Research on Adaptive Optimization Trajectory Tracking Algorithm based on MPC

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Abstract: Aiming at the problems of large trajectory tracking error and poor adaptability to different road conditions of the traditional constant predictive time-domain model predictive control algorithm, a new variable predictive time-domain adaptive trajectory tracking controller is proposed by fusing the traditional model predictive control algorithm and fuzzy control algorithm. The fuzzy control algorithm is used to adjust the predictive time-domain required by the predictive model controller according to the two parameters of vehicle speed and ground adhesion coefficient. The Gaussian membership function is used to formulate fuzzy rules. Through the operation of fuzzy control rules, the predictive time-domain parameters matching the current system are automatically output and transmitted to the model predictive controller to control the vehicle for trajectory tracking to improve the adaptability of trajectory tracking to different road conditions. Simulink and CarSim joint simulation show that the comprehensive use of the proposed adaptive trajectory tracker can effectively improve the accuracy and stability of vehicle trajectory tracking on low adhesion wet and slippery roads.

Keywords: application of automation technology; intelligent car; fuzzy control; model predictive control; trajectory tracking
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1. Introduction

With the increasing number of cars, car accidents, urban congestion, and other problems are becoming more and more serious. Intelligent driving assistance and unmanned driving technology have become effective response measures [1]. Automatic driving can deal with different road conditions, reduce accidents and improve vehicle operation efficiency. It has become a research hotspot in the field of automatic control at home and abroad [2]. The research on trajectory tracking control is a very important link in autopilot technology and the key to the successful realization of autopilot [3,4]. At present, the main methods for intelligent vehicle trajectory tracking control include PID control, sliding mode control, intelligent control, linear quadratic regulator control, and model predictive control [5-9]. MPC algorithm has the characteristics of good robustness and adaptability. It can freely add multiple required constraints to the established model and find the current optimal control quantity by rolling solving the optimization problem [10-12]. Cui Q et al. added vehicle state parameters to the MPC controller to realize vehicle trajectory track at a constant speed, but the control effect is not ideal when the vehicle speed changes [13]. Prediction time-domain (N_p) is a very important parameter in the MPC algorithm. Many scholars have made a lot of research results on the selection of prediction time domain in the MPC algorithm. Zhang Y et al. and used a genetic algorithm (GA) to optimize the prediction time domain of the MPC, which only improved the adaptability of the controller to speed without considering the influence of ground adhesion coefficient [14]. Based on the traditional model predictive control algorithm, Bai G X et al. improved and optimized the parameters, to design a trajectory tracking controller with variable predictive time domain, which improved the adaptability of the controller to speed but also did not consider the influence of ground adhesion coefficient on the selection of predictive time domain [15]. At present, in the research of variable time-domain predictive control, most model predictive controllers for vehicle trajectory tracking only consider the influence of speed on the prediction time-domain but ignore the influence of ground adhesion coefficient on the prediction time domain, especially on wet and slippery roads, the reduction of ground adhesion coefficient will increase the vehicle trajectory tracking error and reduce the vehicle driving stability. Therefore, based on the fuzzy control algorithm and model predictive control algorithm, a new adaptive trajectory tracking controller is proposed in this paper. Considering the influence of ground adhesion coefficient and vehicle speed on trajectory tracking effect, the two parameters are input into the fuzzy control algorithm first, the predictive time domain of the best matching system required by the model predictive control algorithm is obtained to optimize the tracking effect. The joint simulation results of Simulink and CarSim show that the adaptive optimal trajectory tracking controller proposed in this paper can reduce the trajectory tracking error on wet and slippery roads and improve the vehicle running stability. The joint simulation results of Simulink and CarSim show that the adaptive optimal trajectory tracking controller proposed in this paper can reduce the trajectory tracking error on wet and slippery roads and improve the vehicle running stability.

The remainder of this paper is organized as follows: the establishment and analysis of the vehicle dynamics model is in Section 2. Section 3 introduces the principle of the model predictive controller and combines fuzzy control and model predictive control to establish an adaptive trajectory tracking controller. The performance of the adaptive trajectory tracking controller is tested by Simulink/ CarSim co-simulation in Section4. Section5 is the conclusion.

2. Vehicle dynamics model

The basis of the model predictive controller in this article is the dynamics model of the vehicle. To reduce the computational complexity of the control algorithm as much as possible, some factors in the vehicle dynamics model need to be ignored. There is vertical movement and no wind resistance during driving. The vehicle dynamics model shown in Figure 1 is established.

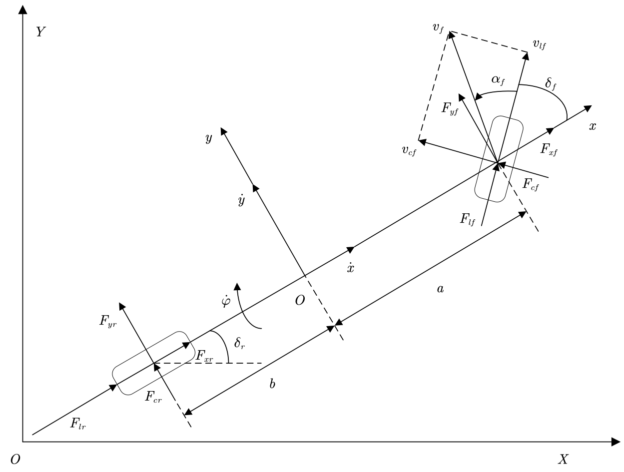

Figure 1. Vehicle monorail dynamic model

Where: the coordinate system XOY is the ground inertial coordinate system; Coordinate system oxy is the vehicle coordinate system; F_{lf}, F_{lr} are the longitudinal force on the front and rear wheels; F_{yf}, F_{yr} are the force in the y-direction of the front and rear wheels; δ_f, δ_r are the deflection angle of front and rear wheels; α_f is the front wheel side deflection angle; $\dot{\varphi}$ is the yaw rate.

According to Newton's second law, the force balance equations along the x-axis, y-axis, and around the z-axis can be obtained as follows:

$$\begin{cases} m\ddot{x} = m\dot{y}\dot{\varphi} + 2F_{xf} + 2F_{xr} \\ m\ddot{y} = -m\dot{x}\dot{\varphi} + 2F_{yf} + 2F_{yr} \\ I_Z\ddot{\varphi} = 2aF_{yf} - 2bF_{yr} \end{cases} \tag{1}$$

Where:

$$\begin{cases} F_{xf,r} = F_{lf,r}\cos\delta - F_{cf,r}\sin\delta_{f,r} \\ F_{yf,r} = F_{lf,r}\sin\delta + F_{cf,r}\cos\delta_{f,r} \end{cases} \tag{2}$$

From the relationship between the ground coordinate system and vehicle coordinate system:

$$\begin{cases} \dot{Y} = \dot{x}\sin\varphi + \dot{y}\cos\varphi \\ \dot{X} = \dot{x}\cos\varphi - \dot{y}\sin\varphi \end{cases} \tag{3}$$

The vehicle dynamics model can be obtained by combining formula (1) (2) (3):

$$\begin{cases} m\ddot{x} = m\dot{y}\dot{\varphi} + 2\left[C_{lf}s_f + C_{cf}\left(\delta_f - \dfrac{\dot{y} + a\dot{\varphi}}{\dot{x}}\right)\delta_f + C_{lr}s_r\right] \\ m\ddot{y} = -m\dot{y}\dot{\varphi} + 2\left[C_{cf}\left(\delta_f - \dfrac{\dot{y} + a\dot{\varphi}}{\dot{x}}\right) + C_{cr}\dfrac{b\dot{\varphi} - \dot{y}}{\dot{x}}\right] \\ I_z\ddot{\varphi} = 2\left[aC_{cf}\left(\delta_f - \dfrac{\dot{y} + a\dot{\varphi}}{\dot{x}}\right) - bC_{cr}\dfrac{b\dot{\varphi} - \dot{y}}{\dot{x}}\right] \\ \dot{Y} = \dot{x}\sin\varphi + \dot{y}\cos\varphi \\ \dot{X} = \dot{x}\cos\varphi - \dot{y}\sin\varphi \end{cases} \tag{4}$$

The state quantity of the system is $\xi = [\dot{y}, \dot{x}, \varphi, \dot{\varphi}, Y, X]^T$, the control quantity of the system is $u = \delta_f$.

3. Adaptive controller design

This section may be divided into subheadings. This section mainly introduces the construction principle and steps of the adaptive controller model.

3.1. Establishment of the prediction model

The vehicle dynamics model established above is a nonlinear dynamics model, which is inconvenient to solve. After processing the nonlinear model, the linear time-varying equation is obtained as follows:

$$\dot{\xi}_{dyn} = A_{dyn}(t)\xi_{dyn}(t) + B_{dyn}(t)u_{dyn}(t) \tag{5}$$

$$A_{dyn}(t) = \frac{\partial f_{dyn}}{\partial u_{dyn}} |_{\xi_t,\ u_t} = \begin{bmatrix} \frac{-2(C_{cf}+C_{cr})}{m\dot{x}_t} & \frac{\partial f_{\dot{y}}}{\partial \dot{x}} & 0 & -\dot{x}_t + \frac{2(bC_{cr}-aC_{cf})}{m\dot{x}_t} & 0 & 0 \\ \dot{\varphi} - \frac{2C_{cf}\delta_{f,t-1}}{m\dot{x}_t} & \frac{\partial f_{\dot{x}}}{\partial \dot{x}} & 0 & \dot{y}_t - \frac{2aC_{cf}\delta_{f,t-1}}{m\dot{x}_t} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{2(bC_{cr}-aC_{cf})}{I_z\dot{x}_t} & \frac{\partial f_{\dot{\varphi}}}{\partial \dot{x}} & 0 & \frac{-2(a^2C_{cf}+b^2C_{cr})}{I_z\dot{x}_t} & 0 & 0 \\ \cos(\varphi_t) & \sin(\varphi_t) & \dot{x}_t\cos(\varphi_t) - \dot{y}_t\sin(\varphi_t) & 0 & 0 & 0 \\ -\sin(\varphi_t) & \cos(\varphi_t) & \dot{y}_t\cos(\varphi_t) - \dot{x}_t\sin(\varphi_t) & 0 & 0 & 0 \end{bmatrix}$$

$$B_{dyn}(t) = \frac{\partial f_{dyn}}{\partial u_{dyn}} |_{\xi_t,\ u_t} = \left[\frac{2C_{cf}}{m}, \frac{2C_{cf}\left(2\delta_{f,t-1} - \frac{\dot{y}_t + a\dot{\varphi}_t}{\dot{x}_t}\right)}{m}, 0, \frac{2aC_{cf}}{I_z}, 0, 0\right]$$

$$\begin{cases} \frac{\partial f_{\dot{y}}}{\partial \dot{x}} = \frac{2C_{cf}(\dot{y}_t + a\dot{\varphi}_t) + 2C_{cr}(\dot{y}_t - b\dot{\varphi}_t)}{m\dot{x}_t} - \dot{\varphi}_t \\ \frac{\partial f_{\dot{x}}}{\partial \dot{x}} = \frac{2C_{cf}\delta_{f,t-1}(\dot{y}_t + a\dot{\varphi}_t)}{m\dot{x}_t} \\ \frac{\partial f_{\dot{\varphi}}}{\partial \dot{x}} = \frac{2aC_{cf}(\dot{y}_t + a\dot{\varphi}_t) - 2bC_{cr}(\dot{y}_t - b\dot{\varphi}_t)}{I_z\dot{x}_t} \end{cases}$$

Equation (5) obtains the discrete state space expression through the interpolation method:

$$\xi_{dyn}(k+1) = A_{dyn}(k)\xi_{dyn}(k) + B_{dyn}(k)u_{dyn}(k) \tag{6}$$

$$A_{dyn}(k) = I + TA_{dyn}(t), B_{dyn}(k) = TB_{dyn}(t)$$

Due to space constraints, the other specific derivation processes are shown in literature [16].

3.2. Optimal solution of the objective function

The solution process of vehicle trajectory prediction needs to be finally completed in the quadratic programming equation. The setting of the upper and lower range of variables is very key. In this paper, constraints are added to the control quantity and control increment to improve the tracking accuracy and stability of vehicle trajectory tracking. The constraints of system input and output are as follows:

$$\begin{gathered} u_{\min}(t+k) \leqslant u(t+k) \leqslant u_{\max}(t+k) \\ k = 1, \cdots, m-1 \end{gathered} \tag{7}$$

$$\begin{gathered} y_{\min}(t+k) \leqslant y(t+k) \leqslant y_{\max}(t+k) \\ k = 1, \cdots, m-1 \end{gathered} \tag{8}$$

To meet the requirements of vehicle trajectory tracking and passenger comfort, some vehicle constraints need to be set in this paper. The constraint range of vehicle track tracking front-wheel deflection angle is set as follows: $-11.46^{\circ} \leqslant \delta \leqslant 11.46^{\circ}$, the constraint range of front-wheel deflection control is: $-2.5^{\circ} \leqslant \Delta\delta \leqslant 2.5^{\circ}$, the yaw angle constraint range is: $-17.197^{\circ} \leqslant \phi \leqslant 17.197^{\circ}$, the constraint range of longitudinal displacement is: $-3 \leqslant l \leqslant 5$.

The complexity of the vehicle dynamics model is high. To simplify the calculation, it is necessary to simplify the model and add many constraints. The optimization objective function adopted in this paper is as follows [17]:

$$J(\xi_{dyn}(t), u_{dyn}(t-1), \Delta U_{dyn}(t)) = \sum_{i=1}^{N_p} \| \eta_{dyn}(t+i\,|\,t) - \eta_{dyn,ref}(t+i\,|\,t) \|_Q^2 + \sum_{i=1}^{N_c-1} \| \Delta u_{dyn}(t+i\,|\,t) \|_R^2 + \rho\varepsilon^2 \tag{9}$$

Where Q and R are system weight matrices, ρ is the weight coefficient, ϵ is the relaxation factor.

In equation (9), the first item reflects the accuracy of the trajectory following of the autonomous vehicle through the 2 norms of the actual output of the system and the reference trajectory error, and the second item considers the control increment of the vehicle to ensure the smooth change of the control quantity in the trajectory following process. The function of the objective function is to keep the accuracy and stability of the vehicle in the track following process, to meet the basic requirements of vehicle track following.

3.3. Design of variable time domain controller based on fuzzy control

In model predictive control trajectory tracking, the choice of prediction time domain has a great impact on the error and stability of vehicle trajectory tracking. The prediction distance of the vehicle is directly related to the sampling time and prediction time domain of the controller:

$$d = v_x \times T_s \times N_p \tag{10}$$

When the sampling time and prediction time domain of the controller remain unchanged, the prediction distance of the vehicle will increase with the increase of vehicle speed, and the excessive prediction distance will lead to the decline of trajectory tracking accuracy near the current vehicle; If the predicted distance is too small, the control quantity and control increment of the controller may exceed the constraints, resulting in the vehicle unable to turn in time, the decline of vehicle stability and the vehicle out of control [18].

To improve the track tracking accuracy at low speed, a smaller prediction time domain should be used to obtain a shorter prediction distance; When the vehicle is in high-speed condition, to avoid losing control of the vehicle, focusing on the stability of the vehicle, a larger prediction time domain should be used to obtain a longer prediction distance [19].

When the vehicle is driving on a slippery road, because the ground adhesion coefficient decreases and the friction decreases, it is unable to track the planned path and is easy to lose stability. At this time, to avoid the vehicle out of control, we should focus on the stability of the vehicle and increase the prediction time domain to increase the prediction distance; When the vehicle is driving on a dry road, we should focus on the vehicle trajectory tracking accuracy and reduce the prediction time domain to reduce the prediction distance.

The fuzzy control algorithm is used to adjust the prediction time-domain required by the prediction model controller according to the vehicle speed and ground adhesion coefficient. The fuzzy rules shown in Table 1 are formulated by using the Gaussian membership function. The corresponding relationship between the two inputs of vehicle speed and ground adhesion coefficient controlled by the fuzzy rules and the output prediction time domain is shown in Figure 2.

Tab 1. Fuzzy control rules

N_p		ν						
		NB	NM	NS	ZO	PS	PM	PB
	NB	NB	NB	NB	NM	ZO	PM	PB
	NM	NB	NB	NB	NM	NS	PS	PM
	NS	NB	NB	NB	NM	NS	ZO	PS
μ	ZO	NB	NB	NB	NM	NS	ZO	ZO
	PS	NB	NB	NB	NB	NM	NM	NS
	PM	NB	NB	NB	NB	NB	NM	NM
	PB	NB	NB	NB	NB	NB	NM	NM

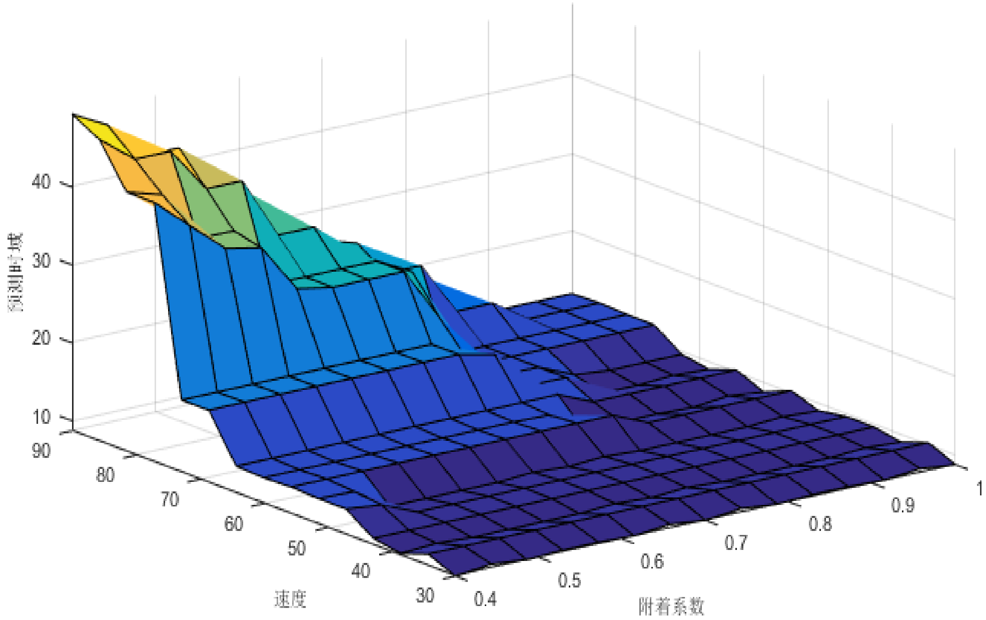

Figure 2. Relationship between adhesion coefficient vehicle speed and control time domain

Figure 3 is an adaptive MPC controller based on a fuzzy control algorithm. The initial state and future reference trajectory of the vehicle are input to the MPC controller. The MPC controller calculates the generated control quantity and transmits it to the CarSim vehicle model for trajectory tracking. CarSim outputs two parameters of vehicle speed and ground attachment coefficient to the fuzzy controller, The fuzzy controller generates the corresponding prediction time domain through the fuzzy control rules and then inputs it into the MPC controller for cyclic calculation, to control the vehicle to complete the trajectory tracking process.

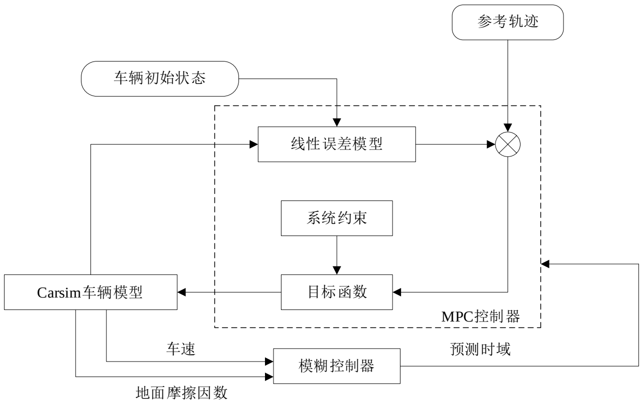

Figure 3. Adaptive MPC controller based on fuzzy control

4. Simulation analysis

To verify the effectiveness of the adaptive model controller proposed in this paper, a joint simulation is constructed based on Simulink and CarSim. The proposed algorithm is simulated and analyzed at different speeds on the dry road with an adhesion coefficient of 0.85 and the wet road with an adhesion coefficient of 0.4 respectively, and the double moving track with high frequency is selected to simulate the model [20].

The vehicle parameters in the simulation model are shown in Table 2.

Tab 2. Vehicle parameters

Vehicle parameters	Value
mass/kg	1723
vehicle length/m	4
vehicle width/m	1.9
front tire cornering stiffness/$N \cdot rad^{-1}$	66900
rear tire cornering stiffness/$N \cdot rad^{-1}$	62700
distance from front wheel to centroid/m	1.232
distance from rear wheel to centroid/m	1.468
moment of inertia about Z-axis/$kg \cdot m^2$	4175

The parameters of the MPC controller are shown in Table 3.

Tab 3. Controller parameters

Controller parameters	Value
sampling period/s	0.02
prediction time domain	20
control time domain	1
Q	$\begin{bmatrix} 1000 & 0 \\ 0 & 30 \end{bmatrix}$
R	10
ρ	1000

4.1. Track tracking of different roads at 30km / h

As shown in Figure 4 and Figure 5, when the low-speed running speed of the vehicle is 30km / h, the traditional controller and adaptive controller can track the planned trajectory well on both dry and wet roads, and the trajectory tracking effect is good. Compared with the traditional controller, the adaptive controller can effectively reduce the lateral error and heading angle error of trajectory tracking, and the trajectory tracking accuracy is higher. It can be seen from Figure 6 and Figure 7 that when the vehicle is running at low speed, the yaw rate of the vehicle is very small in the track tracking process of the two controllers; The front wheel deflection angle is in a small range, and the variation range is relatively gentle, indicating that the vehicle stability is good at this time.

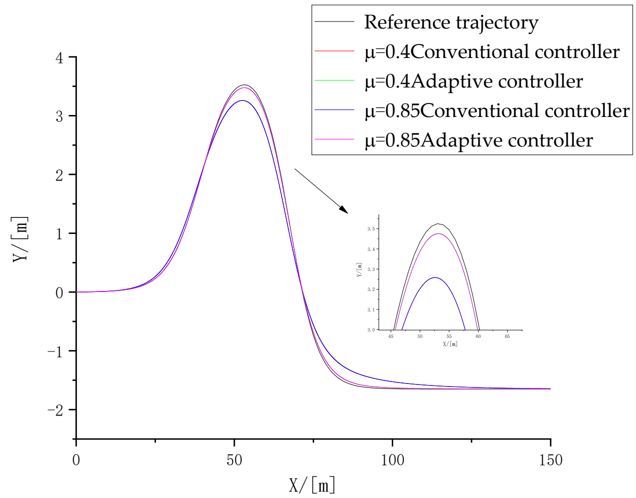

Figure 4. Lateral position

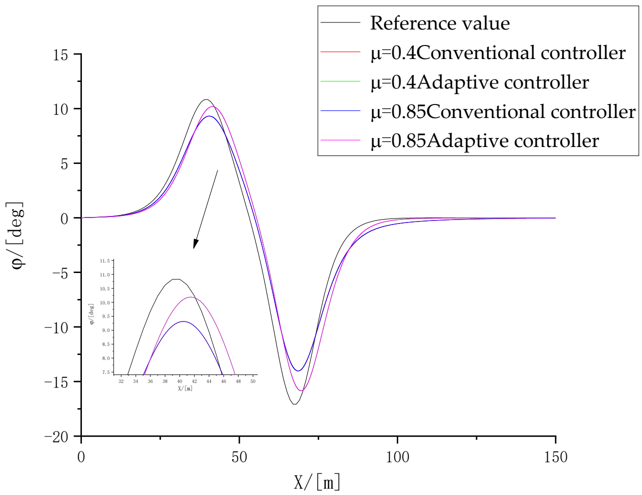

Figure 5. Heading angle

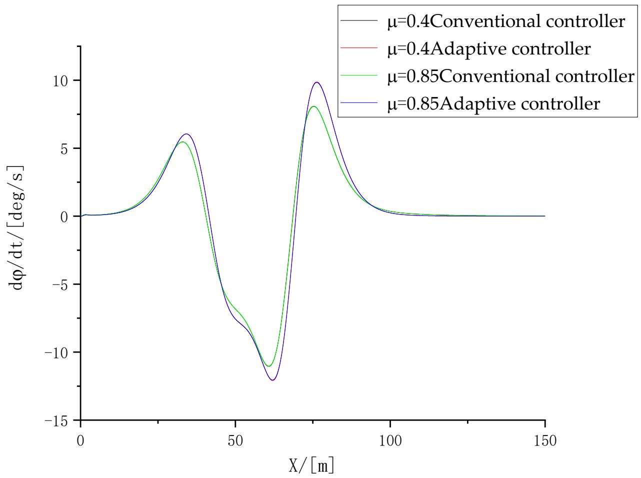

Figure6. Yaw rate

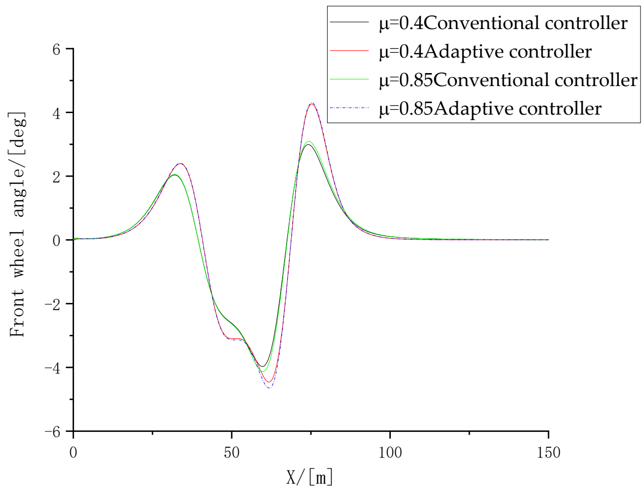

Figure7. Front-wheel angle

4.2. Track tracking of different roads at 60km / h

As shown in Figure 8 and Figure 9, when the medium speed driving speed of the vehicle is 60km / h, due to the increase of speed, the traditional controller has certain errors in the transverse position and heading angle of track tracking on both dry and wet roads. Compared with the traditional controller, the tracking errors of transverse position and heading angle of the adaptive controller is significantly reduced, which can effectively track the expected track, Improve trajectory tracking accuracy. It can be seen from Figure 10 and Figure 11 that at this time, the vehicle yaw rate in the track tracking process of the two controllers is not offset. At 70m, the vehicle is in the position where the track direction changes rapidly, and the yaw rate changes rapidly, but there is no offset; The front wheel deflection angle of the vehicle almost reaches the constraint critical value at 70m, but they are all within the constraint range, and finally return to 0deg, indicating that the vehicle is still in a stable state.

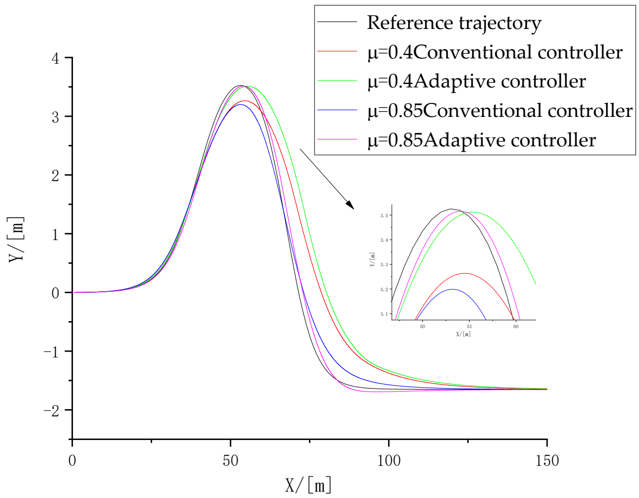

Figure 8. Lateral position

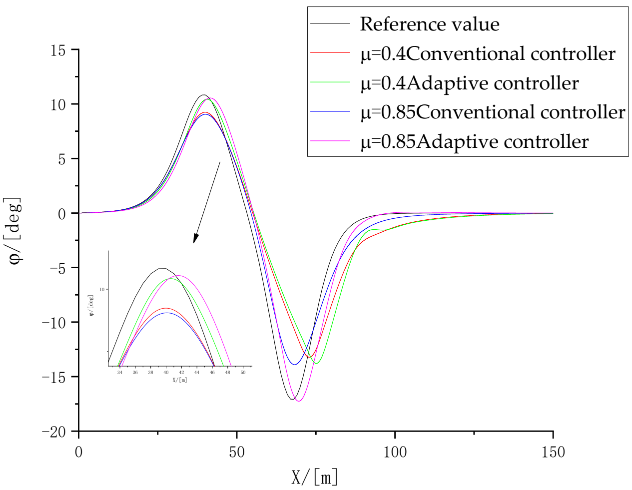

Figure 9. Heading angle

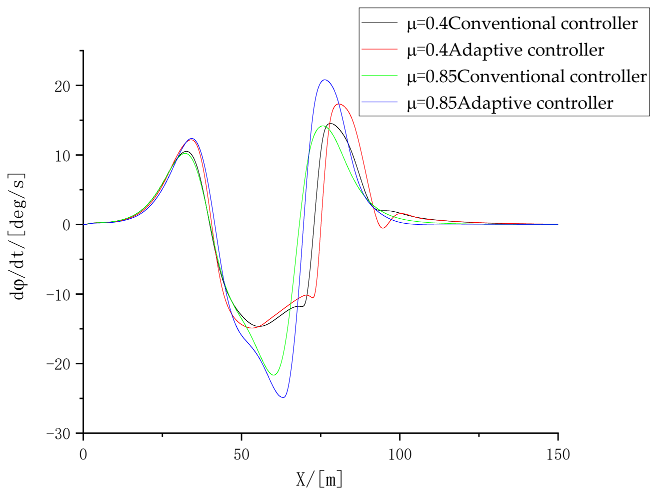

Figure10. Yaw rate

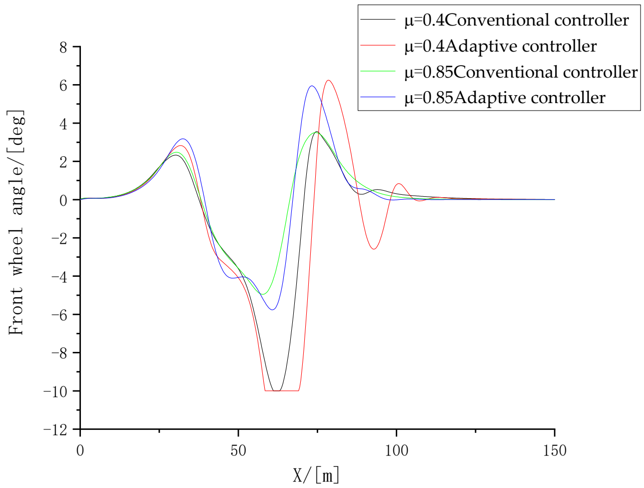

Figure11. Front-wheel angle

4.3. Track tracking of different roads at 90km / h

As shown in Figure 12, Figure 13, Figure 14, and Figure 15, when the vehicle's high-speed driving speed is 90km / h and the driving road is the dry road, both the traditional controller and the adaptive controller can still track the track. Compared with the traditional controller, the tracking transverse error and heading angle error of the adaptive controller are smaller, indicating that the adaptive controller has a good tracking effect on a dry road, It is helpful to improve the trajectory tracking accuracy; At this time, the vehicle is running at a high speed, so more consideration should be given to the vehicle driving stability. There is no imbalance in the change of vehicle yaw rate, and the steering wheel angle is also within the constraint range. At this time, the vehicle stability is good; When the driving road is wet and slippery, the traditional controller can not track the track, the vehicle yaw rate is maladjusted and can not converge, the steering wheel angle has always been at the critical value of the constraint range and changes violently, and finally can not return to 0DEG, the vehicle is out of control and is in danger, while the adaptive controller can effectively adjust the vehicle attitude, At this time, the yaw rate of the vehicle is not maladjusted, the steering wheel angle is within the constraint range and the change is relatively gentle. It can still effectively track the planned trajectory, effectively avoid the vehicle being out of control, and the vehicle has good stability.

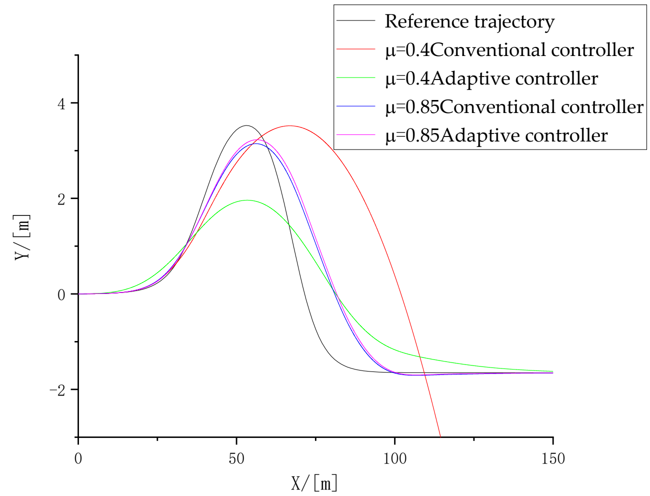

Figure 12. Lateral position

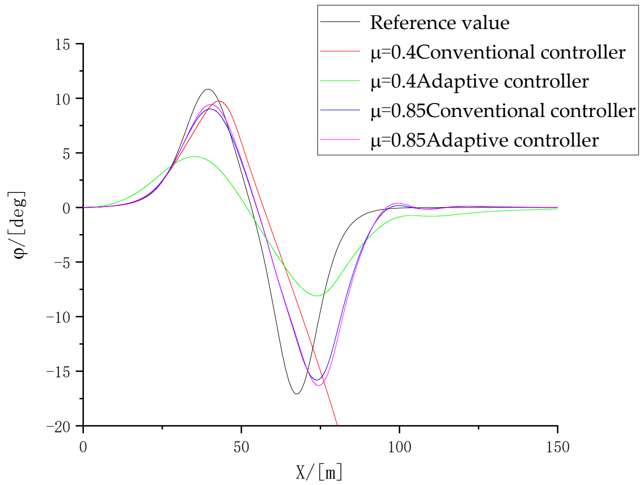

Figure 13. Heading angle

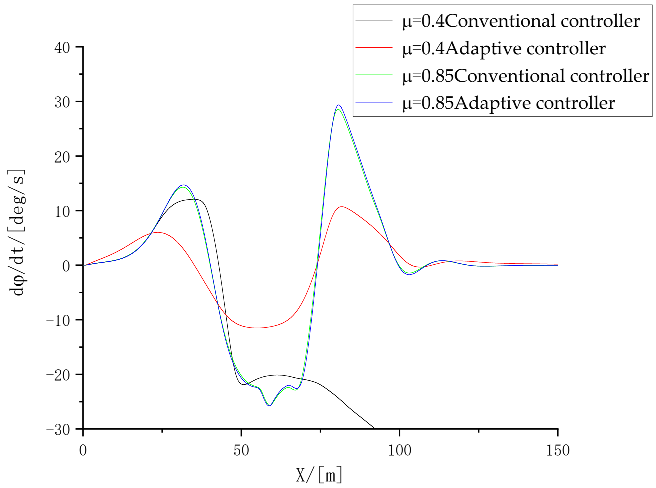

Figure14. Yaw rate

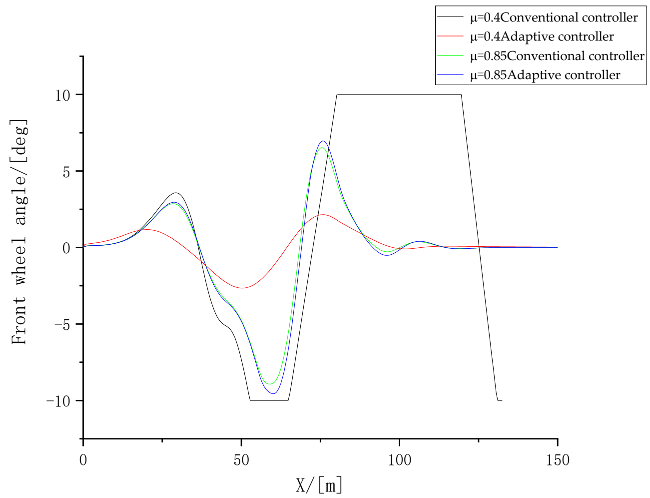

Figure15. Front-wheel angle

5. Conclusions

To solve the problem that the traditional controller has poor adaptability to the trajectory tracking of roads with different adhesion coefficients, a new adaptive trajectory tracking control algorithm is obtained by integrating fuzzy control and traditional model predictive control algorithm. The vehicle speed and ground adhesion coefficient are input into the fuzzy controller, the Gaussian membership function is used to formulate the fuzzy rules, and the prediction time domain of the best matching system is output and transmitted to the model prediction controller to complete the trajectory tracking. The joint simulation results of Simulink/CarSim show that the algorithm can improve the adaptability of the controller to different roads, reduce the trajectory tracking error and improve the trajectory tracking accuracy. When driving on wet and slippery roads, the vehicle state can be adjusted in time to avoid the vehicle being out of control and improve the driving stability and stability of the vehicle. It has certain practical value.

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and editing, Y.J.Guo.; funding acquisition, J.P.Xin. All authors have read and agreed to the published version of the manuscript.
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