Numerical Modeling of Thermal and Flow Analysis of a Conduction Magneto-Hydrodynamic Pump

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Abstract

Magnetohydrodynamic (MHD) is a scientific discipline which describes the behavior of a conducting fluid (Liquid or ionized gas called plasma) in the presence of electromagnetic fields, The MHD conversion is one of the applications of this discipline, it relates to the mechanical energy transformation of the movement of a fluid into electric power. This article is concerned with the study of a coupling between the stationary Maxwell equations, the transient state Navier Stokes and thermal equations in conduction magneto hydrodynamic pumps. The model developed computes the magnetic field and calculates the velocity and the temperature using the finite volume method. The paper focuses on the analysis of the flux density, the electromagnetic thrust, the electric power density, the velocity, and the temperature in the channel of the MHD pump.

Keywords: Magnetohydrodynamics; Maxwell equations thermal equations; Finite volume method; Electrical power; Temperature.

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introduction:

The interaction of moving conducting fluids with electric and magnetic fields provides for a rich variety of phenomena associated with electro-fluid-mechanical energy conversion.

Effects from such interactions can be observed in liquids, gases, two-phase mixtures, or plasmas. Numerous scientific and technical applications exist, such as heating and flow

control in metals processing, power generation from two-phase mixtures or seeded high temperature gases, magnetic confinement of high-temperature plasmas — even dynamos

that create magnetic fields in planetary bodies. Several terms have been applied to the broad field of electromagnetic effects in conducting fluids, such as magneto-fluid mechanics, magneto-gas-dynamics, and the more common one used here — magnetohydrodynamics,or "MHD".[1].

Magnetohydrodynamics or simply (MHD) is the field of science that studies the movement of conductive fluids subjected to electromagnetic forces. This phenomenon brings together concepts of fluid dynamics and electromagnetism. Formally, MHD is concerned with the mutual interactions between fluid flows and magnetic fields. Electrically conducting and non-magnetic fluids must be used, which limits the applications to liquid metals, hot ionized gases (plasmas) and electrolytes.

Over the years, MHD has been applied to a wide spectrum of technological devices, directed, for example, to electromagnetic propulsion or to biological studies.

Application arises in astronomy and geophysics as well as in connection with numerous engineering problems, such as liquid metal cooling of nuclear reactors, electromagnetic casting of metals, MHD power generation and propulsion [1].

The pumping of liquid metal may use an electromagnetic device, which induces eddy currents in the metal. These induced currents and their associated magnetic fields generate the Lorentz force, and allow the pumping of liquid metal [2,3]. Magnetohydrodynamics is widely applied in various domains, such as metallurgical industry, to transport or the liquid metals in fusion and the marine propulsion [4,5]. The advantage of these pumps, which ensure the energy transformation, is the absence of moving parts.

The interaction of moving conducting fluids with electric and magnetic fields allows for a rich variety of phenomena associated with electro-fluid-mechanical energy conversion [6,7].

The schematic of the MHD pump is shown in (fig.1). The basic principle is to apply an electric current across a channel filled with electrically conducting liquids and a dc magnetic field orthogonal to the currents via permanent magnets.

This paper presents the numerical modeling of the coupling electromagnetic; hydrodynamic and thermic phenomena using the finite volume method in a DC MHD conduction pump. The resolution of the equations is obtained by introducing the magnetic vector potential A, and the temperature T.

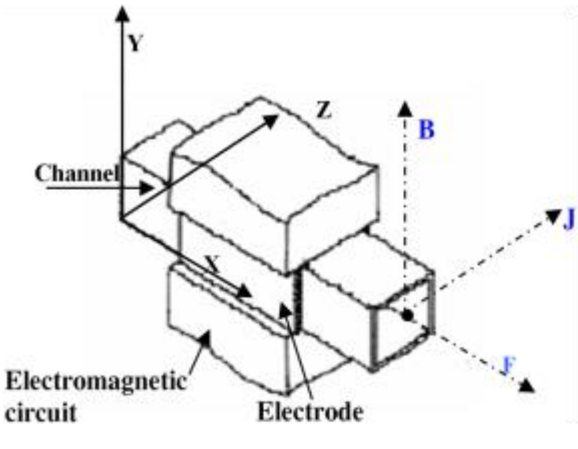


Figure. 1. Scheme of a DC MHD pump [3].

Ii. Mathematical Analysis Of Problems

Ii.2 Lectromagnetic Problem

The schematic structure of the pump is shown in figure (1). In the pump, the electromagnetic forces are obtained from the Lorentz forces induced by interaction between the applied electrical currents and the magnetic fields, [3,5]. The electromagnetic model of the MHD pump is as follows:

$$\overrightarrow{rot}\left(\frac{1}{\mu}\overrightarrow{rot}\vec{A}\right) = \vec{J}_{ex} + \vec{J}_a + \sigma(V.\frac{\partial \vec{A}}{\partial x}) \quad (1)$$

The magnetic induction and the electromagnetic force are given by:

$$\vec{B} = \overrightarrow{rot}\vec{A} \quad (2)$$

$$\vec{F} = \vec{J} \wedge \vec{B} \quad (3)$$

Following the two-dimensional (2D) developments in Cartesian coordinates, where the current density and the magnetic vector potential are perpendicular to the longitudinal section of the MHD pump, the equation becomes:

$$\frac{-1}{\mu}\left(\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}\right)=J_{ex}+J_a+\sigma(V_x\frac{\partial A}{\partial x}) \quad (4)$$

Ii. 2 Thermal Problem

The thermal phenomena are studied only in the channel of the MHD pump. So, the governing thermal equation is given by

$$\rho C_p\left(\frac{\partial T}{\partial t}\right) = div(Kgrad(T) + P_s \quad (5)$$

Where ρ is the density of the fluid, Cp the specific heat, K the thermal conductivity, T the temperature and Ps the thermal source (electric power density) induced by eddy current such as:

$$P_s = \frac{1}{2\sigma} J_i^2 \quad (6)$$

After developments in Cartesian coordinates, replacing the source term Ps, we obtain:

$$\rho C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial t}\left(K\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)+\frac{1}{2\sigma}J_i^2 \quad (7)$$

II.3 Hydrodynamic Problems

The MHD flow of an incompressible, viscous and electrically conducting fluid in a transient state condition is governed by the Navier-Stokes equations [8]:

$$\frac{\partial \vec{V}}{\partial t}+(\nabla.\vec{V})\vec{V}=-\frac{1}{\rho}\nabla p+\nu\wedge\vec{V}+\frac{\vec{F}}{\rho} \quad (8)$$

$$div\vec{V} = 0 \quad (9)$$

Where p the is the pressure of the fluid, υ the kinemactic viscosity of the fluid, F the electromagnetic thrust and ρ the fluid density, [12,14].

The development of the equation of the flow in Cartesian coordinates gives, [11,15]

$$\frac{\partial V_x}{\partial t}+V_x.\frac{\partial V_x}{\partial x}+V_y\frac{\partial V_x}{\partial V_y}=\frac{-1}{\rho}\frac{\partial p}{\partial x}+\nu\left[\frac{\partial^2 V_x}{\partial^2 x^2}+\frac{\partial^2 V_x}{\partial^2 y^2}\right]+\frac{1}{\rho}F_x$$

$$\frac{\partial V_y}{\partial t}+V_x.\frac{\partial V_y}{\partial x}+V_y\frac{\partial V_y}{\partial V_y}=\frac{-1}{\rho}\frac{\partial p}{\partial y}+\nu\left[\frac{\partial^2 V_y}{\partial^2 x^2}+\frac{\partial^2 V_y}{\partial^2 y^2}\right]+\frac{1}{\rho}F_y$$

$$\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial V_y}=0 \qquad (10)$$

The real difficulty is the calculation of the velocity lies in the unknown pressure. To overcome this difficulty is to relax the incompressibility constraint in an appropriate way. So, the elimination of pressure from the equations leads to a velocity-stream function

The velocity vector is defined by:

$$\zeta=\overrightarrow{rot}\ V \qquad (11)$$

The stream function is given in 2D Cartesian coordinates as:

$$\frac{\partial\psi}{\partial y}=V_x\ ;\quad \frac{\partial\psi}{\partial x}=V_y \qquad (12)$$

Where Vx and Vy the components of the velocity V.

We eliminate the pressure from the equation (12) and we use the two new dependent variables ξ and Ψ to obtain the following equation:

$$\frac{\partial\zeta}{\partial t}+V_y\frac{\partial\zeta}{\partial y}+V_x\frac{\partial\zeta}{\partial x}=\nu\left[\frac{\partial^2\zeta}{\partial^2 x^2}+\frac{\partial^2\zeta}{\partial^2 y^2}\right]+\frac{1}{\rho}\left(\frac{\partial F_x}{\partial y}+\frac{\partial F_y}{\partial y}\right) \qquad (13)$$

Iii. Numerical Method And Results

There are several methods for the determination of the electromagnetic fields and the velocity; the choice of the method depends on the type of problem, [10, 11].

In our work, we thus choose the finite volume method; its principle consists on subdividing the field of study (Ω) in a number of elements. Each element contains four nodes of the grid. A finite volume surrounds each node of the grid (Figure.2), [12, 13].

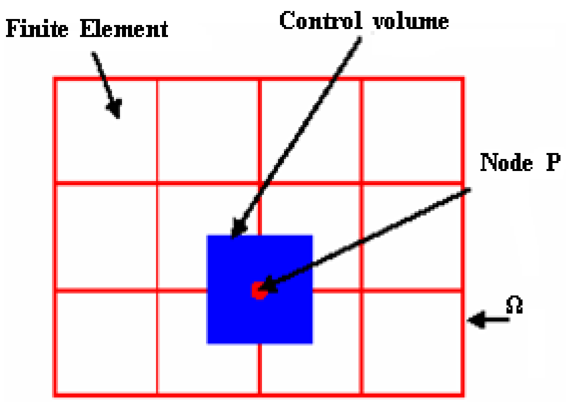


Figure 2. Grid of the domain.

The method consists of discretizing differential equations by integration on finite volumes surrounding the nodes of the grid. In this method, each principal node P is surrounded by four nodes N, S, E and W located respectively at North, South, Est and West (Figure.3)

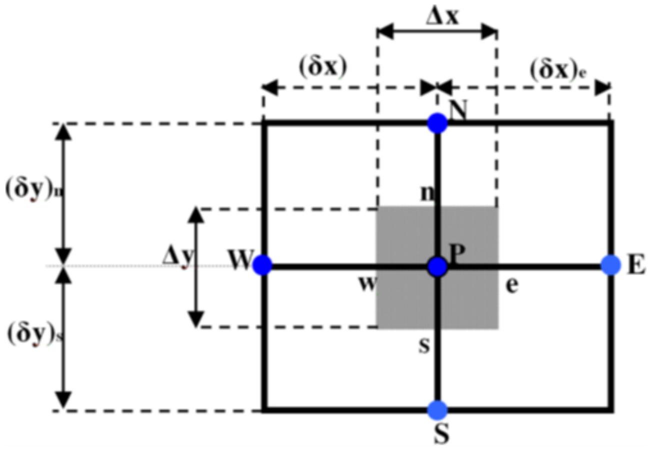


Figure 3. Discretization in finite volume method.

We integrate the electromagnetic thermal and hydrodynamic equations in the finite volume method delimited by the surfaces E, W, N and S, [14]. Finally we obtain the algebraic equation which is written as:

$$\int_w^e\int_s^n \left[\frac{1}{\mu}\left(\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}\right)\right]dx\,dy = \int_w^e\int_s^n \left(J_{ex}+J_a+\sigma V_x\frac{\partial A}{\partial x}\right)dxdy \quad (14)$$

$$\rho C_p \int_t\int_s^n\int_w^e \frac{\partial T}{\partial t}dxdydt = \int_t\int_s^n\int_w^e \frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)dxdydt + \int_t\int_s^n\int_w^e \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)dxdydt + \int_t\int_s^n\int_w^e P_s\,dxdydt \quad (15)$$

After integration, the final algebraic equation will be:

$$a_p A_p = a_e A_e + a_w A_w + a_n A_n + a_s A_s + d_p \quad (15)$$

$$a_E = \frac{\Delta y}{\mu_e(\delta x)_e},\ a_W = \frac{\Delta y}{\mu_w(\delta x)_w},\ a_N = \frac{\Delta x}{\mu_n(\delta y)_n},\ a_S = \frac{\Delta x}{\mu_s(\delta y)_s}, \quad (16)$$

$$c_E = \frac{K\Delta t\Delta y}{(\delta x)_e},\ c_W = \frac{K\Delta y\Delta t}{(\delta x)_w},\ c_N = \frac{K\Delta t\Delta x}{(\delta y)_n},\ c_S = \frac{K\Delta t\Delta x}{(\delta y)_s},$$

We use the same steps for the hydrodynamic problem:

$$\int_t\int_s^n\int_w^e \left(\frac{\partial\zeta}{\partial t}+V_y\frac{\partial\zeta}{\partial y}+V_x\frac{\partial\zeta}{\partial x}\right)dxdydt = \iiint \left(\nu\left[\frac{\partial^2\zeta}{\partial^2 x^2}+\frac{\partial^2\zeta}{\partial^2 y^2}\right]\right)dxdydt + \iiint \left(\frac{1}{\rho}\left(\frac{\partial F_x}{\partial y}+\frac{\partial F_y}{\partial y}\right)\right)dxdydt \quad (17)$$

$$\int_s^n \int_w^e \left(\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2}\right) dx dy = -\int_s^n \left(\int_w^e \zeta dx dy\right. \qquad (18)$$

$$b_P \zeta_P = a_e \zeta_e + b_w \zeta_w + b_n \zeta_n + b_s \zeta_s + b_0 \zeta_0 + d_P \qquad (19)$$

The resolution of the electromagnetic, thermic and the hydrodynamic equations makes it possible to determine the magnetic potential vector, magnetic induction ($\vec{A}, \vec{B}$) the Electromagnetic force F, temperature and the velocity in the channel of the conduction pump.

4. Application And Results

We consider the following figure (Fig 4) which represents the transverse section of MHD pump with the following characteristics:

-The liquid in the channel is mercury with the conductivity is $\sigma_{\text{Mercure}} = 1.66*10^6$ [S/m] ;

- Current source density is $Jex=1.8*10^6$ [A/m²]

- Current density in the electrodes is $Ja=1.5*10^6$ [A/m²].

The figures (5) and (6) represent respectively the equipotential lines and the distribution of the magnetic vector potential in the MHD pump.

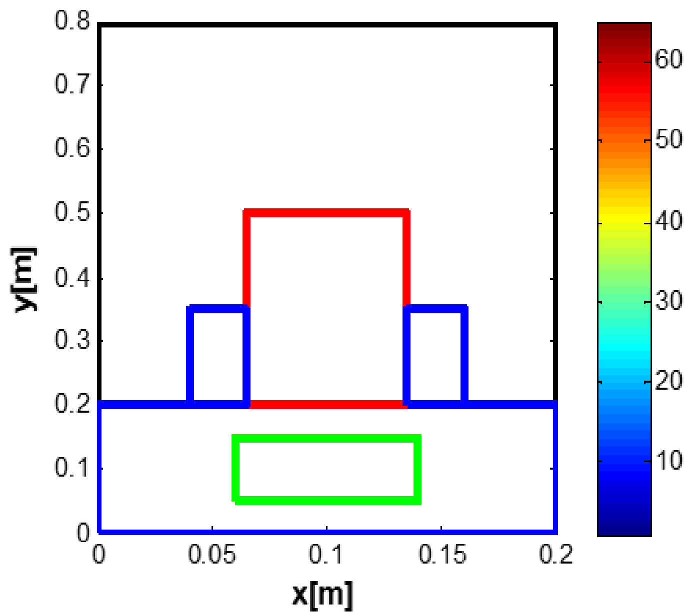


Figure 4. A conduction MHD pump configuration

Figure. 5 – Equipotential lines in a DC MHD pump

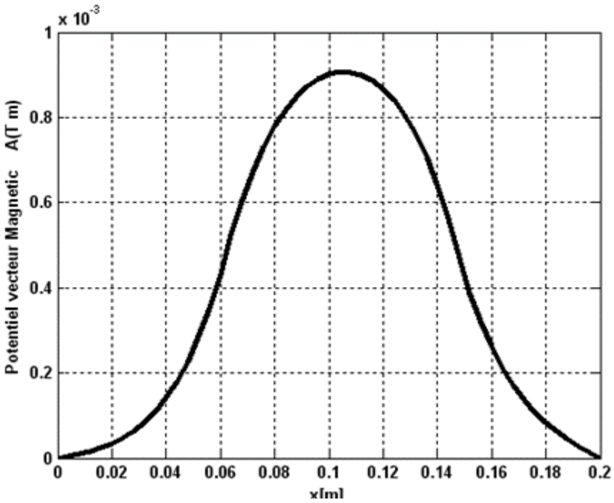
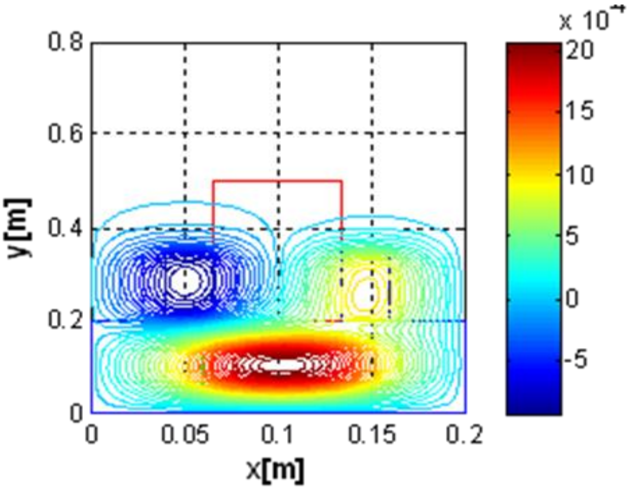


Figure. 6 – Magnetic vector potential in a Dc MHD pump



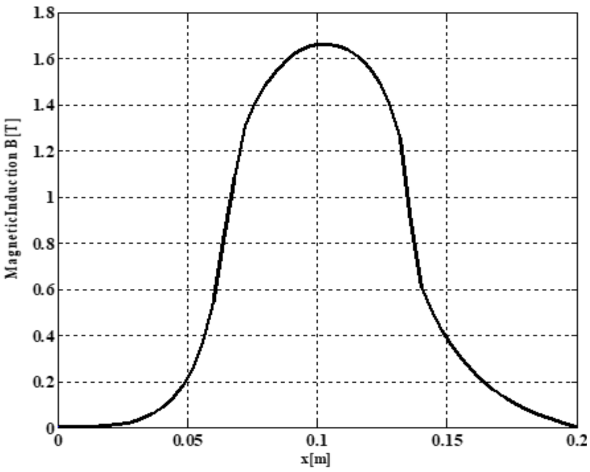


Figure. 7 – Magnetic induction in the MHD pump

The figure (7) represents the magnetic induction in the channel. It is shown that, the magnetic induction reaches its maximum value at the inductor and in the medium of the channel.

This figure (8) represents the electromagnetic force in the channel; it is note that, the maximum value in the medium of the channel of the MHD pump.

The figure (9) represents the velocity in the channel of the MHD pump. It is noticed that the velocity of the fluid flow passes by a transitory mode then is stabilized like all electric machine and the steady state is obtained approximately after ten seconds. The results obtained are almost identical qualitatively to those obtained by [6, 14].

The figure (10) shows the electric power density in the channel. The maximum induced power reaches $2.157*10^{6} W / m^{3}$. The pace obtained is directly related to that of the eddy current density. This characteristic of the heat source is used in the numerical calculation of the temperature.

The figure (11) shows the distribution of the temperature in the channel of the MHD pump. It is noticed that the temperature passes by a transitory mode then is stabilized.

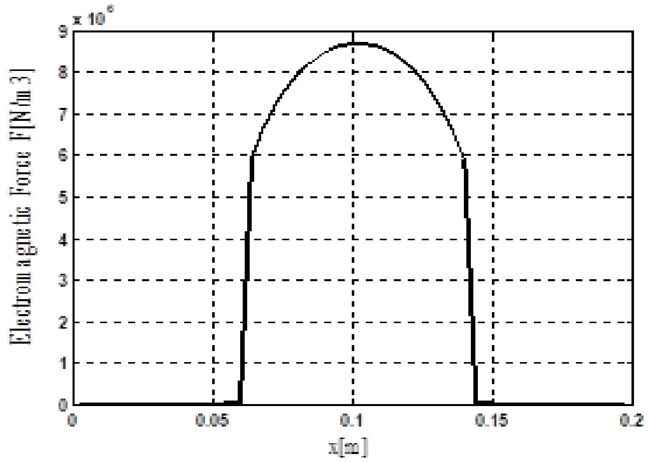


Figure. 8 – Electromagnetic force in The MHD pump

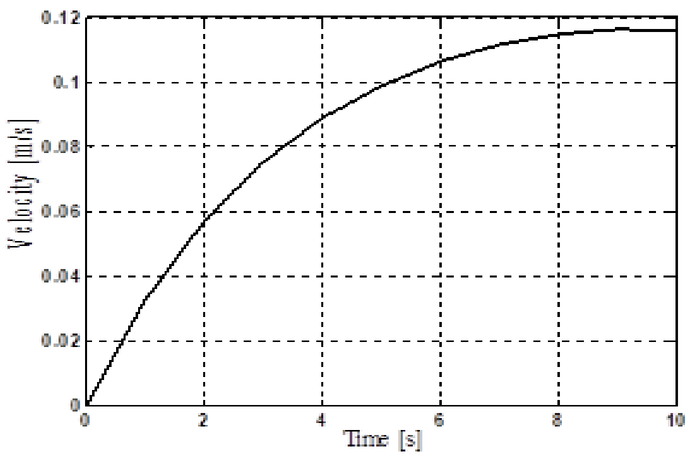


Figure. 9 –Velocity in the channel of the MHD pump

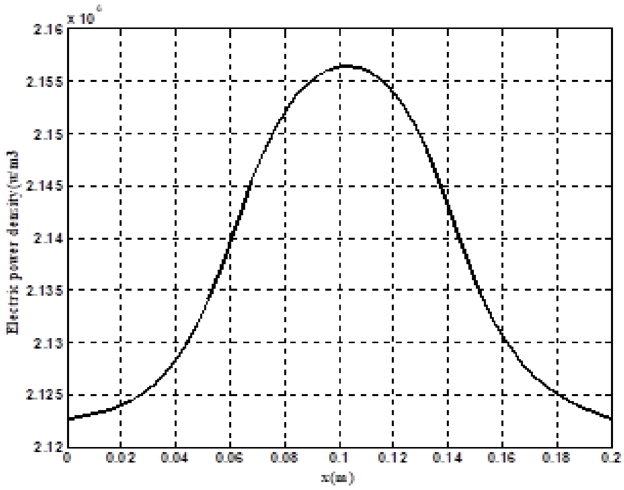


Figure 10. The electric power density in the channel

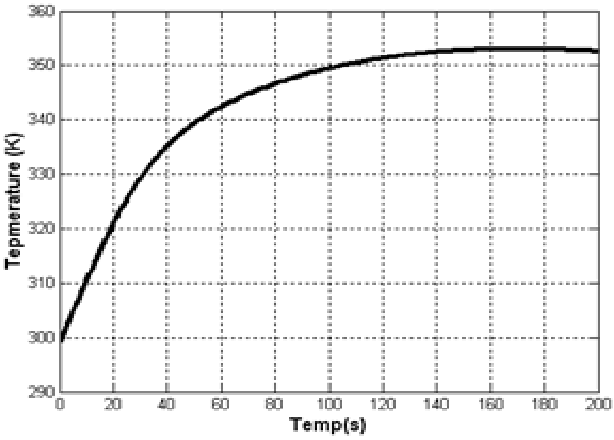


Figure. 11 – The temperature in the channel of the MHD pump

5. Conclusion

In this paper we have studied the coupled magneto hydrodynamic and thermal problems using 2D finite volume method taking into account the movement of the fluid.

Various characteristics such as the distribution of the magnetic vector potential, the magnetic flux density, the electromagnetic force, the velocity and temperature in the channel are given. The results of velocity obtained are almost identical qualitatively to those obtained by Majid Ghassemi and P.J. Wang.

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