulting from the propeller rotation combined with the body rotation, and also the actuators behavior are three concepts that are included in the quadrotor mathematical model. With transformation of the dynamic equations into E gives translational dynamics as follows:

$$\begin{cases} \ddot{x} = \frac{u_x}{m} u_1 - \frac{k_x}{m} \dot{x} \\ \ddot{y} = \frac{u_y}{m} u_1 - \frac{k_y}{m} \dot{y} \\ \ddot{z} = \frac{u_1}{m} \left[\mathrm{c}(\phi)\,\mathrm{c}(\theta) - \frac{k_z}{u_1} \dot{z} \right] - g \end{cases} \quad (3)$$

The and rotational dynamics, as follows

$$\begin{cases} \ddot{\phi} = \frac{1}{I_x} \left[\left(I_y - I_z \right) qr - J_r \Omega_r q + l u_2 - (\dot{\phi}^2 - 2\dot{\phi}\dot{\psi} \mathrm{s}(\theta)^2) \tau_x \right] \\ \ddot{\theta} = \frac{1}{I_y} \left[(I_z - I_x) pr + J_r \Omega_r p + l u_3 - (\dot{\theta}^2\, \mathrm{c}(\phi)^2 + 2\dot{\phi}\dot{\psi}\, \mathrm{s}(\phi)\,\mathrm{c}(\phi)\,\mathrm{c}(\theta) + \dot{\psi}^2 \mathrm{s}(\phi)^2 \mathrm{c}(\theta)^2) \tau_y \right] \\ \ddot{\psi} = \frac{1}{I_z} \left[\left(I_x - I_y \right) pq + J_r \Omega_r p + u_4 - (\dot{\theta}^2\, \mathrm{s}(\phi)^2 - 2\dot{\phi}\dot{\psi}\, \mathrm{s}(\phi)\,\mathrm{c}(\phi)\,\mathrm{c}(\theta) + \dot{\psi}^2 \mathrm{c}(\phi)^2 \mathrm{c}(\theta)^2) \tau_z \right] \end{cases}$$

(4)

with

$$\begin{cases} u_x = \mathrm{c}(\phi)\,\mathrm{s}(\theta)\,\mathrm{c}(\psi) + \mathrm{s}(\phi)\,\mathrm{s}(\psi) \\ u_y = \mathrm{c}(\phi)\,\mathrm{s}(\theta)\,\mathrm{s}(\psi) - \mathrm{s}(\phi)\,\mathrm{c}(\psi) \end{cases} \quad (5)$$

where m is the total mass of the quadrotor, l its half span, and I_x, I_y and I_z represent the inertias around (x, y, z) axis. g is the gravitational acceleration. k_x, k_y, and k_z are frictions aerodynamics coefficients; τ_x, τ_y, and τ_z denote translation drags coefficients; J_r denotes the propeller inertia along the x-axis.

$\Omega_r = \upsilon_1 - \upsilon_2 + \upsilon_3 - \upsilon_4$ is the total relative angular speed of the propellers, with υ_i being the speed of the i^{th} propeller. According to (3)-(4), the quadrotor's control inputs are described by u_1, which determines the lifting force generated by the rotation of the propellers on the body in the $z - axis$. u_2, u_3, u_4 are, respectively, the roll, pitch, and yaw input torques. The relationships between υ_i and u_i, are expressed as (6):

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} \vartheta & \vartheta & \vartheta & \vartheta \\ 0 & -\vartheta & 0 & \vartheta \\ -\vartheta & 0 & \vartheta & 0 \\ \varsigma & -\varsigma & \varsigma & -\varsigma \end{pmatrix} \begin{pmatrix} \upsilon_1^2 \\ \upsilon_2^2 \\ \upsilon_3^2 \\ \upsilon_4^2 \end{pmatrix} \quad (6)$$

where ϑ and ς are the thrust/lift and drag coefficients, which depend on the air density, the radius of the propeller, and the geometry. The description of the quadrotor parameters is given in Table. 1.

Table. 1. Description of the quadrotor parameters.

Parameter	Description	Value	Unit
I_x	moment inertia with respect to the axes	0.0075	$kg.m^2$
I_y		0.0075	
I_z		0.013	
J_r	The inertia of the rotors	6×10^{-5}	$kg.m^2$
k_x	The friction aerodynamic coefficients	5.567×10^{-4}	$Ns.m^{-1}$
k_y		5.567×10^{-4}	
k_z		6.354×10^{-4}	
τ_x	The translation drag coefficients	5.567×10^{-4}	$Ns.m^{-1}$
τ_y		5.567×10^{-4}	
τ_z		6.354×10^{-4}	
ϑ	Thrust coefficient	3.13×10^{-5}	$kg.m.rad^{-2}$
ς	Drag coefficient	7.5×10^{-7}	$kg.m.rad^{-2}$
m	quadrotor mass	0.65	kg
g	Acceleration of gravity	9.806	$m.s^{-2}$
l	length	0.23	m

In quadrotor control system, roll and pitch (ϕ and θ)angles are generally stabilized by separate controllers. Therefore, small angle assumption around the hover position is made, where $\phi\approx\theta\approx0$and choosing $u_1=u_{10}+mg$,the dynamic equation of the altitude motion can be approximated in linear form as:

$$\ddot{z}=\frac{u_{10}}{m}\quad(7)$$

2.2 Problem statement

According to the dynamical model presented in (3) and (4). The quadrotor control system can be divided into two subsystems, since control inputs are represented by altitude(z), attitude (ϕ,θandψ), andtheir temporal derivatives. The dynamics of altitude and attitude are included in the second subsystemΠ_2, whereas linear translations in the $x$and $y$axes are a part of the first subsystemΠ_1(underactuated sub-system). The two subsystems are given as follows

$$\Pi_1:\begin{cases}\ddot{x}=\frac{u_x}{m}u_1-\frac{k_x}{m}\dot{x}\\ \ddot{y}=\frac{u_y}{m}u_1-\frac{k_y}{m}\dot{y}\end{cases}\quad(8)$$

$$\Pi_2:\begin{cases} \ddot{z}=\frac{u_1}{m}\left(c(\phi)c(\theta)-\frac{k_z}{u_1}\dot{z}\right)-g \\ \ddot{\phi}=\frac{1}{I_x}\left[\left(I_y-I_z\right)qr-J_r\Omega_r q+lu_2-\left(\dot{\phi}^2-2\dot{\phi}\dot{\psi}\,\mathrm{s}(\theta)^2\right)\tau_x\right] \\ \ddot{\theta}=\frac{1}{I_y}\left[(I_z-I_x)pr+J_r\Omega_r p+lu_3-(\dot{\theta}^2\,\mathrm{c}(\phi)^2+2\dot{\phi}\dot{\psi}\,\mathrm{s}(\phi)\,\mathrm{c}(\phi)\,\mathrm{c}(\theta)+\dot{\psi}^2\mathrm{s}(\phi)^2\mathrm{c}(\theta)^2)\tau_y\right] \\ \ddot{\psi}=\frac{1}{I_z}\left[\left(I_x-I_y\right)pq+J_r\Omega_r p+u_4-(\dot{\theta}^2\,\mathrm{s}(\phi)^2-2\dot{\phi}\dot{\psi}\,\mathrm{s}(\phi)\,\mathrm{c}(\phi)\,\mathrm{c}(\theta)+\dot{\psi}^2\mathrm{c}(\phi)^2\mathrm{c}(\theta)^2)\tau_z\right] \end{cases}$$

(9)

To guarantee that the quadrotor can follow the desired position and attitude trajectories asymptotically and steadily in spite of modeling errors. In other words, the high-level control approach presented in this study should provide that the position tracking errors between the actual and the desired trajectories of the quadrotor ($e_x=x_d-x$, $e_y=y_d-y$, $e_z=z_d-z$)and the attitude tracking errors between the actual and the desired angles ($\phi_x=\phi_d-\phi$, $\theta_y=\theta_d-\theta$, $\psi_z=\psi_d-\psi$)converge to zero. Where$[x_d,y_d,z_d]^T$and$[\phi_d,\theta_d,\psi_d]^T$are the desired position and attitude angles respectively.

3. Interval Type-2 Fuzzy PD Controller design and analysis

The interval type-2 fuzzy logic system was used in this paper to made quickly and easily related calculations. The basic structure of T2-FLSs is depicted in Fig. 2, and consists of five major elements as follows:

— **Fuzzification:** It transformed inputs (actual values) into membership function values of fuzzy by using triangular or guassian membership functions (MFs). These membership degrees make up type-2 fuzzy input sets. [31].

— **Knowledge Base:** In this section, it consisted of a set of fuzzy *If-Then* rules called the basic rules and a set of membership functions called the database. The rule-base is considered as the heart of fuzzy logic systems[32].

— **Inference Engine:** The Interval Type-2 FLC have used the mechanism of fuzzy reasoning to produce a fuzzy output.

— **Type-Reduction:** The function of the reducer type was to transform the interval type-2 fuzzy set to type-1 fuzzy set.

— **Defuzzification:** The function of the defuzzifier was to change the fuzzy output for precise values, Karnik-Mendel Algorithm is used as defuzzifier on the interval type-2 fuzzy logic controller (IT2-FLC) that use centroid method.

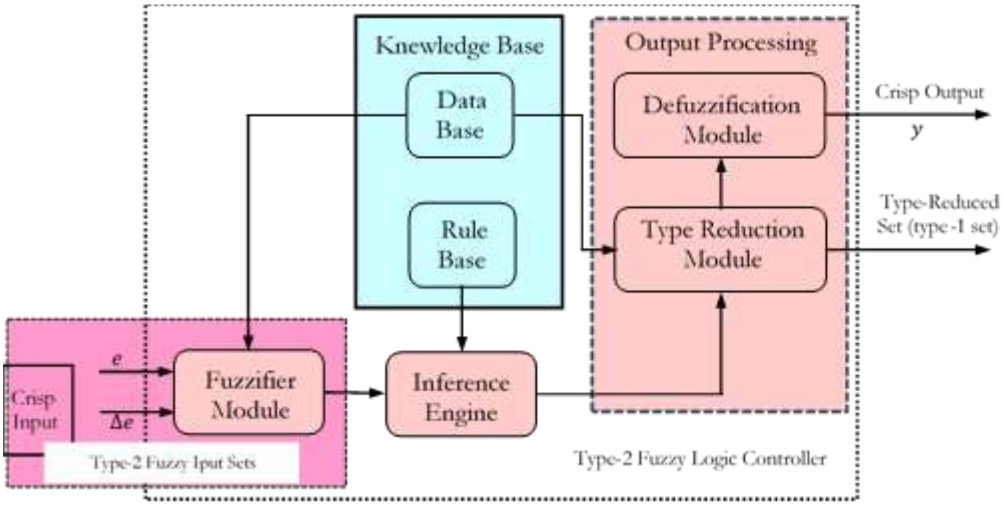


Figure. 2. The fundamental structure of Interval Type-2 Fuzzy System

To develop practical applications using T2-FLSs, it becomes vital to obtain a crisp value for all the fired fuzzy sets. To do this, a type-reduction is implemented where T2-FLSs are reduced to T1-FLSs [32]. There are many type-reduction approaches in the literature such as iterative Karnik-Mendel (KM) algorithm and other alternative type-reduction algorithms which mostly have a closed-form representation[31].

Uncertainties can affect decision-making in various ways, as the available data may be imprecise, incomplete, vague or fragmented. From a control perspective in FLCs, uncertainties can occur from input devices to the FLCs, which can be translated into uncertainties in the antecedents' membership functions.

3.1 Interval Type-2 Fuzzy Set

A type-2 fuzzy set is defined as an object $\tilde{A}$which is characterized by the membership function[33, 34-36]:

$$\tilde{A} = \{\langle (x,u), \mu_{\tilde{A}}(x,u) | \forall x \in X, \forall \mu_{\tilde{A}} \in J_x \subseteq [0,1] \rangle\} \quad (10)$$

in which $0 \leq \mu_{\tilde{A}}(x,u) \leq 1$is a type-1 fuzzy set known as the secondary set. Another expression for $\tilde{A}$ is,

$$\tilde{A} = \int_{x\in X} \int_{u\in J_x} \mu_{\tilde{A}}(x,u)/(x,u), \quad J_x \subseteq [0,1] \quad (11)$$

$J_x \subseteq [0,1]$represents the primary membership of x,$u$the secondary variable, has domain$J_x$at each is called the primary membership of$x$.

$$\tilde{A} = \int_{x\in X} \int_{u\in J_x} 1/(x,u) \quad (12)$$

In this equation the secondary grades of$\tilde{A}$all equal to 1. Uncertainty about$\tilde{A}$is transmitted by the union of all the primary memberships, which is called the footprint of uncertainty (FOU).

$FOU(\tilde{A}) = \cup_{x \in X} J_x$(13)

All points in foot of uncertainty (FOU) with unity supplementary membership functions (MFs) are part of the general framework of interval type-2 fuzzy sets [36, 37], as shown in Fig. 3. It is important to note that the triangular interval type-2 fuzzy MF is carried out using the upper membership function (UMF), lower membership function (LMF), and with 20% foot of uncertainties (FOU). Here, in this presented IT2-FPD designing, the control actions for the fuzzy PD controllers are realized via the Mamdani type inference technique. A rule base containing 25 rules is generated for IT2-FPD. Interval type-2 triangular MF is used to define the antecedent and consequent MFs. The MFs with five input and output fuzzy sets, which encompass the standardized operating range [-1, 1], are shown in Fig. 6(a).

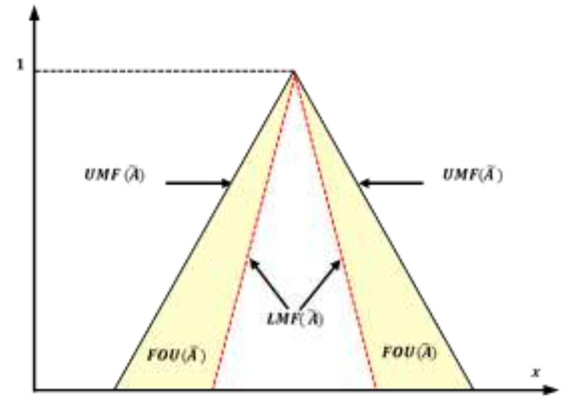


Figure. 3An Interval type-2 fuzzy set obtained by blurring the width of a triangular type-1 fuzzy set.

The *FOU* was bound by two type-1 MFs (membership functions) defined as the upper membership function (UMF) and lower membership function (LMF) of $\tilde{A}$. The LMF is associated with the lower bound of $\underline{FOU(\tilde{A})}$ and is defined as $\underline{\mu}_{\tilde{A}}(x), \forall x \in X$, whereas the UMF is associated with the upper bound of $\overline{FOU(\tilde{A})}$and is denoted as $\overline{\mu}_{\tilde{A}}(x), \forall x \in X$[36]:

$\overline{\mu}_{\tilde{A}}(x) = \overline{FOU(\tilde{A})}, \quad \forall x \in X$(14)

$\underline{\mu}_{\tilde{A}}(x) = \underline{FOU(\tilde{A})}, \quad \forall x \in X$(15)

$J_x = \left[\overline{\mu}_{\tilde{A}}(x), \underline{\mu}_{\tilde{A}}(x)\right], \quad \forall x \in X$(16)

If$\forall x \in X$,$\overline{FOU(\tilde{A})} = \underline{FOU(\tilde{A})}$then$\tilde{A}$is a type-1 set, and its membership values are degenerated intervals.

3.2 Design of Interval Type-2 Fuzzy PDTracking Controller

In this section, we will introduce a new controller for the quadrotor attitude and position subsystems. The aim of this controller is to track the reference of the position track and stabilize the attitude of the UAV.

The new IT2-FPD control approach developped is summarized in two parts, the first is related to the inner loop and the second to the outer loop, as rsented in the schematic diagram in Fig. 4. The designed controller is not only able to obtain null steady-state error tracking as well as achieving a faster convergence rate, but it is also capable to guesstimate the unknown boundaries of the uncertainties.

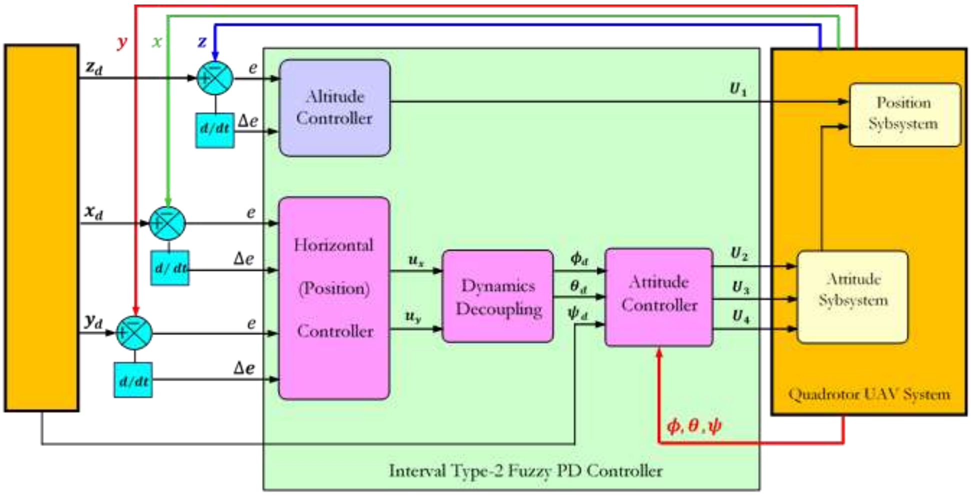


Figure. 4 The proposed IT2-FPD Control framework.

From Fig. 4, the control scheme consists of two loops: the loop (Attitude) and the (Position). The attitude loop is based on the nonlinear robust IT2-FPD Controller that is applied to perform the UAV attitude stabilization. This loop gives the yawing, pitching, and rolling torques to control the angular and the velocity of the rotational subsystem. While the position control loop is used to obtain a robust path tracking. The reference angles (ϕ_d, θ_d) are generated by this part. The proposed control enhances the tracking performances of the path reference and increases the robustness of the IT2-FPD control system compared with the classical PID controller, and type-1 fuzzy PD (T1-FPD) control methods.

The structure of the proposed IT2-FPD controller is shown in Fig.5. The antecedents as well as the consequent parts of the rule base are both interval type-2 fuzzy sets. Thus, the controller is better able to handle both model uncertainties and measurement noise in a more efficient manner[32]. As shown in Fig.5, the controller is made up of choosing E and ΔE as inputs and the control action (U) as an output of the IT2-FPD Controller. The position error (e) and its rate of change (Δe) are shown as controller inputs.

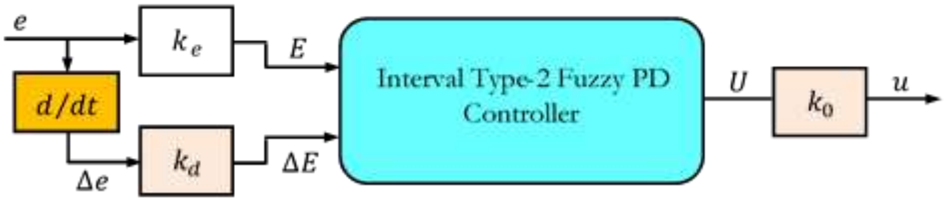


Figure.5. Illustration of the IT2-FPD controller.

As shown in Figure 5, $k_e$and k_d presents the input scaling factors (SFs), while k_0 is the output scaling factor (SF).

$E = k_e e, \quad \Delta E = k_d \Delta e$(17)

The FLC inputs are represented by $E$and ΔE. The output (U) is transformed to the actual control signal of the IT2-FPD controller (u) as follows:

$u = k_0 U$(18)

The handled IT2-FLC structureis designed and presented with symetrical (5×5) rule base as shownin Table. 2. The MFs of IT2-FPD are characterised by triangular MFs, the control surface of the IT2-FPD is given in Fig.6(a). Moreover, they are represented with five linguistic fuzzy terms, which are defined as follows: PB—(positive big); PS—(positive small); ZE—(zero); NS—(negative small); and NB—(negative big).

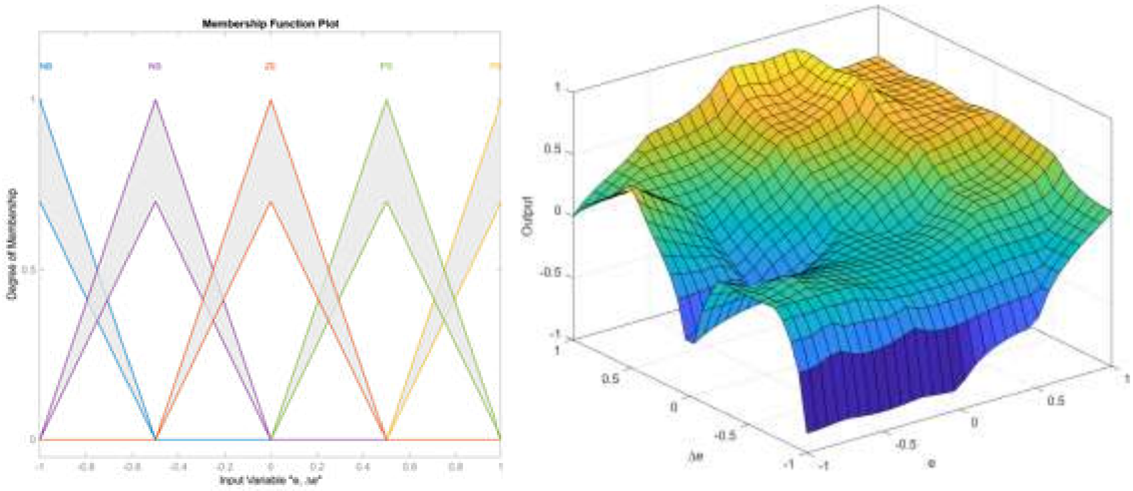
Table.2. The rule table of the IT2-FPD controller

Derivative of error signal ΔE	Error signal E				
	NB	NS	ZE	PS	PB
NB	c_1=NB	c_2=NB	c_3=NB	c_4=NS	c_5=ZE
NS	c_6=NB	c_7=NB	c_8=NS	c_9=ZE	c_{10}=PS
ZE	c_{11}=NB	c_{12}=NS	c_{13}=ZE	c_{14}=PS	c_{15}=PB
PS	c_{16}=PS	c_{17}=ZE	c_{18}=PS	c_{19}=PB	c_{20}=PB
PB	c_{21}=ZE	c_{22}=PS	c_{23}=PB	c_{24}=PB	c_{25}=PB

The rule structure of the IT2-FPID is as follows:

$R^n: IF\ E\ is\ \tilde{A}_{1i} and\ \Delta E\ is\ \tilde{A}_{2j} THEN\ u\ is\ c_q, \quad i,j = 1, \dots, 5; q = 1, \dots, 25$(19)

The antecedent MFs part is defined with triangular interval type-2 fuzzy sets $\tilde{A}_{1i}$ and $\tilde{A}_{2j}$ for the inputs $E$and ΔE, respectively. Where c_q is the consequent crisp set of each rule and q is the total number of rules.Here, the footprint of uncertainties of IT2-FPD controller is created only by the value of the parameterm_n[38].We employ the following parameters: $m_1 = m_3 = 1 - \alpha$and $m_2 = \alpha$as suggested in [38,39–41]. Therefore, the selection of the parameter (α) is vital because it directly affects U.



(a) (b)

Figure.6. (a) Interval type-2 fuzzy antecedent MFs (b) Control surface.

The controller output (U) has been determined by using a several methods [38, 41]. The Biglarbegian-Melek-Mendel approach is applied in this study [38]. The following equation applies to this approach:

$$\begin{cases} \overline{y} = \dfrac{\sum_{i=1}^{q} f_h^i y^i}{\sum_{i=1}^{q} f_h^i} \\ \underline{y} = \dfrac{\sum_{i=1}^{q} f_l^i y^i}{\sum_{i=1}^{q} f_l^i} \end{cases} \tag{20}$$

$$U = \beta \underline{y} + (1-\beta)\overline{y} \tag{21}$$

where y^i shows consequent MFs, q is the total number of rules (see Table. 2), β is a weighting parameter for the type reduced set ($[\underline{y}, \overline{y}]$), and U is the output of FLC. Lastly, f_l and f_h are defined as the lower and upper firing functions of IT2-FPD, respectively. In Figure 6(a), the control surface, which selected $\alpha = \beta = 0.3$ for IT2-FPD, is illustrated.Parameters of IT2-FPD are selectedaggressively by using rule-based FLCs and MFs. The β and α parameters are selected by trial and error so that an aggressive control surface is created for IT2-FPD controller (see Fig. 6(b)).

In the design of the conventional controller, the optimal choice of the gains of the system is difficult. One of the most applicable methods to choose these parameters is the trial-and-error method, which is time consuming. However, in the proposed approach the responsibility of the PD controller is only to keep the system stable for a while. The intelligent controller can add more degrees of freedom to the system to enhance the performance of the system. The stability analysis of the adaptation laws of the IT2-FPD are guaranteed using Lyapunov stability theory.

Consider error, $e \equiv \chi_d - \chi$ and its rate of change Δe, are defined as

$$\Delta e \equiv e(kT) - e((k-1)T) \tag{22}$$

where $\chi_d = [x_d, y_d, z_d, \phi_d, \theta_d, \psi_d]^T$ represents the desired reference vector and $\chi = [x, y, z, \phi, \theta, \psi]^T$ denotes the actual vector of the quadrotor system. T is the sampling period of the discrete system, and k is an integer. The above rules allow us to model the uncertainties encountered in the antecedents. Therfore, The output control vector of IT2-FPD controller will be as follows:

$$\begin{cases} u_1 = \frac{m u_z}{c(\phi)c(\theta)} \\ u_2 = k_e \phi_x + k_{\Delta e} \dot{\phi}_x \\ u_3 = k_e \theta_y + k_{\Delta e} \dot{\theta}_y \\ u_4 = k_e \psi_z + k_{\Delta e} \dot{\psi}_z \end{cases} \quad (23)$$

The following expressions can be used to obtain the desired angles (Roll, Pitch):

$$\begin{cases} \phi_d = arcsin\left(\frac{u_x s(\psi_d) - u_y c(\psi_d)}{\sqrt{u_x^2 + u_y^2 + u_1^2}}\right) \\ \theta_d = arctan\left(\frac{u_x c(\psi_d) + u_y s(\psi_d)}{\sqrt{u_x^2 + u_y^2 + u_1^2}}\right) \end{cases} \quad (24)$$

The parameters k_e (Proportional action) and $k_{\Delta e}$ (Derivative action) are input scaling factors, respectively.

The control actions realising the deisred trajectories of the Euler angles are determined by the weighted average:

$$u_i = \frac{\sum_{j=1}^{25} \vartheta_q c_q}{\sum_{j=1}^{25} \vartheta_q}, \quad i = 2,3,4 \quad (25)$$

While the altitudecontrol action of our IT2-FPD controller is given by:

$$u_1 = \frac{\sum_{j=1}^{25} \vartheta_q c_q}{\sum_{j=1}^{25} \vartheta_q} + mg \quad (26)$$

with ϑ_q is the truth value of c_q, calculated by the following algebraic product method:

$$\vartheta_q = \mu_{e_q}(e(k)) \times \mu_{\Delta e_q}(\Delta e(k)) \quad (27)$$

μ_{y_q} is the membership degree of the input variable y evaluated in rule q by the corresponding interval type-2 membership function (IT2-MF).

4. Simulation Results and discussion

To evaluate the effectiveness of the proposed approach, numerical simulations study of the UAV system will be presented in this section. The physical parameters for the quadrotor used in the simulation are illustrated in Table. 1. The simulations were performed using MATLAB/Simulink software.

Furthermore, comparisons with the conventional PID control approach and the type-1 fuzzy PD controller are done to highlight the superiority and the improvement obtained of the offered control approach.A time step of 0.01 secis applied to simulations in Matlab/Simulink environments.

The purpose of the controller design is to track the following altitude and position desired path:

$$\begin{cases} x_d = \lambda * \cos(\omega * t) \\ y_d = \lambda * \sin(\omega * t) \\ z_d = 0.1 * t \end{cases} (28)$$

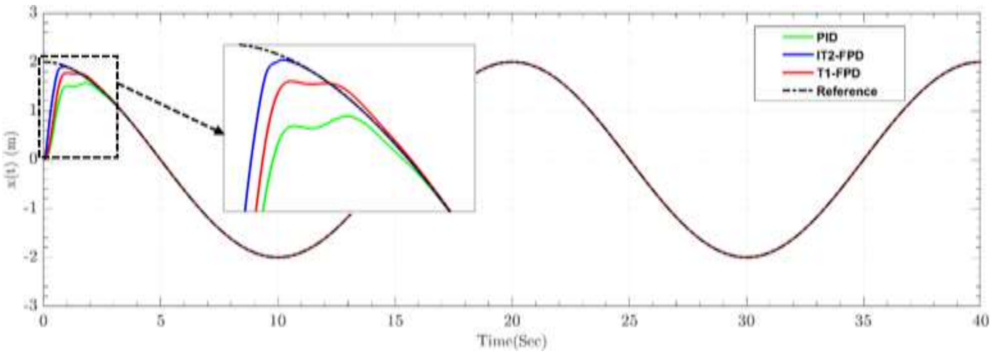
and the desired yaw path:

$$\psi_d = a * \sin(\omega * t) (29)$$

with$\lambda = 2$;$\omega = 2\pi/40$ and$a = \text{deg2grad}(5)$.The initial conditions for attitude and position of the quadrotor are chosen as $\psi_d(0) = 0$ (rad) and $[x_d(0), y_d(0), z_d(0)]^T = [0,0,0]^T$ (m), respectively. All other initial conditions are zero. The intended trajectory and proposed control strategy were successful in efficiently preserving the quadrotor's position and attitude path, as seen in Figs. 7-8.

Remark 1. The simulations are performed on MATLAB 2022 software, which is installed on a DELL computer comprising 2.60 GHz;Core i5-6440HQ-(4 CPUs) processor, with 16 GB of RAM and a 256 GB SSD.

In order to highlight the superiority of the nonlinear IT2-FPD control laws, the simulation results are shown in Figures. 8, 9, 10, 11 and 12. The desired position and actual tracking results of variables $x(t)$, $y(t)$, and $z(t)$are presented in Fig. 7. The initial detailed part of the angle change is shown through a partially enlarged view. It can be seen that the proposed method allows for precise and fast trajectory tracking, differently from the cases when the T1-FPD control and PID control methods is adopted, the behavior can be observed at $t = 1$ sec for the $x(t)$ position tracking of the quadrotor system.



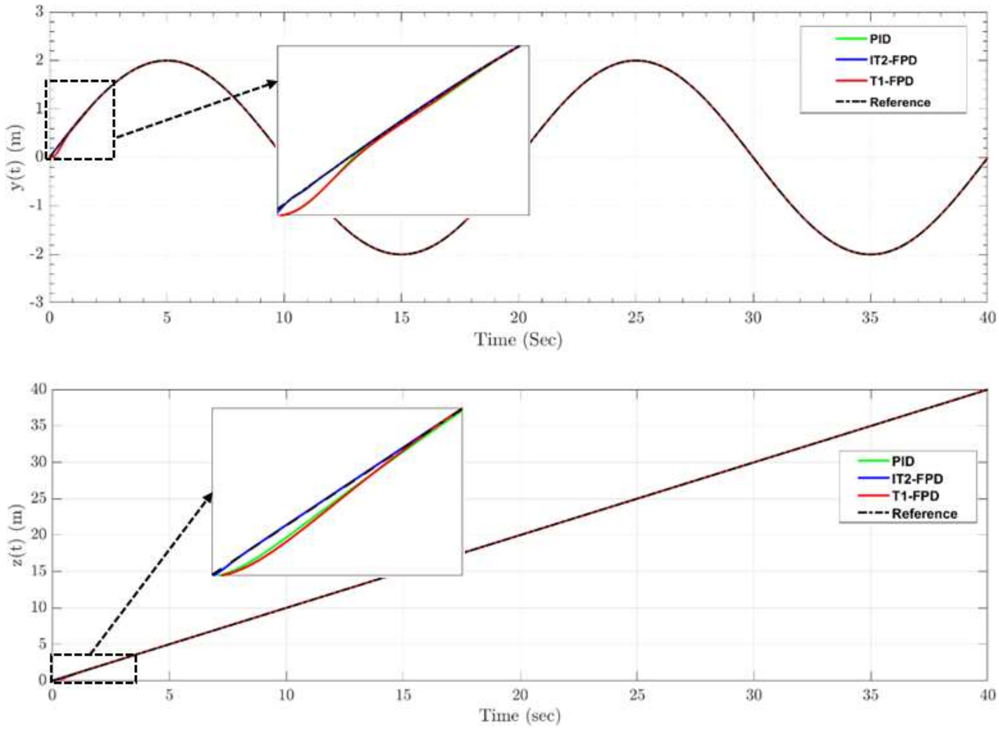
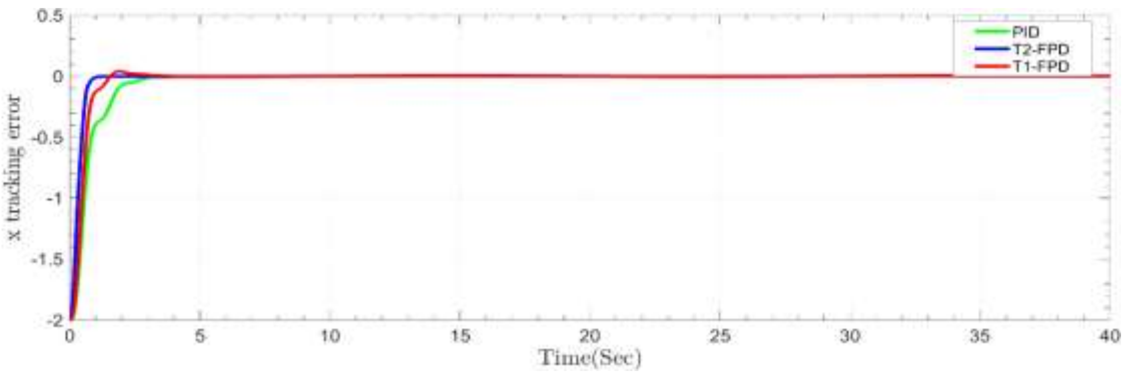


Figure. 7. The quadrotor position tracking response in direction of $\left(x(t), y(t), z(t)\right)$

Indeed, the latter determines a major delay of the convergence time in the position outputs, which is not able to handle the rule uncertainties at the beginning of the simulation. Meanwhile, the proposed IT2-FPD utilizes the interval type-2 FLS. The results show that the interval type-2 FLS is able to handle rule uncertainties [34].

Fig. 8 refers to the response curve of the quadrotor position tracking errors for each control approach. In order to have a comparison between the proposed approach (IT2-FPD) and other counterpart, we can be clairely seen that the performance of our controller is better than the other two mentioned approachs (e_x, e_y, and e_z are zeroed as expected and displayed in Fig. 8).



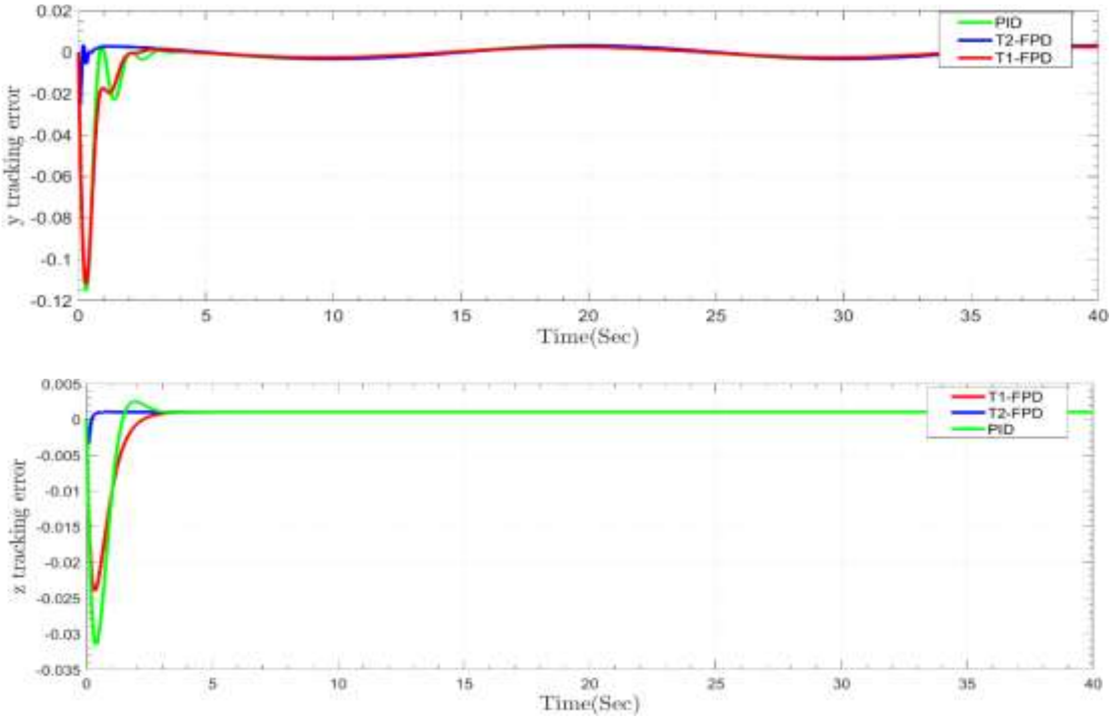
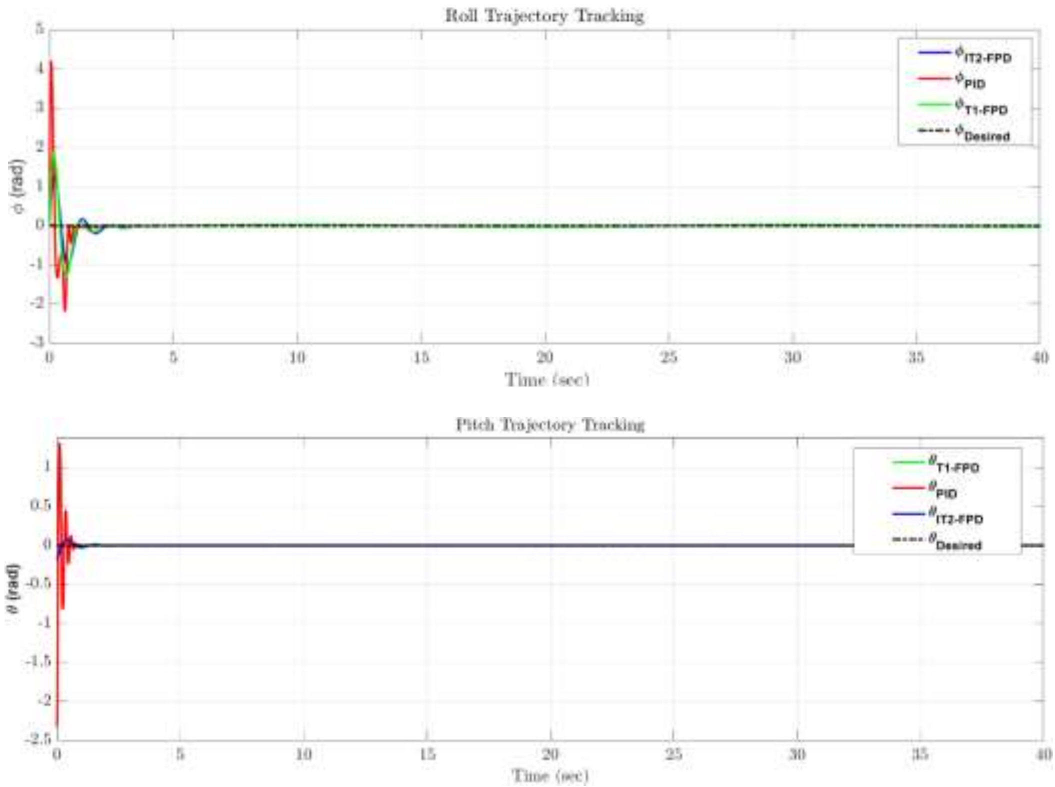


Figure 8. The position tracking error response$(e_x(t), e_y(t), e_z(t))$

However, Fig. 9 describes the attitude performance, the relationship tracking among the angle of the quadrotor UAV motion and the desired trajectory. We can see that the roll and pitch angles computed by the outer loop trajectory following the proposed control approach and the yaw angle follow the reference values in a finite time. Although the roll and pitch angles are not controlled directly, the proposed controller is able to stabilize the orientation of the quadrotor.



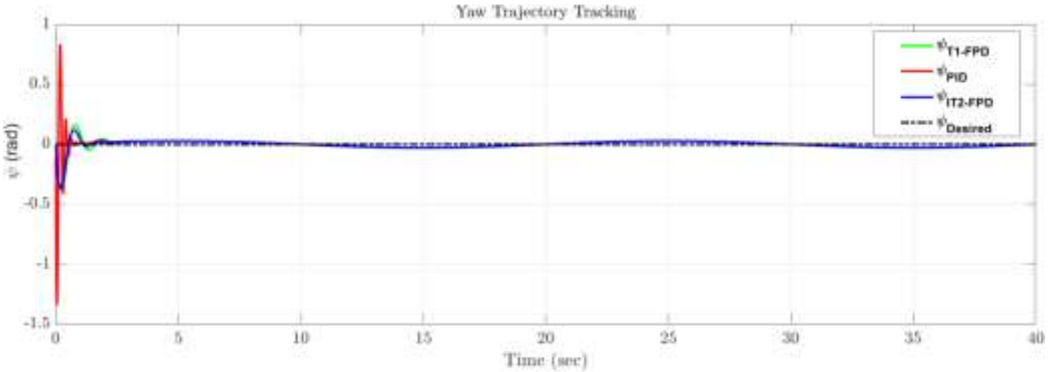


Fig. 9. The quadrotor attitude tracking response $(\phi(t), \theta(t), \psi(t))$.

Moreover, Fig. 10 describes the performance of the proposed method in tracking the desired trajectory in 3D space, the performance of our proposed controller is provide better. We can observe how, starting from an initial condition of the position away from the reference, the IT2-FPD control strategy is able to make the quadrotor follow the desired trajectory.

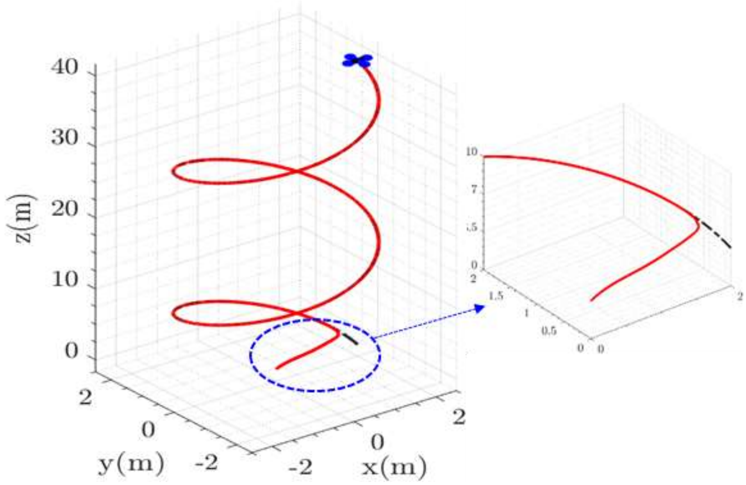


Figure10.Trajectory tracking of the quadrotor in 3D space using IT2-FPD Controller.

Fig. 11illustrates the values visible in the position histogram are a consequence of the static error in the following of input trajectories for each controller. Nevertheless, the maximum instances of error for all of position and altitude coordinates tracking states correspond approximately to zero, which is symptomatic of the capacity of the IT2-FPD control system to maintain these coordinates at a minimal constant value. The inertial coordinate z presented the higher deviations, which was expected since that was the subsystem more solicited. The z-axes obtained values demonstrate that the goal defined for this Euler direction was achieved.

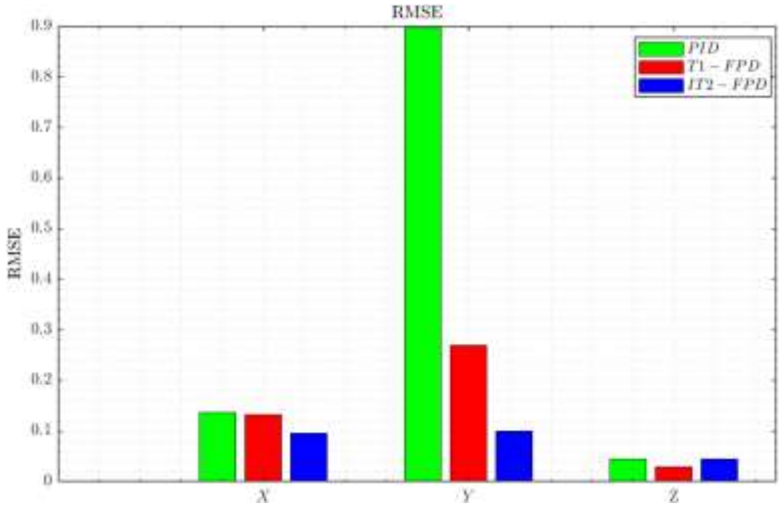


Figure 11. RMSE Histogram of quadrotor position and altitude for different controllers.

For better comparison of these methods, the simulation results on each axis are analysed. Root-mean-square-error (RMSE) approach is chosen as a performance index for the comparison of the controllers. From Fig. 7, the results of RMSE on the x-axis, y-axis and z-axis are summarized inTable 3. for all the reference signals, where lowest RMSE values are achieved from the proposed IT2-FPD controller in all the scenarios. IT2-FPD approach shows almost eight times better results in y-axis. We can clearly indicate that the performances of the controller proposed in this article provides a faster response and excellent tracking.

Table 3.Controllers performance indexes (RMSE values on xyz-axis).

	PID	T1-FPD	T2-FPD
Reference Tracking on x-axis (m)	0.1389	0.1323	0.0912
Reference Tracking on y-axis (m)	0.896	0.2724	0.1012
Reference Tracking on z-axis (m)	0.0482	0.0315	0.0451

In summary, IT2-FPD has the best performance results. Considering the aim of the testing for IT2-FPD, the minimum task time, and RMSE values, maximum accuracy is obtained. However, although T1-FPD is not as good as IT2-FPD, it has better results than the compared PID controller.

Regardless of the kind of curve given, the IT2-FPD control method has a shorter convergence time in terms of attitude and position tracking than the T1-FPD method and conventional PID approach. In general, according to the simulation results and compared with T1-FPD and PID controllers, we may safely draw the conclusion that the Interval type-2 fuzzy PD method can realize the control of the quadrotor system trajectory tracking in a finite amount of time, with a high control quality and good robustness.

5. Conclusion

A novel closed-loop interval type-2 fuzzy proportional derivative (IT2-FPD) control strategy has been proposed throughout the paper, and the effectiveness of the developped method has been applicated for nonlinear quadrotor UAV system. To effectively handle the underactuated characteristics of the quadrotor, the full dynamics of the UAV system was separated into the altitude, position, and attitude subsystems. Furthermore, to deal with the uncertainties of its membership functions, those systems were modeled via interval type-2 fuzzy models.There were two designing conditions; one is to garantiees both the asymptotic stabilisation as well as satisfying tracking performance of the quadrotor motion simultaneously, and the other is for the reachability of the tracking error dynamics and its derivative onto the input of designed IT2-FPD controller.

Triangular interval type-2 membership functions are applied in this study. To further extend the design degrees of freedom and improve control performance, the system error and its derivatives can also be included. The proposed controller topology is also quite simple. The dynamical model of the quadrotor along with the controllers was simulated within Matlab/Simulink. Despite the challenges of both understanding and designing interval type-2 fuzzy controllers contrasted with other controllers, the advantages of the former remain a favored research zone for its robustness through uncertainties and nonlinearities.

Finally, we have provided numerical simulations, highlighting the effectiveness and the impact of the proposed position and attitude tracking control methodology compared with conventional PID and tyep-1 fuzzy PD control approaches. The simulation results demonstrate that the proposed controller outperforms both the conventional PID controller and its fuzzy type-1 counterpart and presents an excellent substitute for the altitude control applications of quadrotors. From all the simulation results that have been done, we can be clairly seen that the designed method was indeed more efficient and robust with less overshoot, faster settling time and have minimum RMSE value against desired altitude and position variations than the T1-FPD and the classical PID controller.

For further research development, the artificial intelligence algorithms will be developed to optimize the proposed IT2-FLPD controller in order to provide more efficient results as well as conducting real-time experiments of a quadrotor using the proposed controller. In terms of future work, the controller design part of this paper does not consider environmental interference or perturbation issues. Thus, the influence of these parameters will be taken into consideration in our next stage of work.

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