Analysis of Instability of Solutions of Nonlinear Dissipative Equations

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Abstract: In order to solve the difficult and unstable problems of solving nonlinear dissipation equations, this paper studies some important nonlinear dissipation equations and related problems in the soliton theory based on the idea of mathematical mechanization. This paper introduces the historical development and current status of the theory of solving nonlinear dissipation equations and its mathematical mechanization. Calculated the initial value problem of the nonlinear dissipation equation, analyzed the integrability and nonlinear eigenvalue problem of the nonlinear dissipation equation, solved the constructiveness of the nonlinear evolution equation system, and finally analyzed the instability of its solution . Through research, apply mathematical methods to related disciplines and become a good mathematical calculation tool for solving practical complex problems.

Key words: nonlinear dissipative equation; instability; soliton theory; initial value; integrability study; Galerk in method

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1 Introduction

According to the principle of complementarily, the dissipation mechanics is established, which is a kind of dissipation theory corresponding to quantum mechanics. This theory can be used to uniformly deal with non-equilibrium thermodynamics and viscous fluid dynamics, and it can be used to deal with various dissipative and irreversible problems in quantum mechanics [1-3]. With the study of solitons and chaos in modern physics and mathematics, a large number of nonlinear evolution equations with nonlinear dispersion or dissipation have emerged continuously, including equations with soliton solutions, nonlinear Schrodinger equations, and sine- Gordon equation, Sakharov equation, Ginzburg-Landau equation, busennesque equation, etc. [4-6]. On the one hand, nonlinear development equations are closely connected with physical problems, chemical reaction problems, biological population problems, fluid dynamics fluctuation problems, etc., and can successfully describe a large number of fluctuation phenomena in nature, and are widely used in various branches. Such as elementary particles, fluid physics, plasma physics, condensed matter physics, superconducting physics, laser physics, biophysics, statistical physics, etc. [7]. On the other hand, the study of evolution equations is closely related to other mathematical fields, such as classical analysis, Lie groups, Lie algebras, infinite dimensional algebras, algebraic geometry, topology, dynamical systems, computational mathematics and functional analysis, and promote each other [8]. The most common and basic method to study nonlinear equations is variable transformation. After transformation, the equation will be relatively simple and even may become linear equation. Sometimes, the variable transformed equation may become one or more equations. Whether these equations are solvable or not, it can help to understand the behavior of the system.

In recent years, the research of infinite-dimensional dynamical systems has attracted widespread attention, but most of the work deals with the dynamic behavior of various types of equations under homogeneous boundary conditions, and for non-homogeneous, especially nonlinear boundary value problems. The research work is not perfect, and the dissipative properties of nonlinear equations under boundary conditions should be discussed. Exact solutions of nonlinear dissipative equations may lead to dissipative structures. Therefore, it is necessary to develop a new mechanical system called "dissipative mechanics". A more complete name should be "dissipative quantum mechanics", but the name "dissipative mechanics" can better reflect the individuality of the problem [9]. The theory of dissipation mechanics established in this article is a preliminary attempt to introduce thermodynamic factors into quantum dynamics to solve the problems of actual dissipation and irreversibility. There is a class of mathematical and physical equations in nonlinear dissipative equations, which are usually used to describe the process of evolution over time, called evolution equations, also called evolution equations or evolution equations such as heat conduction equations, acoustic and elastic wave equations, reaction diffusion and convection-diffusion equations, fluid and gas mechanics equations, equations in infinite-dimensional dynamic systems, such as generalized equations, all belong to the category of evolution equations, belong to the research of exact solutions of evolution equations, and are closely related to development and soliton theory .

In this paper, the basic nonlinear dissipative equations, especially the basic equations of non-equilibrium thermodynamics and viscous fluid dynamics, are unified into the integrability conditions of the basic equations of dissipative mechanics. The instability of the solutions of the nonlinear dissipative equations is analyzed and their exact solutions are obtained.

2 Research and development status of methods for seeking exact solutions of nonlinear dissipative equations

In the development of science and technology, various methods for solving nonlinear dissipation equations have been proposed, such as using plane vector field theory to solve them. However, the solution of nonlinear dissipative equations is much more difficult than that of linear differential equations, because some of the basic properties of the latter do not affect the former. Again, it is difficult to deal with the former in a unified way [10, 11]. In most cases, the solution of nonlinear equations can only rely on numerical solutions, but the numerical solution cannot include the global characteristics of the infinite situation represented by the original equation solution. Therefore, the work of solving nonlinear dissipative equations has shown very important theoretical and application value [12]. Since the equations were proposed and their exact solutions were obtained, the construction of solutions for a large number of nonlinear dissipative equations has aroused people's great interest. After the continuous efforts of mathematicians and physicists, it has been discovered that there is a hidden element in the soliton theory. A series of effective methods for constructing accurate solutions, such as backscattering method, transformation method, homogeneous balance method, singularity analysis method, etc. With the emergence of various solving methods, not only the equations that were difficult to solve in the past have been solved, but also new and important solutions with important physical significance have been continuously discovered and applied, and there has been an endless momentum [13].

From the perspective of physical symmetry, people have directly deduced quantum electrodynamics and quantum field theory from non-relativistic quantum mechanics, but they have not done anything about irreversible problems with dissipation, such as macroscopic non-equilibrium thermodynamics or fluid thermodynamics. Similar deduction. Therefore, it is physically necessary to develop the theory of dissipation mechanics.

3 Calculation of initial value problem of nonlinear dissipative equation

Galerkin method, also known as the Reitz averaging method, is a common numerical analysis method in modern mathematics, which is currently widely used in the engineering calculation of mathematics and physics.Using this method, we should select finite polynomial functions, and then require the result in the domain and the edge to satisfy the original equation, thus yielding a set of linear algebraic equations. Use the Galeridn method to study the existence of solutions to a class of equations with dissipative boundaries. The formula is:

$$\begin{aligned}&(u(t),w(t))+(\Delta u,\Delta w)-d(t-i)\Delta u(t)\\&=d(t-i)\Delta u(t)(u(k)+d(k)-u(\Delta u,w)\end{aligned} \tag{1}$$

In the process of exploring the solution of the equation, it will be affected by the boundary term. Therefore, it cannot be solved by a special basis. In order to avoid this problem, assume that the initial value is:

$$a(s,0)=a_0=0 \tag{2}$$

The basic idea of finding a numerical solution is to replace an infinite-dimensional Sobolev space with a finite-dimensional space to meet the variational requirements.

According to the existence of the solution with the dissipative boundary equation, a nonlinear dissipative equation can be established, which can reasonably define the nonlinear dissipative equation in the sense of the classical solution [14, 15], according to what the function belongs to select the value interval, select the initial value problem of the nonlinear dissipation type equation, and realize the calculation of the initial value problem of the differential equation through the initial value calculation formula.

The nonlinear dissipative equation is as follows:

$$\begin{cases}[r(i)x(i)]+f(i,x(i-\gamma))=0 & i\ge i_0, i\ne i_k, k\in N\\ x^i(i_k^t)=g_k^t(x^t(i_k)) & i_k+\tau\in C(x^t), t=0,1,2,k\in N\\ x(i)=\varphi(i) & i\in[i_0-\tau,i_0],\varphi\in C^2([i_0-\tau,i_0])\end{cases} \tag{3}$$

Where, $C(x^t)$ represents the set of all continuous points in the domain of the function $x(i)$; N represents the set of positive integers; C represents the entire real number; $k\in N$ represents the definite number; γ is a positive number; $r(i)$ is continuous on $[i_0,+\infty)$, and $r(i)>0$; $f(i,x)$ is continuous on $[i_0,+\infty)$, and $f(i,x)>0(x\ne 0)$; $i_0<i_1<\text{L}<i_k$, $i_{k+1}-i_k>\gamma$, $k=0,1,2,\text{L}$, $x^0(i)=x(i)$.

Define $x^t(i_k^-)$, $x^t(i_k^+)$ and $x^t(i_k)$ as $x^t(i_k^-)=\lim_{i\to i_k^-}x^t(i)$, $x^t(i_k^+)=\lim_{i\to i_k^+}x^t(i)$ and $x^t(i_k)=x^t(i_k^-)$ respectively, then $C(x^t)$ is the set of all continuous points in the domain of function i .

Suppose $x(i):[i_0-\gamma,+\infty)\to R$, and $i_0\ge 0$ is called the overall solution of the current equation. If the current equation satisfies the condition:

(1) $x(i)=\varphi(i)$ $i\in[i_0-\gamma,i_0]$;

(2) $i\in[i_0,+\infty)$ $i\ne i_k, i_k+\gamma(k\in N)$;

(3) $x(i)$ meets $[r(i)x^2(i)]' + f(i, x(i-\gamma)) = 0$;

(4) $x^t(i)$ and $r(i)x^2(i)$ are continuous on $[i_0, +\infty)$, continue left at $i = i_k$, and $x^t(i_k)$ satisfy $x^t(i_k^+) = g_k^t(x^t(i_k))$, $k \in N$.

By observing it, it can be found that the above method avoids the absolute continuity concept of other methods [16], and the value range of the overall solution can be preliminarily determined.

Assuming that $r(i)$ is continuous on $[i_0, +\infty)$, and $f(i, x) > 0 (x \neq 0)$, the initial value calculation method of $x(i)$ is:

$$\begin{cases} (r(i)x^2(i)) + f(i, x(i-\gamma)) = 0 & i \geq a \\ x(i) = \psi(i) & i \in [a-\gamma, a], \psi \in C^2([a-\gamma, a], R) \end{cases} \tag{4}$$

If any $a \geq i_0$ in formula (4), then the initial value problem of $\psi \in C^2([a-\gamma, a], R)$ will have at least one solution to $[a-\gamma, +\infty)$, When $\varphi \in C^2([i_0-\gamma, i_0], R)$ is the initial condition, formula (3) will have at least one solution on $[i_0-\gamma, +\infty)$.

When $x_0(i)$ is the initial value problem, the calculation formula of the initial value problem will be changed. The changed initial value calculation formula is as follows:

$$\begin{cases} (r(i)x_0^2(i)) + f(i, x_0(i-\gamma)) = 0 & i \geq i_0 \\ x_0(i) = \varphi(i) & i \in [i_0-\gamma, i_0] \end{cases} \tag{5}$$

When a solution on $[i_0-\gamma, +\infty)$ satisfies condition $\varphi^t(i_0) = x_0^t$, let $x_1(i)$ be the initial value problem. At this time, the initial value calculation method is as follows:

$$\begin{cases} (x_0^2(i) - \frac{1}{r(i)} r(i_1) x_0^2(i_1) + \frac{1}{r(i)} r(i_1) g_1^t(x_0^2(i_1)) \\ x_1^t(i_1^+) = g_1^t((x_0^t(i_1)) \end{cases} \tag{6}$$

The result of formula (6) belongs to $(t_1, t_1+\tau]$. Further suppose that $y_1(i)$ is the initial value problem, and the initial value calculation method is as follows:

$$\begin{cases} (r(i)y_1^t(i)) + f(i, y_1(i-\tau)) = 0 & i \geq i_1+\gamma \\ y_1(i) = x_1(i) & i \in (i_1, i_1+\gamma], y_1(i_1) = x_1(i_1^+) \end{cases} \tag{7}$$

The result of formula (7) is a solution on $(i_1, +\infty]$, and a function is defined as follows:

$$z_1(i) = \begin{cases} x_0(i) & i \in [i_0-\gamma, i_0] \\ x_1(i) & i \in (i_1, i_1+\gamma] \\ y_1(i) & i \in (i_1+\gamma, i_2] \end{cases} \tag{8}$$

In the formula, $z_1(i)$ satisfies formula (3) in interval $[i_0-\gamma, i_2]$. If $i \in [i_0-\gamma, i_1]$, then $z_1(i) = x_0(i)$, from formula (5), it can be concluded that $z_1(i)$ also satisfies formula (3) in interval $[i_0-\gamma, i_1]$. If $i \in (i_1, i_1+\gamma]$, then

$z_1(i) = x_1(i)$. Then can get $z_1^t(i_1^+) = g_1^t(z_1^t(i_1))$, and $i \in (i_1, i_1 + \gamma]$ at that time, the calculation formula is:

$$r(i)z_1^t(i) - r(i)z_1^t(i_1^+) = r(i)x_1^t(i) - r(i_1)g_1^t(x_0^t + (i_1)) \quad (9)$$

From the result of formula (9), can get:

$$(r(i)z_1^t(i)) = -f(i, z_1(i-\gamma)) \quad i \in (i_1, i_1 + \lambda) \quad (10)$$

If $i \in (i_1 + \gamma, i_2]$, then $z_1(i) = y_1(i)$. When $i \in (i_1 + \lambda, i_2]$, the calculation formula is:

$$r(i)z_1^t(i) - r(i)z_1^t(i_1 + \gamma) = r(i)y_1^t(i) - r(i_1)y_1^t(i_1 + \gamma) \quad (11)$$

According to the definition of $z_1(i)$, can see that $z_1(i)$ is continuous on $i_1 + \gamma$. From formula (9) and formula (11), it can be concluded that $z_1^t(i)$ is also continuous on $i_1 + \gamma$. Calculating formula (11) can be obtained when $i \in (i_1 + \gamma, i_2)$, $(r(i)z_1^t(i))$ is:

$$(r(i)z_1^t(i)) = f(i, z_1(i-\lambda)) \quad i \in (i_1 + \gamma, i_2). \quad (12)$$

Using formulas (1)-(12), the initial value of the nonlinear second-order neutral differential equation can be calculated.

The dissipative boundary is selected for the boundary of the equation, and appropriate assumptions are made on it. The classical Galeridn method is used to prove the existence of the solution. First, the infinite-dimensional space is replaced by a finite-dimensional space to obtain a sequence of convergence. It is possible to transform the partial differential problem and obtain the equation solution according to the ordinary differential equation theory. Then it is estimated to get the initial value solution.

4 Integrability of nonlinear dissipative equations and nonlinear eigen value problems

The research of integrable systems is one of the mainstream of contemporary nonlinear science, and it is still the direction of people's efforts to find more integrable systems [17, 18]. Whether a system is integrable can be reflected by its mathematical structure and is closely related to the existence of solitons. As for integrability, there is no definite and unified definition so far. In the process of exploration and research on integrable systems, some basic framework theories have gradually emerged: such as Liouvill theory, Lax theory, Burchnall-Chaundy-Krichever theory. Among them, the general theoretical framework of Liouvill's complete integrability is one of the basic starting points of modern integrability theory, and its core is the finite-dimensional Hamilton integrable system [19, 20].

For the study of integrable systems, finding the explicit solution of the soliton equation is always a very important task. In fact, seeking an explicit solution of the soliton equation not only helps to further understand the essential properties and algebraic structure of the soliton equation in theory, but also can reasonably explain related natural phenomena in application [21]. Algebraic geometric solutions are also called quasi-periodic solutions or finite-band potential solutions. They are an important display solution of nonlinear dissipative equations. The nonlinear method is extended to find algebraic geometric solutions of nonlinear soliton equations [22].

Nonlinear eigenvalue problem (NLEVP) is a hot research topic in computational science, including polynomial eigenvalues, symmetric nonlinear eigenvalues and nonlinear eigenvalue location，etc. The usefulness of its matrix-valued function will be limited by the difficulty of solving the problem, accompanied by some unique problems. For example, its eigenvalues are usually not It forms the basis of the multi-dimensional vector space, and the specific form of the matrix-valued function will affect the efficiency of the solution method. However, the nonlinear eigenvalue problem has the same core problem as the linear eigenvalue problem: an effective solution method when the value is small may not be well extended to high-dimensional problems. This type of problem usually focuses on finding a small number of specific "eigenvalue-eigenvector" pairs with important physical significance.

Most methods for solving such high-dimensional problems need to generate an initial guess close enough to the expected feature pair to ensure convergence. At the same time, it is necessary to use a method that can calculate continuous feature pairs with close feature values, but does not converge to the same feature pair repeatedly. Especially when a large number of feature pairs are expected, these problems are more difficult to solve. The NLEVP algorithm further increases the difficulty of calculating a large number of feature pairs, resulting in not well adapted to the eigenvalue problem solved in the subspace. The amount of calculation required to solve a nonlinear eigenvalue problem is at least equal to that of solving a single linear eigenvalue problem. Therefore, it is important to keep the dimensionality small to ensure that it can accommodate larger problems.

Given a non-empty open set and a matrix-valued function, consider a nonlinear eigenvalue problem: find a scalar and a non-zero vector such that:

$$T(\lambda)x = 0 \tag{13}$$

This article only discusses the nonlinear correlation between T and λ characteristic parameters, without considering the nonlinear correlation between eigenvectors and multi-parameters. Similar to the linear case, assuming that ρ is the eigenvalue, x and y correspond to the right and left eigenvectors of T, respectively. The set of all eigenvalues ρ of T is called the "spectrum" of T and denoted as $\rho(T)$, that is:

$$\rho(T) := \{\rho \in (T(\rho)) = 0\} \tag{14}$$

Call (ρ, x, y) the feature triple of T, and both (ρ, x) or (ρ, y) are called feature pairs. For $T(\rho) = \rho I - A$ and $T(\rho) = \rho B - A$, they are simplified into standard eigenvalue problem $Ax = \rho x$ and generalized eigenvalue problem $Ax = \rho Bx$ respectively. The properties of generalized nonlinear eigenvalue problems are quite different from linear eigenvalue problems. For example, even if T is a regular value, there may be infinite eigenvalues. The eigenvectors belonging to different eigenvalues are not necessarily linearly independent, and the algebraic multiplicity of an isolated eigenvalue is limited, but it is not limited by the size of the problem. All these characteristics make it very difficult to solve the generalized nonlinear eigenvalue problem.

For simplicity, this paper focuses on the eigenvalue problem of Polynomials: finding scalars and nonzero vectors such that:

$$P(\rho)x = 0,\ y \times P(\rho) = 0 \tag{15}$$

Where, $P(\rho)$ is a matrix polynomial of $n \times n$

$$P(\rho) = \sum_{i=0}^{k} \rho^i A_i \tag{16}$$

If P is regular, then there are r finite eigenvalues and $kn - r$ infinite eigenvalues (complex eigenvalues). Through linearization, can know that the k-th $n \times n$ matrix polynomial $\rho(\lambda)$ has kn eigenvalues (finite or infinite), maximum kn right correlation eigenvectors, and maximum kn left correlation eigenvectors.

5 Constructive solution of nonlinear dissipative equation

In the field of nonlinear science, mathematical and physics researchers often apply nonlinear dissipation equations because it can accurately describe the corresponding physical phenomena. In order to explore the nature of the problem, it is particularly important to conduct structural research on the mathematical model used to describe its phenomenon. The development of ancient mathematics has always been characterized by construction, focusing on methods and practicality. This kind of structural mathematics research often produces new mathematical ideas, methods and theories, and it also directly promotes other disciplines [23]. However, due to the complexity of nonlinear dissipative equations, a large number of existing equations cannot be solved accurately. Even if it can be solved, a lot of skills are needed, and there is no unified method. Moreover, a new solution with physical meaning needs to be further constructed and discovered. Fortunately. After the continuous efforts of mathematicians and physicists. It is found that there are a series of effective methods for constructing accurate solutions in soliton theory, such as transformation, two transformation, similarity reduction, backscattering method, bilinear method, variable separation method, first integration method, function expansion method, etc. With the emergence of various solving methods, not only the equations that were difficult to solve in the past have been solved, but also new and important solutions with important physical significance have been discovered and applied, and there has been an endless stream of momentum [24].

The hyperdiscretization method has been widely used in solving nonlinear dissipative equations. The core of this method lies in a limit formula, that is:

$$\lim_{\varphi \to 0} \varphi \log(v^{\frac{A}{\varphi}} + v^{\frac{B}{\varphi}}) = \max\left[A, B\right] \tag{17}$$

When, $\varphi \to 0$ it can be known that the addition, multiplication, and division operations of the original variable are replaced by the addition and subtraction operations under the new variable.

The hyperdiscretization method can be used not only to obtain soliton solutions of equations, but also to obtain some special types of solutions. The functional solution of the nonlinear dissipative equation is obtained by hyperdiscreteization.

6 Exact solutions of nonlinear dissipative equations

It is an important and difficult task to construct the analytic solution and the transformation between the nonlinear dissipative equations. For a long time, many mathematicians and physicists have done a lot of work in this field and put forward many solving methods. However, there are still many important and practical nonlinear dissipative equations which can not get their explicit analytical solutions or very few solutions. Even if some solutions have been found, there are different approaches to different equations, and there is no unified model. The content of this section introduces a model for solving a large class of nonlinear dissipative equations, and uses this model to give a constructive method for solving the transformation between two equations. It is an idea for solving problems. It emphasizes transformation and simplification, and the central idea is to turn difficult problems into easy-to-solve problems [25]. Based on this idea, a class of operators is studied, and more forms of solutions of a class of equations are given, and this equation is used as an auxiliary equation to extend a class of auxiliary equation expansion method.

The basic mode of mathematics mechanization is: one is to solve equations by mechanization; second, to prove theorems by solving equations[26]. Based on the idea of mathematical mechanization, the "AC=BD" model for solving differential equations is proposed, the unified theory and formula for solving nonlinear dissipative equations are given, and the important concept of "appropriate solution" is put forward, which combines the stress in elastic mechanics in the past. The construction method of the function displacement function is connected with other field theory problems, and a unified method for finding an appropriate solution is obtained, and it is extended to the generalized linear operator equations theory[27].

7 Instability analysis of solutions of nonlinear dissipative equations

Set in equation (1,1)

(1) The roots $\theta_1(i)$, $\theta_2(i)$, and $\theta_3(i)$ of the characteristic equation (1, 4) satisfy

$\mu_1 < \theta_2(i) < \mu_2$, $-\mu_4 < \theta_1(i) < \mu_3$

Where, μ_i $(i=1,2,3,4,5)$ is the normal number of i matter element, and $\mu_1 < \mu_2 < \mu_3 < \mu_4 < \mu_5$;

(2) $a(i)+c(i)+b(i)c(i) < 0$, It holds for all $i \geqslant i_0$.

Then the zero solution of equation (1,1) is unstable.

Proof: the stability of the equivalent system (1,2) of equation (1,1).

Condition 1:

$$\theta_1(i)\theta_2(i)\theta_3(i) = -|\theta_1(i)||\theta_2(i)||\theta_3(i)| < \mu_3\mu_1\mu_5 < 0 \tag{18}$$

$$\theta_1(i)+\theta_2(i) < -(\mu_3+\mu_1) < 0 \tag{19}$$

$$\theta_3(i)+\theta_1(i) = \mu_3 - \mu_2 > 0 \tag{20}$$

From formulas (18)-(20), there are:

$$\begin{aligned}\Delta(i) &= |\theta_1(i)\theta_2(i)\theta_3(i)||\theta_1(i)+\theta_2(i)||\theta_2(i)+\theta_3(i)| \\ &> \mu_1\mu_3\mu_5(\mu_3-\mu_1)\end{aligned} \tag{21}$$

According to formula (21), take:

$$\begin{aligned}D(x_1,x_2,x_3) &= D_{11}(i)x_1 + D_{12}(i)x_1x_2 + D_{13}(i)x_1x_3 \\ &+ D_{23}(i)x_2x_3 + D_{33}(i)x_1x_2x_3\end{aligned} \tag{22}$$

According to condition 1, it can be proved that $D(x_1,x_2,x_3)$ has an infinitesimal upper bound, then when x_3 ≠0:

$$D(i_3,0,0,x_3) = D_{13}(i)x_{23} > 0 \tag{23}$$

Function $D(x_1,x_2,x_3)$ can take a positive value when the value of x_i is arbitrarily small.

Considering equation (1,2) again, similar to Theorem 1:

$$\begin{aligned}\Delta(i)' &= (x_1^2+x_2^2+x_3^2) - \rho\Delta(i)(x_1^2+x_2^2+x_3^2) \\ &- \mu\Delta(i)(x_1^2+x_2^2+x_3^2) \\ &= \Delta(i)(x_1^2+x_2^2+x_3^2)(2-\mu-\theta) \\ &= (2-\mu-\theta)\theta_1\theta_3\theta_5(x_1^2+x_2^2+x_3^2)\end{aligned} \tag{24}$$

Therefore, the solution of equation (1,2) is positive definite.

According to the instability theorem of unsteady motion, it can be known that the zero solution of equation (1,2) is unstable, and the zero solution of equation (1,1) is unstable.

8 Main conclusions

The basic concept of the nonlinear dissipative equation in Menger PN space is: define M , M^+ , and O as the set of real numbers, the set of non-negative real numbers and the set of positive integers, respectively. Function $f: M \to M^+$ is a distribution function. When the function is non-minus left continuous and meets the following requirements: $\inf_{i\in M} f(i)=0$, $\sup_{i\in M} f(i)=1$, and M are the set of distribution functions, then set the distribution function as shown in formula (25):

$$C(i)=\begin{cases}1, i>0\\0, i\le 0\end{cases} \tag{25}$$

Prerequisite knowledge

Suppose the ordered triple (E,F,Δ) is set, E represents the real linear space, and F is the mapping from E to M. The definition f_x meets the following requirements:

(1) $f_x(0)=0, \forall x\in E$;

(2) $f_x(i)=H(i), \forall i\in B$, Only true when $x=\varepsilon$;

(3)Random $x,y\in E$, $i_1,i_2\in M^+$, $f_{x+y}(i_1+i_2)\ge \Delta\left(f_x(i_1), f_y(i_2)\right)$ exists.

Define (E,F,Δ) as Menger PN space, $i-$ norm Δ meets the following requirements: $\Delta(i,i')\ge i$, $\forall i\in[0,1]$; M represents an open subset of E. Define $T:\bar{M}\to E$ to mean tightly continuous operator, $W=N-T$ and $u\in E-\gamma M$ at the same time. Previous studies defined the Leray-Schauder degree for W, denoted by $Deg(W,M,u)$, and $Deg(W,M,u)$ has many properties, as detailed below:

(1) $Deg(N,M,u)=1, \forall u\in M$;

(2)When $C(i,x)$ is a continuous compact operator in $[0,1]\times\bar{M}$ and satisfies the conditions of $u\notin\gamma\left(N-C(i)\right)$ and $\forall i\in[0,1]$, then $Deg\left(N-C(i),M,u\right)$ and $i\in[0,1]$ have no relationship;

(3)Define that there are open subsets M_1 and M_2 in M, and they are disjoint, then there is $Deg(W,M,u)=Deg(W,M_1,u)+Deg(W,M_2,u)$;

(4)Define M_0 to represent the open subset of M, then $Deg(W,M,u)=Deg(W,M_0,u)$;

(5)Define $u\notin W(\gamma M)$, then $Deg(W,M,u)=Deg(W-u,M,\varepsilon)$.

Lemma 1

Define (E,F,Δ) to represent Menger PN space, and the probability distribution function f is the lower semi-continuous, namely random $i\in B$. If $n\to\infty$ exists $q_n\to q$ and $p_n\to p$, then:

$$\lim_{n\to\infty} f_{p_n,q_n}(i) = f_{p,q}(i) \tag{26}$$

Lemma 2

In this paper, define the existence of M open subsets in infinite dimensional Menger PN space (E,F,Δ). the random $i\in[0,1]$, $i-$ norms Δ meet the following requirements: $T:M\to E$ is a continuous compact operator, which meets two requirements:

(1) $\varepsilon \notin \overline{T(\gamma M)}$;

(2)Random $\eta\in[0,1]$, $x\in\gamma M$, there is $Tx\neq\eta x$.

Therefore, $Deg(N-T,M,\varepsilon)=0$ 。

Lemma 3

It is defined that there are open subsets M, $\forall i\in[0,1]$ in Menger PN space (E,F,Δ). Compact continuous operator of $T:\bar{M}\to E$, When T meets the following requirements:

$$(Z_3)\ f_{Tx}(t)\geq f_{Jx+u}(t) \tag{27}$$

Then the nonlinear dissipative equation has a solution in $\bar{M}$.

Prove

If $Tx=Jx+u$ does not have a solution in γM, because $J\neq 0$, defines $T_1=\frac{1}{J}T$, $T_2=\theta$ (zero operator), it is obvious that T_1 and T_2 are compact continuous operators, then there exists:

$$f_{Tx}(|J|t) = f_{x_1}(i) = f_{x_2}(i) = f_{x_3}(i) \tag{28}$$

$$f_{Jx+u}(|J|t) = f_{x_1}(i) = f_{x_2}(i) = f_{x_3}(i) \tag{29}$$

Because $Tx=Jx+u$ has no solution in γM, so $Tx\neq Jx+u$, $\forall x\in\gamma M$, the instability of the solution of nonlinear dissipative equation is established.

According to the above calculation, it can be seen that the different values of the three parameters x_1,x_2,x_3 of the functional solution of the nonlinear dissipative equation have great influence on the properties of the solution. According to the values of the three parameters, the stable form of the functional solution of the nonlinear dissipative equation can be modified by using the mechanical modification method. Through a large number of computer numerical simulations of various types of functional solutions, it is found that:

If the seed solution chooses the function solution, the solution will be unstable; The variable coefficient function of the solution has an effect on the long-time behavior of the principal function. If the variable coefficient function of the solution is not selected properly, the properties of the main function will be submerged and the influence will be called qualitative. The stability of traveling wave solution and non traveling wave solution obtained by constant coefficient transformation is better than that by variable coefficient transformation. The stability of the traveling wave solution obtained by constant coefficient transformation is better than that of the non traveling wave solution.

9 Conclusion

Adjoint differential equation is widely used in practice. Because of its important physical background and mathematical model, more and more people are interested in the nonlinear dissipative differential equation. By calculating the initial value problem of nonlinear dissipative equation, the integrability research and algebraic geometric solution of nonlinear dissipative equation are analyzed. The constructibility of nonlinear evolution equation system is solved, and the instability of its solution is analyzed. According to the obtained calculation results, it provides a calculation tool for the research of mathematics and other disciplines. Due to the large amount of calculation in this paper, further refinement is needed in the future to reduce the amount of calculation and improve the accuracy of operation.

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