Road to Chaotic Natural Convection in a Square Enclosure from a Constant Partially Heated Walls and Internal Heat Source

Hakim Fatmi[1,2*], Salah Laouar[1], Antar Bouhank[2], Djoubir Debbeh[3], Mohamed Cherif Benachour[2], Djamel Belfennache[2]

[1] Physique Department, Sciences Faculty, University Skikda, B.P. 26, 21000, Algéria

[2] Research Center in Industrial Technologies CRTI, P.O. Box 64, Cheraga, 16014 Algiers, Algeria

[3] Department of Mechanical and Electromechanical Engineering, Abdelhafid Boussouf University Center in Mila

*** Corresponding author**: hakim_fatmi85@yahoo.fr

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Abstract

In the present paper, natural convection behavior in a square enclosure, occupied with air, has been examined numerically. Horizontal walls are in an adiabatic state. The temperature in each vertical wall is distributed one on top of another wherein the two opposite half lower walls are hot and constant, whereas the upper half walls are cold. This study is mainly focused on the effect of Rayleigh number values on the flow regime in the cavity from a stable towards chaos. Equations governing the natural convection have been resolved by the finite difference method derivation. Obtained outcomes present a symmetric flow around a declined define axe with low values of Rayleigh number (less than 10^5), at Ra_e=3x10^5 the critical Rayleigh number is found by the loss of the symmetry and finally the chaos is reached at 1x10^6. Some interesting results which are shown in terms of streamlines and isotherms are found and presented.

Keywords*:* Natural convection, Internal heat source, Finite volume method, Transient regime, Critical Rayleigh number, Bifurcation, Chaotic

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1. Introduction

The study of natural convection in closed enclosures is old and dates back to 1950s, many researches have been done to investigate the behavior of this kind of heat transfer in the desired enclosure. Heat transfer by the natural convection in closed enclosures plays an important role in various engineering fields, such as thermal design of the habitat [1], thermal energy storage [2], collectors' solar panels [3], the cooling of electronic components [4] and so on. Most of them studied the effect of internal heat source and walls temperature inside a cavity. These studies reached an overheating of the enclosure. Many researches attempt to overcome this problem. Kazmierczak and Chinoda [5], have studied the effect of the periodically changing wall temperature oscillation into the enclosure, they find the time-averaged heat transfer across the enclosure is quite insensitive to the time-dependent boundary condition. They confirm the average heat transfer through the enclosure and the fixed temperature walls are roughly the same. Lakhal et al [6], have showed that periodic heating of the walls must be considered if the amplitude varies greatly and the strength of the convection is large. In 1999, the phenomenon of the heat transfer in a square cavity has been investigated by Abourida et al [7] with hot and cold horizontal walls varied sinusoidally. They found that the heat loss is less when the cold wall temperature is constant. Another mode of heat has been added inside the cavity as an internal heat generation. This case has been studied by H. Oztop and E Bilgen in 2006 [8] .The outcome of the latter research showed that the heat transfer was generally reduced particularly when the ratio of internal and external Rayleigh numbers is weak. In the same year (2006), the internal heat generation was used, by J. Mohammed et al. [9], as a centered cylinder. The result obtained shows that the average the ratio of heat transfer at the hot and cold walls varies linearly with Temperature with a constant internal heat generation. Oztop et al. [10], have extended their work by using of a volumetric heat source in an enclosure. The changes of the internal to external Rayleigh number ratio values affect the heat transfer. Furthermore, the heat transfer decreases according to waviness of the opposite walls. The aim of the present work is a Rayleigh number effect investigation on the regime flow behavior in an enclosure from stable to chaos. To this end, a square enclosure containing air (Prandtl number equal to 0.71) and exposed to different wall temperatures has been used. The latter has a lower half side walls that are maintained hot and constant, whereas, the upper half side walls are kept cold. The bottom and top walls are adiabatic.

2. Mathematical formulation

In this work, we study the two-dimensional natural convection heat transfer numerically in an air-filled square enclosure. The flow was supposed to be bi-dimensional, time dependent, quasi-incompressible and laminar. The enclosure geometry and thermal boundary conditions have been chosen randomly. Horizontal walls are assumed adiabatic; the lower half side walls are maintained hot with constant value, whereas, the upper half side walls are kept cold (see figure 1). Note that, for the satisfactory of the Boussinesq approximation, the difference in temperature inside the enclosure tends to zero. Internal Rayleigh Ra_I is constant in the whole of this paper.

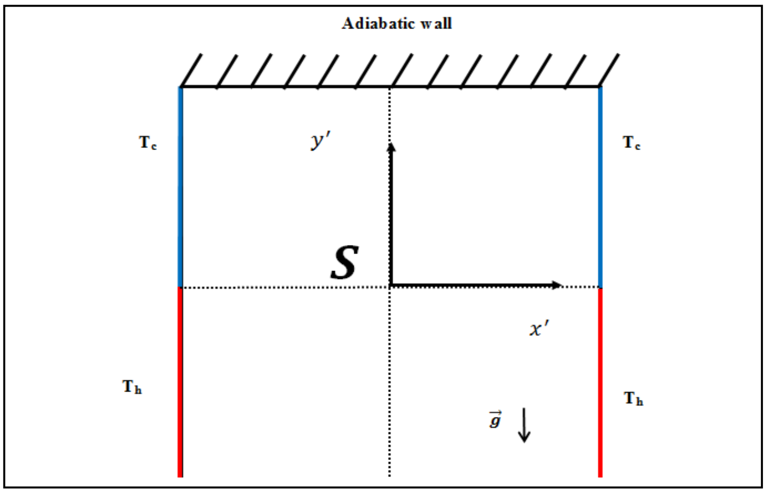

Figure 1: Schematic of the enclosure configuration

The flow and thermal fields are expressed by the partial differential equations and their initial and at the boundary conditions in dimensionless form as follows:

Initial condition:

At $t=0$

$$U=0, V=0, \phi=0 \tag{1}$$

For $t>0$

The continuity equation written as:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{2}$$

The horizontal momentum equation is:

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2} \tag{2}$$

The vertical momentum equation:

$$\frac{\partial V}{\partial \tau}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}+\frac{Ra_E}{Pr}\phi \tag{4}$$

The energy equation:

$$\frac{\partial \phi}{\partial \tau}+U\frac{\partial \phi}{\partial X}+V\frac{\partial \phi}{\partial Y}=\frac{1}{Pr}\left(\frac{\partial^2 \phi}{\partial X^2}+\frac{\partial^2 \phi}{\partial Y^2}\right)+S \tag{5}$$

$$S=\begin{cases}0 & \text{Without heat source}\\ \dfrac{Ra_I}{Ra_E Pr} & \text{With heat source}\end{cases} \tag{6}$$

The boundary conditions:

The velocity and the temperature terms are expressed according to each boundary condition as follows:

At: for the left wall ($X=-0.5$ and $-0.5\le Y\le 0$) and ($X=+0.5$ and $-0.5\le Y\le 0$) for the right wall

We have:

$$U=V=0 \tag{7}$$

And

$$\phi=\begin{cases}1 & \text{hot temperature in steady state}\\ 1+a\sin\left(2\pi f\tau\right) & \text{hot temperature in unsteady state}\end{cases} \tag{8}$$

With $a=0.8$ and $f=0.08$.

At : ($X=-0.5$ and $0<Y\le 0.5$) for the left wall, and ($X=+0.5$ and $0<Y\le 0.5$) for the right wall

$$\begin{aligned}&U=0, V=0\\ &\phi=0\end{aligned} \tag{9}$$

The velocity and the temperature terms at side walls ($Y=0$ and $Y=1$) are:

$$\begin{aligned}&U=0, V=0\\ &\frac{\partial \phi}{\partial Y}=0\end{aligned} \tag{10}$$

The following equation gives the average Nusselt number formula for the heated walls:

$$Nu = \int_{-0.5}^{+0.5} -\frac{\partial \phi}{\partial X} \mathrm{dY} \tag{11}$$

3. Numerical resolution

Equations governing the simulation of natural convection problem, in a square cavity, have been discretized according to specific boundary conditions using the finite volume method yielding a set of algebraic equations Patankar [11]. Different numerical schemes have been tried to find out the central differencing scheme. The latter approximates the combined convection and diffusion terms and gives a physically valid solution for unsteady state problems. However, the velocity and pressure field coupling presents a problem for the resolution of continuity equation. To fix this drawback, the continuity equation has been transformed into a pressure correction equation according to the SIMPLE algorithm. The set of algebraic equations has been solved by using the Tri-Diagonal Matrix Algorithm (TDMA) with the implicit scheme to compute the transient terms in momentum and energy equations. The square cavity used in this study has been discretized according to a uniform mesh of (80 x 80) with a step of (Δt = 2.5x10^{-5}s) of time.

4. Numerical validation

For achieving the grid-independent solutions, some grids refinement studies have been checked (40x40, 60x60, 80x80 and 100x100). The variation of the velocity component and the temperature at the center of the cavity and for different meshes mentioned above are depicted in the figure 2. It can be seen that a high element number of mesh is, a good convergence is. Furthermore, the outcomes obtained for the meshes (80x80 and 100x100) are similar. Thus, the result obtained with the mesh 80x80 is preferred. The latter takes a less computing time and gives us the same results with higher refinement grids.

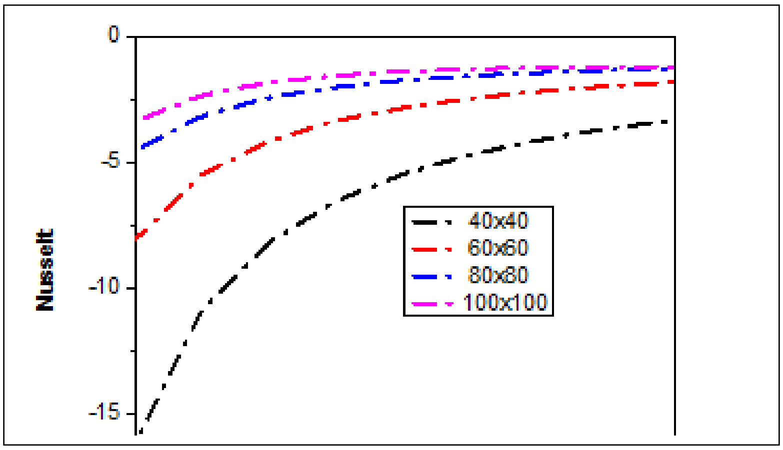

Figure 2 Grid sensitivity check. for $R_{ai}=10^3$, $R_{ae}=10^5$

The obtained results by the present simulation are validated by comparing with the numerical data of [12]. A good agreement was found for isolines of stream function and temperature. For the natural convection of air in the cavity (Pr = 0.71), more validation have been performed with a local isothermally-heated element placed at $X = -0.5$ (the bottom), isothermally-cooled right and left walls, adiabatic upper wall and other parts of the lower wall. A good agreement of temperature profiles was found comparing with experimental data of [13] (see figure 3).

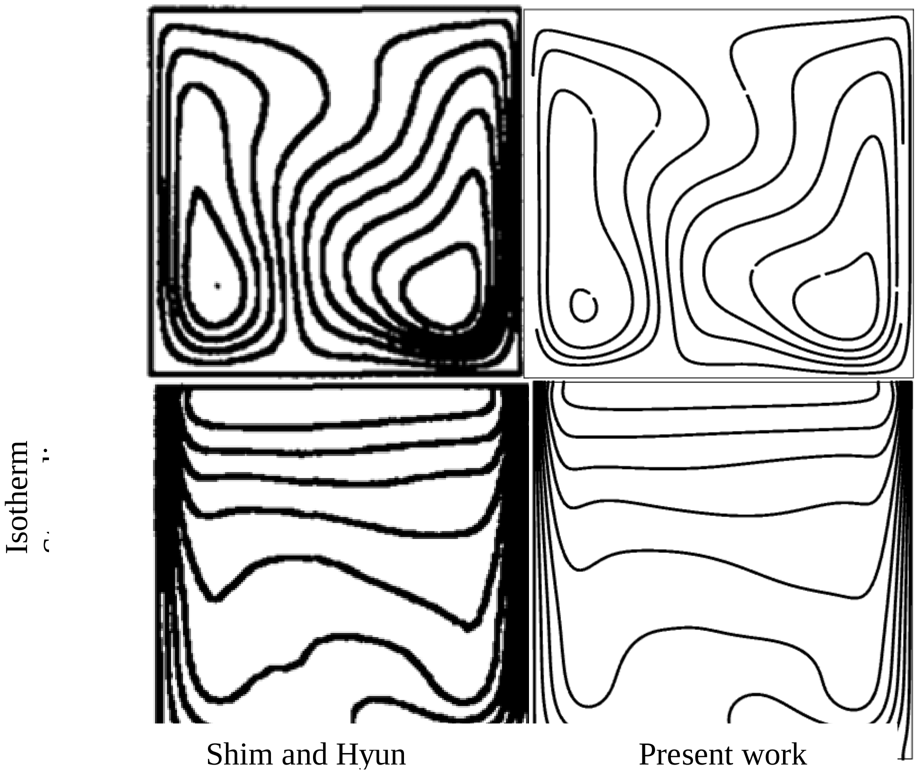

Figure 3: Comparison of the flow and thermal fields

In this part, we will present and discuss the numerical simulation performed for air in a square cavity where its geometric is mentioned above. The outcomes are in the form of velocity and temperature profiles, streamline and isothermal contours for many cases. The latter includs two flow regimes (symmetric and asymmetric). The investigation depends on the chosen case:

Case 1: Less than or equal to $\mathbf{Ra_e=1x10^5}$(Stable flow regime)

For the low Ra_e number value, the flow behavior in the cavity is stable and symmetric around a declined axe with the following coordination (x=0, y=0.3) and (x=1, y=0.7). Three cells are formed inside; wherein a principal cell (chamber temperature) is clockwise rotate centered in the middle, and two secondary cells (the result of hot walls) are counterclockwise rotate. The two cells are repeated on up and down of the principal cell duo to the streamline direction (see figure 4a). The stability of the regime flow is confirmed by Figure 4c where the temperature tends to a constant value. Figure 4b illustrates the heat spread from hot walls to the upper zone in the enclosure.

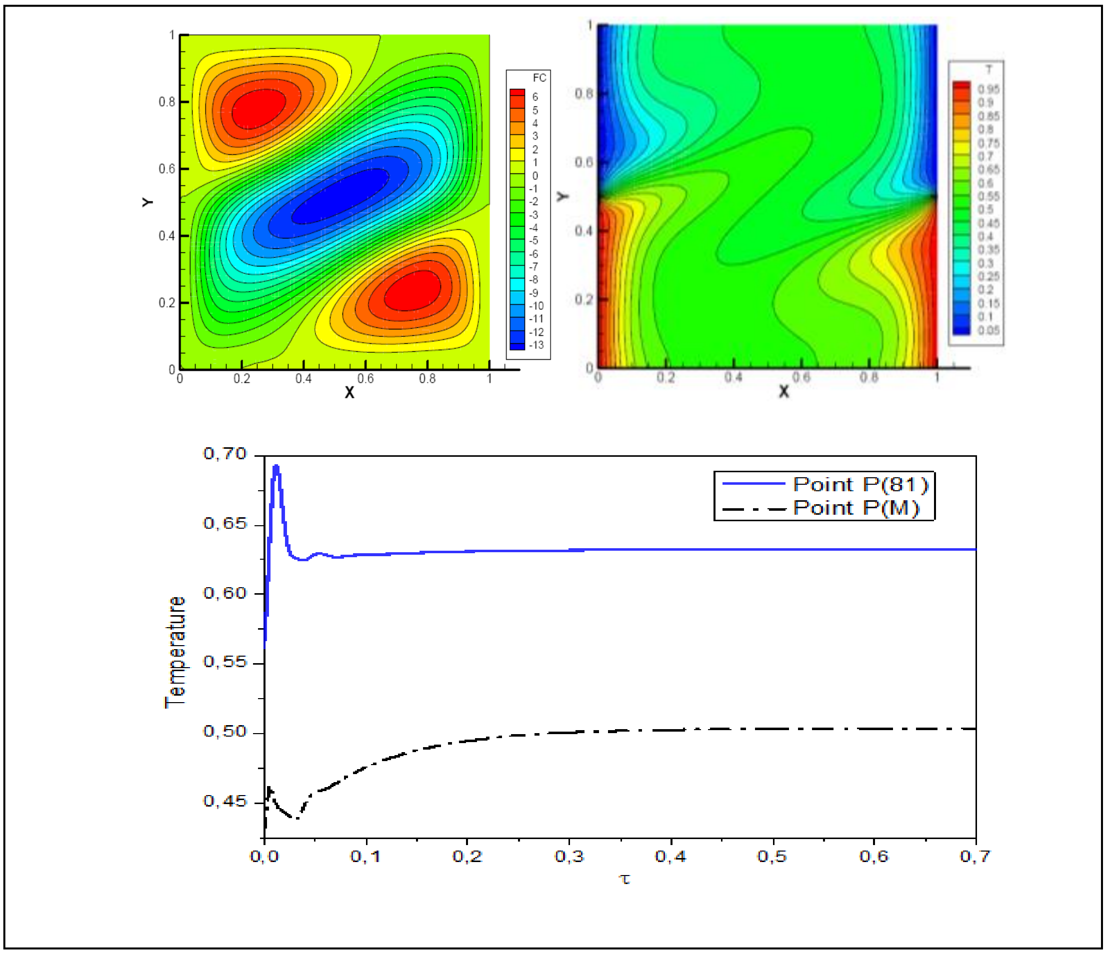

Figure 4: 4a- The streamline, Isotherms, Time series of temperature

Case 2: **Ra_e=3x10^5 (**Oscillatory flow regime)

By increasing the Rayleigh number from the 4c ılue (**Ra_e=1x10^5)**, we saw the change in the size of the secondary vortices (see figure 5a) when the symmetry in this case is lost. Thus, we can conclude that the oscillatory flow regime is reached, that mean the critical Rayleigh number is found and equal to **Ra_ecr=3x10^5.** At this point, the flow regime is almost stable. However, the latter is asymmetric by the vortices repartition with different sizes and different intensities. Especially, the principal cell when begins to divide into two secondary cells (figure 5b). This phenomenon is due to the high Rayleigh number value and the vortices' movement direction. By increasing Rae value, the flow regime is quasi-periodic when the principal vortex is divided and gave us two secondary vortices due to the increasing of ΔT inside the enclosure. With a high value of Ra (**1x10^6**), the system is chaotic.

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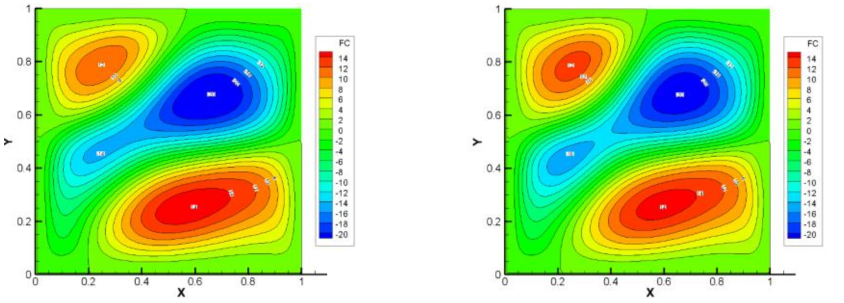

Figure 5a Figure 5b

Figure 5: Streamlines contours before and after for **Ra_eCr=3x10^5**

Case 3: **Ra_e=1x10^6** (chaotic regime)

The chaotic is the final steps of naturel convection regime when the stationary regime transfer to time-dependent convection for a high Rayleigh number value, it has been found over a limited range of **1x10^6 .** Figure 6 shows the temperature evolution for all regimes (Stable, Oscillating, quasi-periodic and chaotic) in function of dimensionless time.

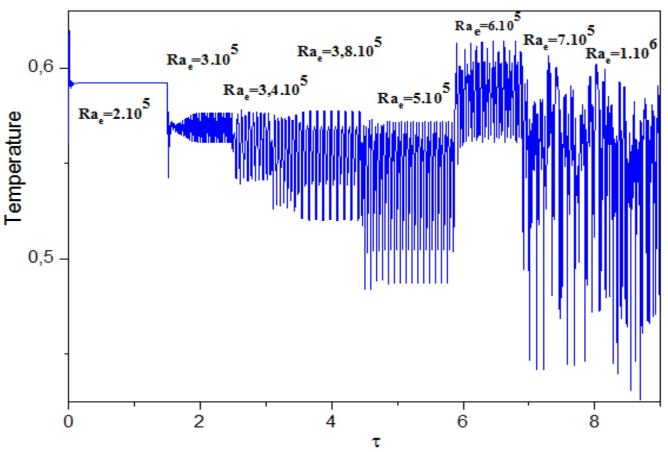

Figure 6: The temperature evolution for all regimes

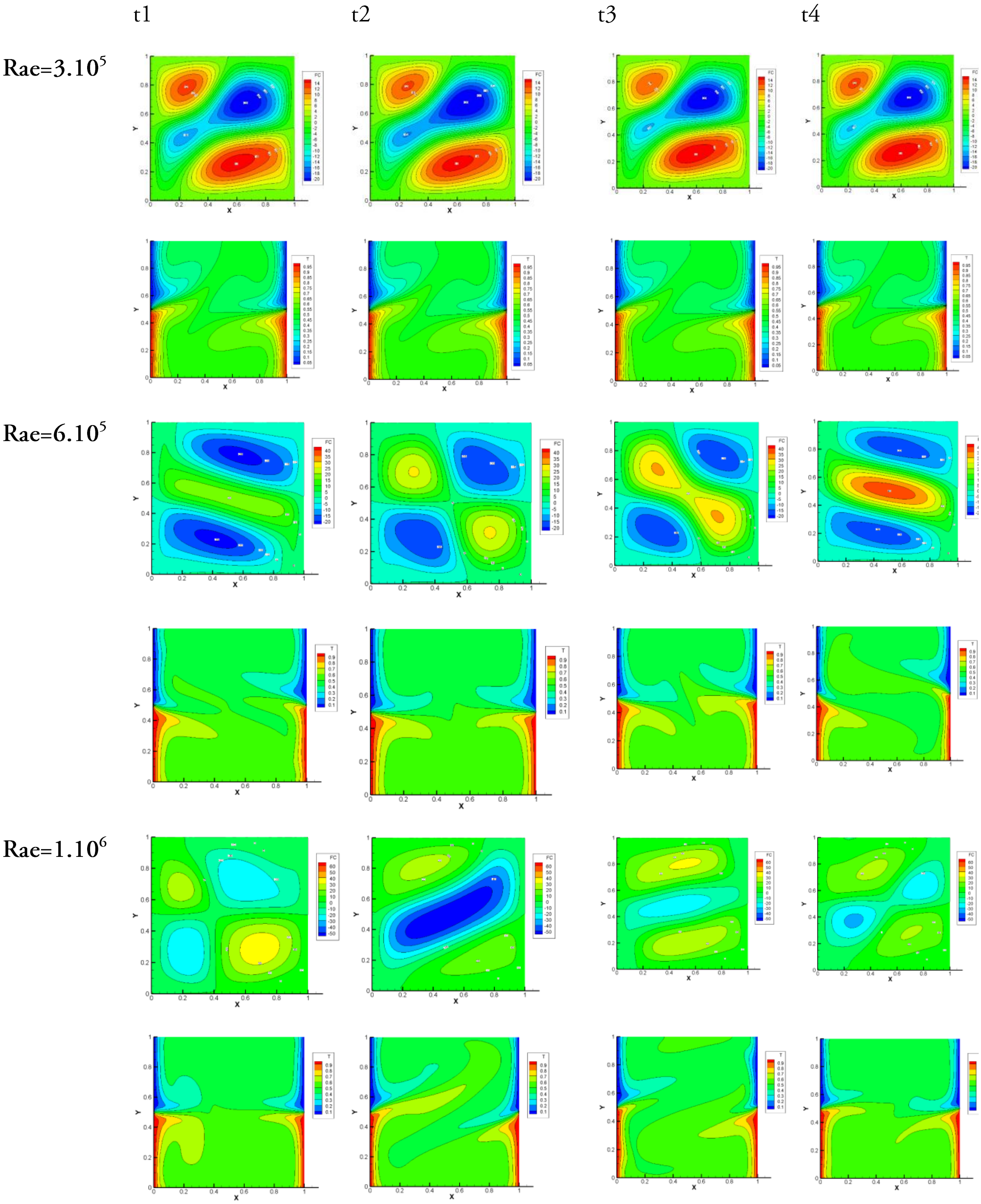

Figure 7: Streamlines and isotherms variation

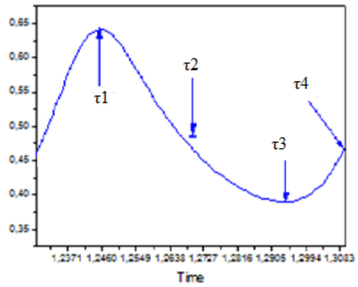

Figure 8: Corresponding points of streamlines and isotherms

5. Conclusion

This work is a numerical study of natural convection in a bi-dimensional square enclosure differentially heated and with internal heat generation. Equations governing the natural convection have been resolved using the finite difference method derivation. The obtained outcomes led us to the following observations:

The velocity components evolution and the average transfer rate, as a function of the Rayleigh number, have been presented and give a very good agreement with the literature.

The temperature inside the enclosure from the beginning is stable and the flow is symmetric, around a declined defined axe, until a Rayleigh number of 3.10^5. This point is assumed critical where the flow loses the symmetric.

An increase in the Rayleigh number ($1x10^6$) led to rise the time necessary for the periodic regime formation. Furthermore, a growth of oscillation amplitudes for the heat transfer rate and fluid flow strength happens with this value. The latter is the chaotic regime.

Throughout this study, the flow in the cavity undergoes bifurcation sequences from the stationary state to a periodic oscillatory state and finally becomes chaotic after passing through a sub harmonic cascade.

Nomenclature

a	Dimensionless amplitude
f	Dimensionless frequency
g	Gravitational acceleration [m/s2]
H	Length of the heat source [m]
k	Thermal conductivity [W/m2.K]
Nu	Nusselt number

Pr	Prandtl number
$Ra_I = g\beta g H^5 / \nu\alpha\kappa$	Internal Rayleigh number
$Ra_E = g\beta (T_h - T_c) H^3 / \nu\alpha$	External Rayleigh number
$Ra_e Cr$	Critical Rayleigh number
S	Internal heat source
T	Temperature [K]
t	Time (s)
X, Y	Dimensionless coordinates
U, V	Dimensionless velocity components
α	Thermal diffusivity [m2/s]
β	Thermal expansion coefficient [1/K]
ν	Kinematic viscosity [m2/s]
ϕ	Dimensionless temperature
τ	Dimensionless period

Subscripts

c	cold wall
h	hot wall

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