Cross Sections for Scattering of Semiclassical Plasma Particles

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Abstract

One of the most fundamental processes in plasma physics is binary-particle interaction, of which the etectron-etectron interaction is the most well-known example. The collision has attracted more attention due to its superior energy-exchange mechanism and numerous significant applications, such as plasma heating and acceleration, in addition to theoretical considerations. We offer a theoretical estimate of the scattering cross sections of collisions in quantum plasmas that are screening in this review paper. In particular, using a newly proposed potential, we concentrate on the electron-electron interactions for semiclassical plasma. Lastly, the behaviors of electron scattering cross sections as a function of various factors are shown.

Keywords: scattering cross section, effective Debye length, quantum effect.

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1. Introduction

In astronomical settings such as planetary interiors, white dwarfs, magnetars, and pulsars, among others, quantum plasmas are widely present [1, 2, 3]. They are also important in applications such as those pertaining to quantum wells [4,5], plasmonics [6], spintronics [7], and ultra-cold plasmas [8,9]. In solids, especially metals, where conduction electrons can be thought of as a mobile plasma neutralized by background ions, quantum plasma effects are also significant [10]. When the average inter-particle distance is less than the de Broglie length λt, which denotes the spatial extension of the particle's wavefunction, quantum degeneracy effects become substantial.

Consequently, it is no longer possible to think of the particle as point like, as in classical plasma, and quantum interference of overlapping particle wave functions must be taken into consideration [10]. The equilibrium distribution function switches from a Maxwell-Boltzmann to a Fermi-Dirac distribution when the Fermi temperature rises above the thermal temperature [11, 12, 13]. In dense plasma, quantum effects can take many different forms, such as quantum statistics. It may be required to adopt a quantum exchange and correlation potential in the modeling due to spin effects, pressure may be dominating (exceeding the thermal pressure), and quantum wave diffraction or tunneling (modeled via a Bohm potential) [14–16].

It is crucial to keep in mind that the standard representation for screening the charge electric field in plasma is the static screening of the Debye. This approach is effective if the colliding particle velocities are close to the thermal velocity. Particles that travel faster than the thermal velocity fail to polarize the surrounding plasma, which reduces screening. These days, dynamic screening-so called since it was dependent on the speed of the particles striking it- is frequently used to look into the properties of non-ideal plasmas.

We have computed a basic scattering cross section for binary interaction in plasma in this paper. Initially, we proposed a novel effective potential that combines the screening and quantum effects. The accounting approach for dynamic screening basically involves replacing the static Debye length with an effective length related to dynamic screening.The scattering cross section of electrons is established according to our new potential. Finally, we have treated a variation of both potential and cross section with main parameters.

2-The interactionpotential of the semiclassical plasma particles

It is convenient to employ the dimensionless parameters defining the plasma state at specificdensities and temperatures for the description of the non-ideal plasma features.

a) The coupling parameter is one of the parameters.

$$\Gamma_{\alpha\beta} = \frac{Z_\alpha Z_\beta e^2}{a K_B T} \tag{1}$$

In this instance, the mean separation between particles is

$$a = \left(\frac{3}{4\pi n}\right)^{1/3} \tag{2}$$

Here, T is the plasma temperature, KB is the Boltzmann's constant, and n=ne+niis the numerical density of electrons and ions.

The density parameter is calculated as :

$$r_s = \frac{a}{a_B} \tag{3}$$

In this case, the Bohr radius is$a_B = \frac{\hbar^2}{m_e e^2}$As the density of the plasma increases, the density parameter decreases.

It is important to remember that the static screening of the Debye is the typical representation for screening of the charge electric field in plasma. If the particle velocities that collide are near the thermal velocity, then this method works.

Particles moving faster than the thermal velocity decrease screening because they are unable to effectively polarize the surrounding plasma. Dynamic screening, so named because it depended on the speed of the hitting particles, is currently widely employed to investigate the characteristics of non-ideal plasmas. The accounting approach for dynamic screening was introduced in [17], and it essentially consists of substituting an effective length associated with dynamic screening for the static Debye length [18].

$$r_0 = r_D \left(1 + \frac{v^2}{v_{Th}^2}\right)^{1/2}$$

(4)

The relative velocity of the colliding particles is denoted by v, whereas the thermal velocity is denoted byv_{Th}

Instead of using the Coulomb potential to obtain more interactions, we take into account the change in quantum effect when the electron is scattered in the vicinity of the emitting ion. This interaction is known as the Deutsch interaction; as it took into account the developments of the quantum effect in plasma and the influence of the quantum effect at short distances between the emitting ion and the colliding electron. Deutsch latency is given by the following expression [19]:

$$V_{Deu} = \frac{k_e Z_\alpha Z_\beta e^2}{r}\left(1 - \exp\left(-\frac{r}{\lambda_t}\right)\right)$$

(5)

In our studies, we are interested in adding both the quantum and collective effects of the plasma. Under this condition, we suggest a new effective interaction that can be translated into the following expression:

$$\frac{1}{\frac{1}{\lambda_D}+\frac{1}{\lambda_t}} = \lambda_t \qquad (6)$$

To simplify the calculation, we use an approximation

$$V_{eff} = \frac{k_e Z_\alpha Z_\beta e^2}{r}\left(1 - \exp\left(-\frac{r}{\lambda_t}\right)\right)\exp\left(-\frac{r}{\lambda_D}\right)$$

(7)

which is valid when

$$\lambda_t < \lambda_D \qquad (8)$$

So we find the new effective potential

$$V_{eff} = \frac{k_e Z_\alpha Z_\beta e^2}{r}\left(\exp\left(-\frac{r}{\lambda_D}\right) - \exp\left(-\frac{r}{\lambda_t}\right)\right)$$

(9)

Then V_{eff} for electron-electron interaction, which takes into accountdynamic screening, is written as

$$V_{eff} = \frac{k_e e^2}{r}\left(\exp\left(-\frac{r}{\lambda_D(1+\delta^2)^{\frac{1}{2}}}\right) - \exp\left(-\frac{r}{\lambda_t}\right)\right)$$

(10)

where:

$$\delta^2 = \frac{v^2}{v_{Th}^2} = \frac{\Gamma_{ee}}{r_s}k^2 a^2$$

(11)

and the wavenumber

$$k = \frac{mv}{\hbar} \qquad (12)$$

3-Scattering cross sections of the electrons

In completely ionized plasma, all collisions result from the interaction between charged particles. In order to treat the interaction of electron-electron collisions, we consider that: The collision is considered a binary (electron-electron) [14]. This condition is met for a fully ionized plasma with weak density. The effect of plasma on an electron-electron collision is represented by the force involved in this collision, which is the Coulomb force. We assume that all electron-electron interactions are effective in the absence of the internal magnetic field (neglected in front of the electric field). This model is an image of a set of electron-electron binary collisions for a detailed study of the collisions.

The formula of the scattering cross section can be defined as [20]

$$\sigma(\theta) = \left| -\frac{m}{2\pi\hbar^2} \int V(r) \exp(-i\boldsymbol{q}.\boldsymbol{r})\, d\boldsymbol{r} \right|^2$$

(13)

Using the formula (10), the scattering cross section of electronscan be calculated with dynamic screeningby

$$\sigma(\theta) = \left| -\frac{k_e m e^2}{2\pi\hbar^2} \int \frac{1}{r} \left(\exp\left(-\frac{r}{\lambda_D (1+\delta^2)^{\frac{1}{2}}} \right) - \exp\left(-\frac{r}{\lambda_t} \right) \right) \exp(-i\boldsymbol{q}.\boldsymbol{r})\, d\boldsymbol{r} \right|^2$$

(14)

The differential element of the distance vector between the electrons (which is the volume element) is:

$$d\boldsymbol{r} = r^2 sin\theta d\theta dr d\varphi$$

(15)

The scalar product between the wave difference qand the distancer is:

$$\boldsymbol{q}.\boldsymbol{r} = qrcos\theta$$

(16)

Where

$$q = 2ksin\frac{\theta}{2} \tag{17}$$

Using the expressions (13), (14), we get:

$$\iiint \frac{1}{r} \left(\exp\left(-\frac{r}{\lambda_D (1+\delta^2)^{\frac{1}{2}}} \right) - \exp\left(-\frac{r}{\lambda_t} \right) \right) \exp(-iqrcos\theta)\, r^2 sin\theta d\theta dr d\varphi$$

(18)

We perform the previous integral over the angle θ by changing the following variable:

$$x = cos\theta \tag{19}$$

We get the following integration result:

$$\int_0^\infty \exp(-iqrcos\theta)\, sin\theta d\theta = -\frac{1}{iqr} (\exp(iqr) - \exp(-iqr))$$

(20)

We determine the our integral by replacing the expression (16) with (18):

$$2\pi \int_0^\infty r\left(\exp\left(-\frac{r}{\lambda_D(1+\delta^2)^{\frac{1}{2}}}\right)-\exp\left(-\frac{r}{\lambda_t}\right)\right)\left(-\frac{1}{iqr}(\exp(iqr)-\exp(-iqr))\right)dr$$

(21)

This last integral leads to four integrals whose solutions are, respectively:

$$\int_0^\infty exp\left(r(iq-\frac{1}{\lambda_D(1+\delta^2)^{\frac{1}{2}}}\right)=-\frac{1}{iq-\frac{1}{\lambda_D(1+\delta^2)^{\frac{1}{2}}}}$$

(22)

$$\int_0^\infty exp\left(r(-iq-\frac{1}{\lambda_D(1+\delta^2)^{\frac{1}{2}}}\right)=\frac{1}{iq+\frac{1}{\lambda_D(1+\delta^2)^{\frac{1}{2}}}}$$

(23)

$$\int_0^\infty exp\left(r(iq-\frac{1}{\lambda_t}\right)=-\frac{1}{iq-\frac{1}{\lambda_t}}$$

(24)

$$\int_0^\infty exp\left(r(-iq-\frac{1}{\lambda_t}\right)=\frac{1}{iq+\frac{1}{\lambda_t}}$$

(25)

Combining the following formulas (20), (21), (22), and (23), we can deduce the integral result:

$$2\pi \int_0^\infty r\left(\exp\left(-\frac{r}{\lambda_D(1+\delta^2)^{\frac{1}{2}}}\right)-\exp\left(-\frac{r}{\lambda_t}\right)\right)\left(-\frac{1}{iqr}(\exp(iqr)-\exp(-iqr))\right)dr$$

$$=4\pi\left(\frac{1}{q^2+\frac{1}{\lambda_t^2}}-\frac{1}{q^2+\frac{1}{(1+\delta^2)\lambda_D^2}}\right)$$

(26)

After we obtain the final result of the integration, we can write the scattering cross section of an electron-electron collision with dynamic screening for our new potential:

$$\sigma(n,T,\delta)=\left|-\frac{2k_e me^2}{\hbar^2}\left(\frac{1}{q^2+\frac{1}{\lambda_t^2}}-\frac{1}{q^2+\frac{1}{\left(1+\frac{\Gamma ee}{r_s}k^2a^2\right)\lambda_D^2}}\right)\right|^2$$

(27)

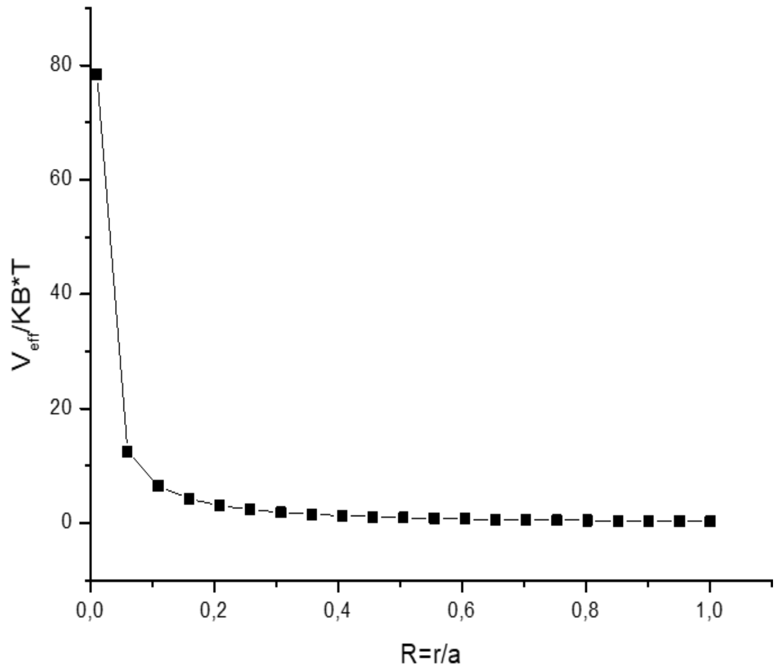

Figure (1): Our effective potentials$\frac{V_{eff}}{K_B T}$of electron-electronpair for $\boldsymbol{\delta}$=1, R_s =10, and$\boldsymbol{\Gamma} = \mathbf{1}$

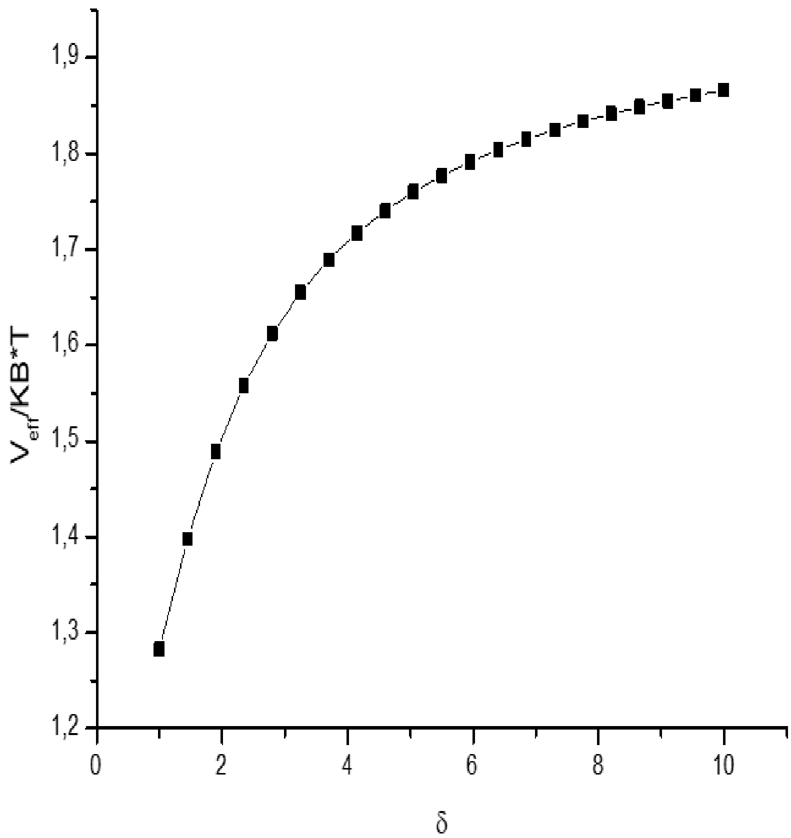

Figure (2): Our effective potentials$\frac{V_{eff}}{K_B T}$of electron-electronpair for R=0.4, R_s=10, and **Γ**=1.

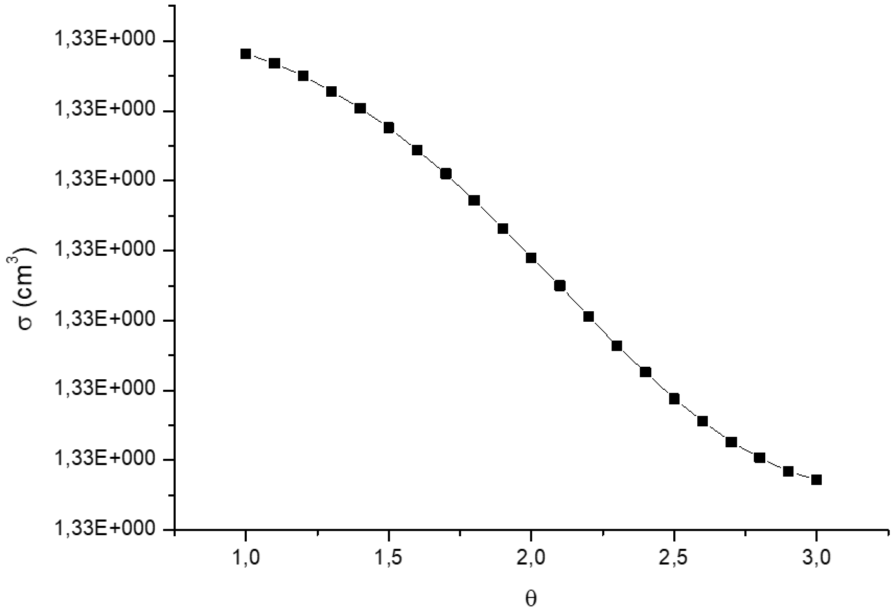

Figure (3):Scattering cross sectionσ of electron-electron pair for k=1, R_s =10, and Γ=2.

The pair interaction potential between electrons is shown in Figures 1 and 2, taking into account both the dynamic and static filtering in potential (10). These numbers seem to indicate that while the potential (10) increases with δ, it decreases with increasing distance (r).

The electron scattering cross section for various wave vector values, as determined by equation (27) is shown in Figure 3. The cross section derived from potential (10) has a finite value at the scattering angle of zero, as demonstrated in Figure 3, because screening in model (10) is independent of the projectile's velocity (momentum).

4-Conclusion

Comprehending the transport characteristics of charged particles holds significance not just from an essential perspective but also because of its applicability in numerous fields. A theoretical model is established in this article to model the interaction of free electron-electron in a semiclassical plasma by employing a scattering cross section calculated according to a suggested effective potential. This new potential takes into account the quantum of the screening effect. We have presented the variations of a screening quantum scattering cross section of electrons as functions of some parameters in plasma. We have found that this section increases with an increase in the fraction of velocity.

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