# Design of Microstrip Filters by Particle Swarm Optimization Algorithm

Dr. Hathat ahmed 1

<sup>1</sup> University of Ziane Achour, Djelfa, Algeria

Email: hathat.ahmed@gmail.com

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#### Abstract

In this paper , we present our contribution in design of microstrip low-pass and band-pass filter by particle swarm optimization (PSO ) algorithm. After the design of these filter by classical methods Butterworth and Chebyshev approximation for low-pass and band-pass respectively to determine the characteristics of these filters , we adapted PSO algorithm to low-pass and band-pass filter design problem . The proposed method for design microstrip filters presents better results compared to classical methods in terms of losses in the pass-band and stop-band .

**Keywords**: low-pass filter, band-pass filter, particle swarm optimization, Butterworth approximation, Chebyshev approximation.

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### 1-Introduction

Filters play an important role in many applications in the microwave field. They are used to separate or combine different frequencies. Emerging applications in the microwaves field continue to challenge filters with stricter requirements, higher performance, smaller size, lighter weight and lower cost [1].

According to the requirements and specification, microwave filters can be designed as circuits to localized or distributed elements the can be designed in various transmission line structures such as waveguide, coaxial line and the microstrip [1-2]

In application where the transport high power signals is not an essential element, the use of planar technology is the solution to address congestion problem and weight volumetric structures [3].

Planar filters are very attractive on two points but they are also on the implementation costs which are lower their good reproducibility and their interconnection facilities with other circuits including the active circuits [4].

One of the planar technology, microstrip technology is widely used in microwave systems, the advantages of this technology is insensitive to manufacturing tolerances, a wide range of bandwidth and easy design process.

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In recent years the research is focused on new methods for optimized design of microwave devices [5-8]. The particle swarms optimization belongs to the family of artificial intelligence algorithms that shows greater reliability in solving optimization problems. This is a method using a population of agents called particles but compared to other algorithms of the same family it has some interesting features including the notion that efficiency is due to the collaboration rather than competition [9].

In this research paper we proposed a procedure for design low-pass and band-pass microstrip filter using the particle swarm optimization algorithms

# 2- Theory of microstrip filter

According to the position of the bandwidth and attenuated bands in the frequency response, filters can be classified into four categories: low-pass, high-pass, band-pass, and band-stop filters [1].

The specifications of a filter are generally provided based on required specifications, which indicates the type of filter and the electrical characteristics (centre frequency, bandwidth, rejection level in the passband, insertion loss). Filters are typically composed of capacitors and inductors, as indicated in Figure 1.

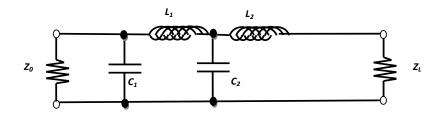


Fig.1 Localized element filter

These localized element filters (C and L) cannot be directly implemented in the microwave domain because the component values are low, and the availability of localized elements is very limited in this domain [4]. Therefore, it is necessary to transform these elements into distributed elements (transmission lines, cavities, etc.) in order to realize microwave filters.

To transform localized elements into distributed elements, there are two tools, namely the Richards transformation and the Kuroda identities [1].

Today, there are several technologies for the physical implementation of microwave filters, each of which will have characteristics in terms of complexity, cost, and electrical performance, making its use particularly suitable for specific applications.

Microstrip technology currently occupies a privileged position in the design of passive microwave circuits such as filters. It is easy to design resonators with interesting performances with reduced dimensions by playing on the dimensions of the lines. The geometry of a microstrip line is shown in Figure 2. It consists of a metal strip located on the upper surface of a dielectric substrate, with the ground plane located on the lower surface.

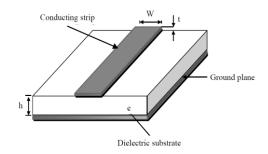


Fig.2: The geometry of a microstrip line

The main parameters that characterize the microstrip structure are the permittivity  $\varepsilon_r$ , and the geometric parameters W, t, and h.

Usually the first step in design of filters is to choice the transfer function that approximates the required specification, the most common transfer functions are Butterworth, Chebyshev, elliptic, quasi-elliptic, and generalized Chebyshev [1].

# 3- Particle Swarm Optimization algorithm

The Particle Swarm Optimization PSO is a member to the family of artificial intelligence algorithms that shows greater reliability in solving optimization problems. This method uses a population of agents named particles but compared to other algorithms of the same family; it has some interesting features including the notion that efficiency is due to the collaboration rather than competition. In the PSO, each particle is a candidate solution in the search space [9-10].

The algorithm is generally randomly initialized and the particles are placed randomly in the search space of the objective function. PSO converges successfully to a global optimum.

The main concept of the PSO is that possible solutions are accelerated towards the best solutions. The particles estimate iteratively the objective function of the candidate solution and remember where they had their best value of the objective function. At each iteration, the particles move, taking into account their best position but also the best position for their neighbours. The goal is to change their path so that they approach as close as possible to the optimum.

#### Steps of PSO algorithm

**Step 1**: we initialize the swarm of particles in the search space. This can be done either randomly or regularly, especially on the boundary.

**Step 2**: we also initialize the velocities randomly.

Step 3: We define the neighbourhood for each particle. There are two main methods;

Either a "geographical" neighbourhood, which must be recalculated at each time step and assumes the existence of a distance in the search space, or a "social" neighbourhood defined once and for all. The social neighbourhood is the most commonly used for several reasons:

- ✓ it is simpler to program,
- ✓ it is less computationally expensive.

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Anyway, in case of convergence, a social neighbourhood tends to become a geographical neighbourhood.

**Step 4**: Evaluation of all particles and finding the best particle in the group and the best particles in the iterations.

Step 5: Modification of the position of each particle by introducing the following equations:

$$\begin{cases} {v_i}^{(t+1)} = w{v_i}^{(t)} + c_1 rand{\left( {p_i} - {x_i}^{(t)} \right)} + c_2 rand{\left( {p_g} - {x_i}^{(t)} \right)} \\ {x_i}^{(t+1)} = {x_i}^{(t)} + {v_i}^{(t+1)} \end{cases}$$

(1)

Such as:

- $v_i^{(t+1)}$ : speed of particle *i*, at iteration t+1
- w: the inertial coefficient,
- c1 : weighting factor (importance of personal best)
- c2: weighting factor (importance of neighbourhood best)
- rand : random number between 0 and 1
- $x_i^{(t)}$ : the position of particle i, at iteration t,
- $p_i$ : best position of the particle i,
- $p_g$ : best position in the group

The inertial weight is defined as follows:

$$w = w_{\text{max}} - \frac{w_{\text{max}} - w_{\text{min}}}{iter_{\text{max}}} iter$$

(2)

- w<sub>max</sub>: maximum inertia weight
- W<sub>min</sub>: minimum inertia weight
- iter<sub>max</sub>: maximum number of iteration
- Iter: current iteration

Step 6: stopping criteria; if the stop criterion checked so close, if not go to step 4.

The basic PSO algorithm has several parameters that the user must define a priori [11]:

- ✓ The size of the swarm (the number of particles)
- ✓ Defining neighbourhoods

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- ✓ Weighting factors
- ✓ Defining the boundaries of the search space
- ✓ Termination criteria for the algorithm (either the objective function or the number of iterations)

#### 4- Problem formulation

# 4.1 Design of the low-pass microstrip filter

The first application considered in this work is the design of a third-order stepped impedance microstrip low-pass filter, the desired characteristics of this filter are:

- Cut-off frequency = 1 GHz
- Attenuation in the passband = 0.1 dB
- Load impedance  $Z_0$  = source impedance  $Z_L$  = 50 ohms.

We used the Butterworth approximation to synthesize the parameters of the desired filter in the first step, then in a second step, PSO is applied to this filter to obtain an optimal filter structure. The synthesized prototype filter is illustrated in Figure 3 with the normalized electrical elements and the layout of this i  $g_{1}=1.0316$  vn in Figure 4  $g_{2}=1.0316$ 

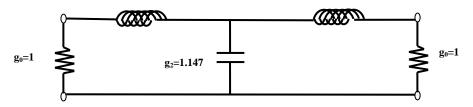
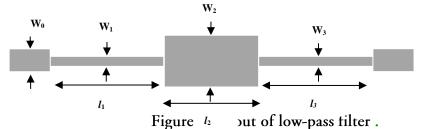


Figure 3. low-pass prototype filter.



The stepped impedance technique utilizes high impedances for inductors and low impedances for capacitors. The electrical lengths for the inductors and capacitors are determined as follows [1]:

$$\beta_l l_l = g_k \frac{Z_{low}}{Z_0}$$

(3)

$$\beta_c l_c = g_k \frac{Z_0}{Z_{low}}$$

(4)

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Where  $Z_0$  is the characteristic impedance of the filter,  $Z_{low}$  low impedance, where  $Z_{high}$  is the high impedance and  $g_k$  is the value of the elements of the prototype low-pass filter obtained using the desired filter approximation methods. For the design of the filter presented in Figure 4, we choose  $Z_{high}$ =93 Ohms,  $Z_{low}$ =24 Ohms and a substrate of height h=1.27 mm and relative dielectric constant  $\varepsilon_r$ =10.8. Using the equations of microstrip line, we find:  $W_l$ =0.193 mm,  $W_c$ =3.936 mm, and using the equations (3) and (4), we find:  $U_l$ =10.309 mm, et  $U_c$ =9.2253 mm.

The layout of this filter is shown in Figure 5.

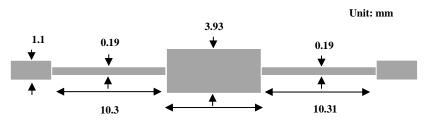
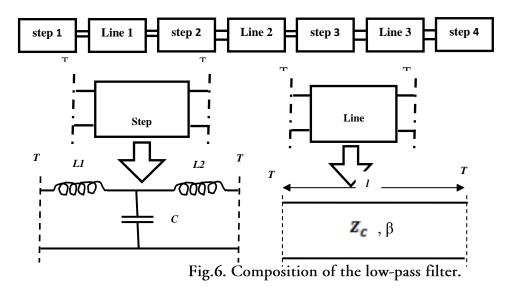


Fig. 5. layout of the low-pass filter sylling dusing the Butterworth technique.

The filter shown in Figure 5 consists of subnetworks: a step discontinuity and a transmission line as shown in Figure 6 [1]:



The ABCD matrices for the step discontinuity and the transmission line are given by:

$$\begin{bmatrix} 1 - \omega^2 C L_1 & (j\omega L_1 + j\omega L_2) - j\omega^3 C L_1 L_2 \\ j\omega C & 1 - \omega^2 C L_1 \end{bmatrix}$$
(5)

$$\begin{bmatrix} \cos\theta & jZc\sin\theta \\ j\sin\theta/Zc & \cos\theta \end{bmatrix}$$

(6)

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where :  $\theta$ = $\beta l$ 

(7)

$$C = 0.00137 h \frac{\sqrt{\varepsilon_{rel}}}{Z_{c1}} \left( 1 - \frac{W_2}{W_1} \right) \left( \frac{\varepsilon_{rel} + 0.3}{\varepsilon_{rel} - 0.258} \right) \left( \frac{\frac{W_1}{h} + 0.264}{\frac{W_1}{h} + 0.8} \right)$$
 (pf

(8)

$$L_1 = \frac{L_{W1}}{L_{W1} + L_{W2}} L$$

(9)

$$L_2 = \frac{L_{W2}}{L_{W1} + L_{W2}} L$$

(10)

$$L_{Wi} = Z_{ci} \sqrt{\varepsilon_{rei}/c} \qquad i=1,2$$

(11)

$$L = 0.000987 h \left( 1 - \frac{Z_{c1}}{Z_{c2}} \sqrt{\frac{\varepsilon_{re1}}{\varepsilon_{re2}}} \right)^2$$
 (nH)

(12)

The ABCD matrix of this filter is calculated as follows:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \prod_{i=1}^{n} \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{i}$$

(13)

And the transmission coefficient  $S_{21}$  is given by:

$$S_{2,1} = \frac{2}{A + B/Z_0 + CZ_0 + D}$$

(14)

According to  $S_{21}$ , the synthesis of the filter can be considered as a function of  $Z_{0C}$ ,  $Z_{0L}$ ,  $\theta_L'$  et  $\theta_C'$ .

where

$$\theta' = \theta / f = \frac{2\pi}{300} \sqrt{\varepsilon_{re}} l$$

(15)

# 4.2 Design of the band-pass microstrip filter

The second application is a third-order microstrip bandpass filter with half-wavelength resonators coupled in parallel, as depicted in Figure 7. The desired characteristics of this filter are:

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- Center frequency = 4.1GHz
- Attenuation in the passband = 0.1 dB
- Fractional bandwidth = 48.7% (Bandpass = [3.1-5.1] GHz)
- Load impedance  $Z_0$  = source impedance  $Z_L$  = 50 ohms.

For simplicity, we assume here that all coupled lines are identical. The filter is subdivided into a cascade of subnetworks, as illustrated in Figure 8. The calculation of the ABCD matrices for the subnetworks of the discontinuity steps is similar to those discussed earlier. The ABCD parameters for each subnetwork of the coupled line can be calculated using [1]:

(16) 
$$A = D = \frac{Z_{0e} \cot \theta_e + Z_{0o} \cot \theta_o}{Z_{0e} \csc \theta_e - Z_{0o} \csc \theta_o}$$

$$B = \frac{j}{2} \frac{Z_{0e}^2 + Z_{0o}^2 - 2Z_{0e} Z_{0o} \left(\cot \theta_e \cot \theta_o + \csc \theta_e \csc \theta_o\right)}{Z_{0e} \csc \theta_e - Z_{0o} \csc \theta_o}$$

$$C = \frac{2j}{Z_{0e} \csc \theta_e - Z_{0o} \csc \theta_o}$$
(18)

where Z0e and Z0o are the characteristic impedances of the even and odd modes.  $\Theta$ e and  $\theta$ o are the electrical lengths of the two modes.

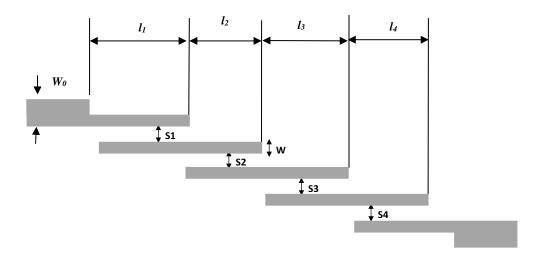


Fig.7. Layout of the microwave bandpass filter.

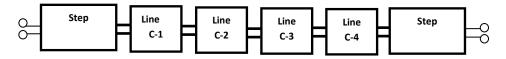


Fig. 8. Structure of the microwave bandpass filter.

The ABCD matrix of this filter is calculated as follows:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \prod_{i=1}^{n} \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{i}$$

(19)

And the transmission coefficient S2,1 is calculated as follows:

$$S_{2,1} = \frac{2}{A + B/Z_0 + CZ_0 + D}$$

(20)

Let's assume that the dielectric is homogeneous,  $\theta = \theta = \theta$ , then the ABCD matrices of the subnetworks of the coupled line can be rewritten as follows:

$$A = D = \frac{\left(Z_{0e} + Z_{0o}\right)\cot\theta}{\left(Z_{0e} - Z_{0o}\right)\csc\theta}$$

(21)

$$B = \frac{j}{2} \frac{Z_{0e}^2 + Z_{0o}^2 - 2Z_{0e}Z_{0o}(\cot^2\theta + \csc^2\theta)}{(Z_{0e} - Z_{0o})\csc\theta}$$

(22)

$$C = \frac{2j}{(Z_{0e} - Z_{0o})\csc\theta}$$

(23)

According to S21, the synthesis of the bandpass filter can be considered as a function of Z0e, Z0o, and  $\theta$ '. Such that:

$$\theta' = \theta / f = \frac{2\pi}{300} \sqrt{\varepsilon_{re}} l$$

(24)

# 4.3 Proposed method for design of microstrip low-pass filters by PSO

The following points describe the adaptation of the particle swarm optimization algorithm for synthesizing a microstrip low-pass filter:

1. The population is the global particles, and each particle is characterized by coordinates in the search space. In our program, the population is the all filters, and each filter is characterized by impedances and electrical lengths. Therefore, the coordinates in our case are the values of impedances and electrical lengths. In the third application, the filter is defined as follows:

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Filter = 
$$(Z_{0L}, Z_{0C}, Z_{0L}, \theta'_{L}, \theta'_{c}, \theta'_{L})$$

And with the symmetry of the filter, we can define the filter as follows: Filter ==  $(Z_{0L}, Z_{0C}, \theta'_{L}, \theta'_{c})$ 

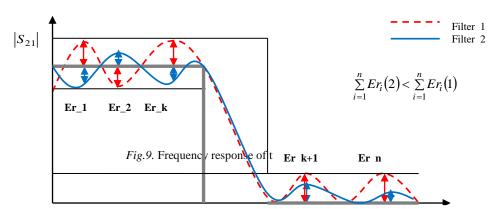
2- The search space in this application is defined as follows:

$$80 \le Z_{0L} \le 150 \text{ (Ohm)}$$
,  $15 \le Z_{0C} \le 45 \text{ (Ohm)}$ ,  $0.2 \le \theta'_L \le 0.7 \text{(rad)}$  et  $0.2 \le \theta'_C \le 0.7 \text{ (rad)}$ .

3- The evaluation function we used is the sum of errors function, which is evaluated after the calculation of  $S_{21}$ . This is calculated by the following expression:  $\sum_{i=1}^{n} Er_i$ 

$$Er_i = \begin{cases} \left| S_{2,1} - 1 \right| & f_i \le f_c \\ \left| S_{2,1} \right| & f_i > f_c \end{cases}$$

(25)



We notice that filter 2 is the better s f n because it show low ripples in f passband and the stopband.

4. Selection of  $P_g$  and  $P_{best}$ :  $P_g$  is the best solution in all iteration for filter i when  $P_{best}$  is the best filter

between two successive iterations.

# 4.4. Proposed method for design microstrip bandpass filters by PSO

Same design procedure as presented in the first application, with the following changes:

1. The filter is defined as follows:

Filtre = 
$$(Z0e, Z0o, \theta')$$

2. The search space in this application is defined as follows:

$$80 \le Z0e \le 200$$
 (Ohms),  $20 \le Z0o \le 80$  (Ohms), and  $0.2 \le \theta' \le 0.7$  (radians).

3. The objective function we used is the error function, which is evaluated after the calculation of  $S_{21}$  and the expression:  $\sum_{i=1}^{n} Er_i$ 

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$$Er_{i} = \begin{cases} \begin{vmatrix} S_{2,1} \\ |S_{2,1} - 1 \end{vmatrix} & f < f_{p1} \\ |S_{2,1} \end{vmatrix}$$

$$f < f_{p1} \le f \le f_{p2}$$

$$f > f_{p2}$$

(26)

4. Selection of  $P_g$  and  $P_{best}$ :  $P_g$  is the best solution in all iteration for filter i when  $P_{best}$  is the best filter

between two successive iterations.

#### 5-Résultat and discussion

## 5.1. Result of low-pass filter

The study of the effect of the number of iterations is very useful for verifying the convergence of the algorithm and for determining the necessary number of iterations in the algorithm in order to achieve minimal error

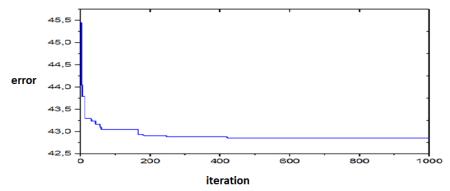
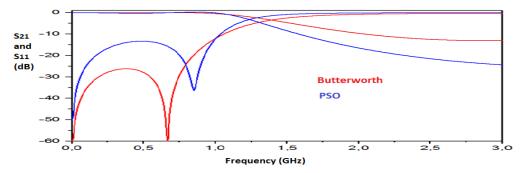


Fig. 10. The effect of the number of iterations on the minimum error

The PSO algorithm converges to the minimum error each time the number of iterations increases up to 420, then the algorithm stabilizes at the value of 42.8566. In our application, this number of iterations has been set to 500.

The best low-pass microstrip filter obtained by OEP has the following characteristics  $Z_l$  = 149.96 (Ohm),  $Z_c$  = 15.24 (Ohm),  $\theta_l'$  = 0.48 (rad) and  $\theta_c'$  = 0.30 (rad) With an error of 42.85. Figure 11 shows the transmission

coefficient  $(S_{21})$  and reflection coefficient  $(S_{11})$  of the two low-pass microstrip filters (the filter designed by PSO and the Butterworth filter)



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# Fig. 11. Performance of the microstrip low-pass filter designed by PSO and Butterworth method

As mentioned earlier, the selection of the best microstrip low-pass filter by PSO is based on the fitness function, such that the optimal filter exhibits minimal error. The Butterworth filter results in an error of 64.66, while the filter designed by PSO yields an error of 42.85. Therefore, the performance in terms of error of the microstrip low-pass filter determined by the particle swarm optimization algorithm is evident.

For the implementation of the filter designed by PSO on a microstrip structure with a substrate height h=1.27mm and a relative dielectric constant equal to 10.8, the dimensions of this filter are illustrated in Figure 12. By using the equations of microstrip line, we find : $W_l = 0.02$  mm,  $W_c = 6.49$  mm, And using the equations (3) and (4) we find:  $l_l = 9.16$  mm and  $l_c = 4.89$  mm

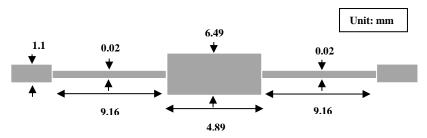


Fig12. Layout of the microstrip low-pass filter designed by PSO

# 5.2 Result of band-pass filter

Figure 13 shows the variation of the minimum error as a function of iterations. As mentioned previously, this study helps to determine the number of iterations in our program to obtain an optimal filter.

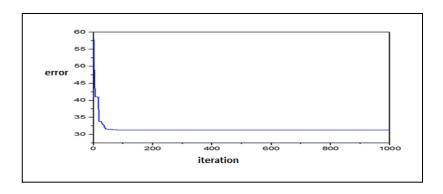


Fig13. Effect of iterations on the minimum error.

The PSO algorithm quickly converges to acceptable solutions within the range of 1-76 iterations, after which the convergence becomes slower between 76 and 294 iterations. After 294 iterations, the algorithm stabilizes at an error of 31.2834. In our case, the number of iterations is set to 400. The best microstrip bandpass filter obtained by the PSO algorithm has the following characteristics: Z0e = 177.46 (Ohms), Z0o = 80 (Ohms), and  $\theta' = 0.3836$  (radians) with an error of 31.28 when the filter desined by chebyshev method present an error of 85.2. Figure 14

represents the transmission coefficient ( $S_{21}$ ) and reflection coefficient ( $S_{11}$ ) of the two microstrip bandpass filters (the filter designed by PSO and the Chebyshev filter).

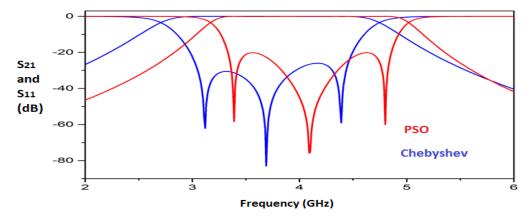


Fig. 14. Performance of the microstrip bandpass filter designed by PSO and the Chebyshev filter

For the implementation of the microwave bandpass filter designed by PSO on a microstrip structure with a substrate height h=0.79 mm and relative dielectric constant  $\varepsilon_r$  = 2.5, The dimensions of this filter are illustrated in Figure 15. For the calculation of the width W and spacing S of the coupled microstrip lines, refer to [1]: W=0.316 mm and S = 0.221 mm. And for the length l, we have:  $\varepsilon_{re}$ =1.89,  $\theta'$ =0.38 and  $l=\frac{300\theta'}{2\pi\sqrt{c}}$  we find l=13.18 mm.

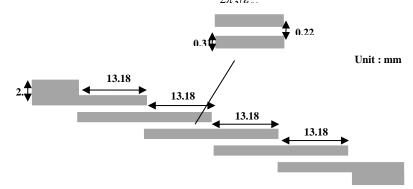


Fig. 15. layout of the microwave bandpass filter obtained by PSO

#### 6-Conclusion

This study presents the design of stepped impedance microstrip filter and parallel coupled microstrip filter by the particle swarm optimization algorithm. The PSO is based on collaboration between the particles, it has some interesting features including the notion that efficiency is due to the collaboration rather than competition, it shows greater reliability in solving optimization problems in continuous or discrete variables.

The particle swarm optimization is easily applied in many fields, you need only to adapt the optimization problem. The three main parameters to be define are; the particle, search space and the objective function.

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The design of filter by a classical method (butterworth, chebyshev,..etc) is firstly done and then we adapt the optimization problem. We choose the particle as a filter and the position of particle represents the characteristic impedances and electrical lengths of the filter.

The search space is chosen from the values of characteristic impedances and electrical lengths of the filter and the objective function we used the function sum of the errors on the magnitude response of filter.

The simulation results showed the effectiveness of the algorithm, the first example has given a filter with 42.85 in the objective function when the filter design by butterworth approximation present a 64.66 in the objective function.

In the second example the filer designed by PSO present a 31.28 in the objective function when the filter designed by chebyshev a 85.2 in the objective function.

The use of PSO for designing microstrip filters it is possible and yields better results.

The generalization of our contribution to the many devices with different structures is possible

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